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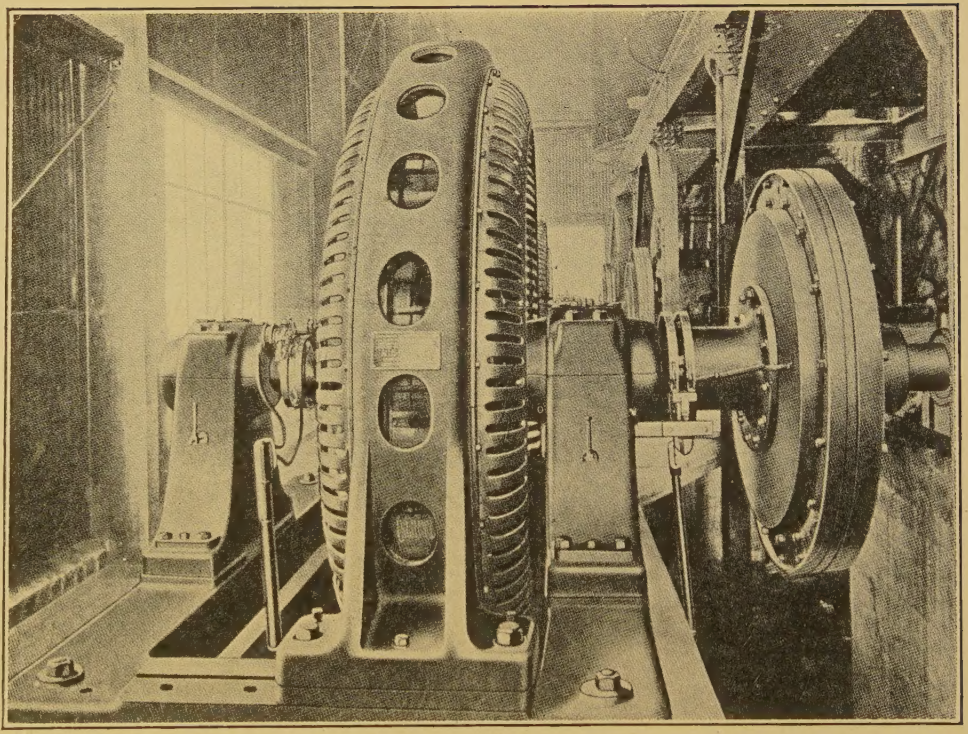
MAGNETS



FRONTISPIECE.—Magnetic Clutch (60″) installed with 500-h. p. synchronous motor driving a compeb mill. Two similar drives shown in the background. Motors are started without load, brought to synchronous speed, and the load then connected through the magnetic clutch. Merely closing the control switch causes the clutch to engage and connect the load to the motor (see page 108).

MAGNETS

A Practical Treatise on Electromagnetic Devices, Their Circuits, and the Specific Forms and Characteristics of Electromagnets and Permanent Magnets Employed.

BY

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TO MY WIFE

ELLA HOWELL UNDERHILL

In recognition of her sacrifice, encouragement,
and assistance in the preparation
of this book.

PREFACE

The object of this book is to treat broadly the general principles of apparatus electromagnetically operated and controlled, but particularly to treat electromagnets and permanent magnets in detail. Hence, the materials composing the structures of magnets, the relative physical dimensions of the various parts, and the phenomena associated with them, are carefully considered. Since electric motors and other electromagnetic devices to be controlled are operated from both alternating-current and direct-current circuits, these general circuits and their phenomena are explained in a simple manner. Many of the data have been obtained through personal experience and research, and a number of the experiments are those used in my series of lectures in American colleges during a period of many years.

Equations are used as sparingly as possible, the mathematical deductions having been translated into the text, with the law of conservation of energy as a basis, and the mathematics then eliminated. Numerous examples are worked out for the benefit of the reader. It is shown that alternating-current electromagnets are almost as easily designed as are direct-current electromagnets when the magnetic flux due to the electric current is separated from the magnetic flux inherent to the magnetic material. The application of the static condenser to power-factor correction is also touched upon.

The scope and utility of the electromagnet will be appreciated when it is considered that electric generators, static transformers and electric motors are electromagnets in combination. In the electric generator, one group of electromagnets is rotated in the magnetic field or fields of another group. In the static transformer, the mutual magnetic flux constantly varies in magnitude and direction in unison with the alternating current established by the generator, and energy is thus transferred from one circuit to another without mechanical rotation in the transformer. In electric motors, electromagnetic members revolve in the fields of stationary electromagnets. The induction motor tends to behave as a static transformer with a short-circuited secondary at the

instant of starting, but it tends to run as a synchronous motor. The synchronous motor has a low starting torque. In nearly all cases where the power is relatively great, starting, controlling and protection problems have confronted engineers who have solved them with the aid of electromagnets. Hence, auxiliary electromagnetic devices are employed for the automatic or semi-automatic starting, control and protection of such machinery and apparatus, and for the protection and sectionalization of power and lighting systems. These chiefly embody automatic or semi-automatic switching means which are operated or controlled by electromagnets, except in certain cases where small electric motors are employed for the purpose.

The electrification of the industries, public utilities, and the instrumentalities of our daily lives, has been paralelled by the development of other electromagnetic devices for the control and operation of various machines and the parts thereof, aside from the complex systems of communication and signalling. Designers are learning the advantages of electric transmission over short distances as well as over relatively great distances. Mechanical devices in various machines are rapidly being replaced by electromagnetic devices, thus eliminating cumbersome parts that cannot be moved quickly unless relatively great mechanical forces are developed, with resulting increases in safety, production, economy and convenience. As Prof. Harris J. Ryan stated in a letter to me, "The electromagnet is fundamental in very much of the equipment that dominates the electric field."

Electromagnetic apparatus has been developed to a point where it is thoroughly reliable, and the annual production of such apparatus is now enormous. The "wrong number" is not prevalent where automatic telephones are used (except when the calling subscriber wrongly manipulates the dial) and this reliability is largely due to the electromagnets employed. Although the human element may sometimes fail, train wrecks are few where the blocks are protected by electromagnetic means. The forthcoming advent of the automatic train stop, operated by means of electromagnets, will make travel by rail even safer. Electromagnets are more dependable than humans. The safety records of the New York subways, which have automatic train stops, are truly remarkable. Besides, the trains cannot be started when a door is open, and the electromagnetic control

of the train is automatic in response to the movement of the control lever in the hand of the motorman. Furthermore, the doors are opened and closed through the operation of electromagnets which control the air engines actually operating the doors.

In the system of Centralized Control, an operator sitting at a switchboard in a business office restores service or disconnects sections in case of trouble by pushing a button or two in response to the flash of a lamp. Thus automatic substations and hydroelectric plants are controlled in a manner similar to that in which telephone connections are handled. What the electric motor has accomplished in individual and group drive is closely followed by the various applications of the electromagnet in the control and operation of various devices, as set forth in the text.

While I am, naturally, indebted to many persons and the various companies with which I have been associated, the work and writings of Prof. Silvanus P. Thompson have ever been an inspiration; Mr. William D. Weaver advised and encouraged me in my early work, and Prof. Charles F. Scott has been a friendly source of assistance for many years. Credit is due Mr. Charles R. Underhill, Jr. in connection with the illustrations.

CHARLES R. UNDERHILL.

October, 1924.

CONTENTS

		Page
Frontispiece		iv
Preface		vii
Chapter		
I.	Introductory	1
II.	Forms of Electromagnets	31
III.	Mechanical Forces of Electromagnets	61
IV.	Time and Polarity Characteristics	79
V.	Portative Electromagnets	101
VI.	Switching, Control, and Protective Apparatus	119
VII.	Relays	139
VIII.	Miscellaneous Electromagnetic Devices	147
IX.	Solenoids	160
X.	Constant-current Solenoids with Plungers	176
XI.	Test Characteristics of Solenoids with Plungers	190
XII.	General Characteristics of Solenoids with Plungers	202
XIII.	Direct-current Electromagnets	221
XIV.	Electric Circuits and Their Phenomena	237
XV.	Constant Impressed-direct-voltage Current-time Characteristics	259
XVI.	Magnetic Circuits and Their Phenomena	281
XVII.	Series and Shunt Magnetic Circuits	297
XVIII.	Magnetic Materials	309
XIX.	Air Gaps and Magnetic Leakage	316
XX.	Alternating-current Phenomena	330
XXI.	Series and Parallel Alternating-current Circuits	347
XXII.	Alternating-current Solenoids	369
XXIII.	Alternating-current Electromagnets and Mechanisms	389
XXIV.	Electromagnets in General	414
XXV.	Permanent Magnets	433
Index		451

MAGNETS

CHAPTER I

INTRODUCTORY

A survey of the subject is herein given, including the definitions of the two general classes of magnets, the materials which enter into their structures, the special properties of these materials, and some of the phenomena associated with them.

1. Magnets.—A magnet is anything that exhibits the phenomenon of magnetism, as by its ability to magnetize and attract iron and other ferromagnetic substances or magnetic materials which resemble iron in magnetic properties. The two general classes of magnets are permanent magnets and electromagnets.

2. Permanent Magnets.—These are found in the natural state as hard black stones (magnetite), notably at Magnesia in Asia Minor. The name lodestone (or "leading stone") was given this ore when it was used as the original magnetic compass needle. A commercial permanent magnet consists of hardened alloy steel, such as tungsten steel or chromium steel, which has been magnetized by means of an electromagnet.

3. Electromagnets.—While a permanent magnet has many uses and will attract and indefinitely hold pieces of magnetic material, an electromagnet possesses among other properties the further advantage that it can release its load almost as readily as it can attract and hold it, the control being either manual or automatic and either local or remote. An electromagnet can demagnetize a permanent magnet so that the phenomenon of magnetism will completely disappear from it.

The simplest form of electromagnet is a solenoid or coil composed of one or more loops or turns of electric conductor. The material used for the electric conductor is generally very pure soft copper. The turns are electrically insulated from one another by means of electric insulation or electric insulating materials, technically known as dielectrics. The electric conductors used in

the great majority of coils are insulated with enamel, silk, cotton, or combinations of enamel and cotton, or enamel and silk, and are known as magnet wires. Magnet wires designed for high temperatures are insulated with asbestos. Enamel, silk, cotton, asbestos, paper, and so forth are dielectrics. Coils also are supported mechanically by various other materials, but the special materials which enter into their construction are electric conductors and dielectrics.

The technical definition of a solenoid is an electrically energized coil or helix. A solenoid greatly resembles a permanent magnet in its magnetic properties but, since it has a core hole throughout its entire length, it can magnetize magnetic materials and attract them so that they shall pass inside of the core hole. A solenoid with a core of magnetic material is the basis of all forms of commercial electromagnets. Therefore, the special materials comprising an electromagnet are as follows:

1. Electric conductors.
2. Dielectrics.
3. Magnetic materials.

4. Matter and Energy.—A permanent magnet is a piece of matter endowed with the ability to do work by attracting another piece of matter and moving it through a distance under the influence of a mechanical force exerted by the permanent magnet. Work can be accomplished only through the expenditure of an equal amount of mechanical energy. In this case, the mechanical energy is converted from the magnetic energy with which the permanent magnet is endowed. After a permanent magnet has accomplished its work by attracting a suitable magnetic material to it, if said magnetic material is forcibly removed from the permanent magnet the mechanical energy expended in the process will be returned to the permanent magnet in the form of magnetic energy.

It is to be noted that a piece of magnetic material does not appear to possess magnetic energy until it has been magnetized. Then the invisible magnetic flux is in evidence, as may be noted by sprinkling iron or steel filings near the magnetic material. Magnetic flux is one of the components of magnetic energy. For instance, mechanical energy = force $\times$ length, but force is not energy. Force is a manifestation of mechanical energy. Magnetic flux is a manifestation of magnetic energy.

The magnetic flux is always in the magnetic material and is made to produce external manifestations through the action of a magnetizing magnetic field of magnetic flux, as when a piece of magnetized magnetic material is placed near a piece of unmagnetized magnetic material. If a piece of magnetic material, either magnetized or unmagnetized, could be separated into its constituent molecules without changing the general form of the structure, each molecule would be a separate magnet. Magnetic flux may be thought of as the linkage between two magnetic bodies that are mutually attracted, the mechanical force being considered as due to the tendency of the magnetic flux to contract and thus tend to pull together the two magnetic bodies with which it is mutually linked.

5. Electricity and Matter.—Matter consists of electricity and energy, if nothing more. Solid electric conductors consisting of elementary metals like copper or aluminum are aggregations of atoms. Science teaches that each atom constitutes an ultraminute solar system consisting of positively and negatively charged particles. These particles, which are common to all forms of matter, may be pure positive and negative electricity, respectively. The simplest form of atom has one proton (the electropositive particle) as a nucleus, and one or more electrons (electronegative particles) describing orbits about the proton at enormous frequencies.

In solid electric conductors, some of the electrons fly from atom to atom in great quantities, at enormous velocities, and at exceedingly frequent intervals. Thus, in the aggregate, there is a definite amount of negative electricity consisting of free electrons in the spaces between the atoms. This aggregation of electrons is sometimes referred to as an "electronic gas," because it behaves like a gas, and is the electricity that is moved along within the structure of an electric conductor under the influence of an electromotive force (e.m.f.) or voltage to form an electric current.

Nothing but moving electricity produces the phenomenon of magnetism. No magnetic flux can exist without an electric current of some sort. Magnetic flux is a manifestation of moving electricity. The magnetic energy of an electromagnetic system is proportional to the product of the electric current strength and the magnitude of the magnetic flux linked with the electric current. It is believed that each molecule of magnetic material

has one or more electrons associated with it, as describing orbits around it, so that the phenomenon of ferromagnetism, that is, the phenomenon of magnetism associated with ferromagnetic materials, or with magnetic materials in general, is really due to electric currents within the structures of magnetic materials.

A dielectric is a material in which the electrons are elastically bound in such manners that, while they can be but slightly displaced from their mean positions to form a displacement current, they cannot be moved continuously in a given direction under the influence of a continuous or direct voltage to form a direct current without breaking down the structure of the dielectric. However, they may oscillate to and fro to form an oscillating current, which is one form of alternating current.

Mutual attraction exists between protons and electrons, and a field of dielectric flux is said to exist between them. Electrons mutually repel one another. A body is said to possess a negative electric charge when it contains or possesses more electrons than are required to make the body electrically neutral, and is said to possess a positive electric charge when it contains less electrons than are required to make the body electrically neutral. Thus, when celluloid is rubbed against wool, the celluloid removes some of the electrons from the wool, with the result that the celluloid receives a negative charge, and the wool becomes positively charged. When a dielectric is strained, due to the displacement of its bound electrons, one side is positively charged and the opposite side is negatively charged.

It is interesting to note that, while there is a force of mutual electrostatic repulsion between two electrons whether or not they are in motion, when two electrons move in parallel paths they produce magnetic flux which tends to link both electrons, and, therefore, tends to pull them together. When two electrons move in parallel paths at the velocity of light, their mutual electrostatic repulsion is exactly balanced by their mutual electromagnetic attraction.

The electrical particles of which matter is composed are so small, with respect to the distances between them, that matter occupies practically no space at all. The forces due to the energy contained in matter give matter its various resistive properties. But electrons may move in the free spaces between atoms and molecules under favorable conditions, any apparent friction being due to the reactions between them.

The temperature of a body is a measure of the quantity of thermal energy or heat per unit mass stored in the body, and is further defined as being proportional to the square of the average velocity of the smallest particles of the body. Electrons are the smallest known particles.

Electricity is not a form of energy, but it is one of the components of electric energy.

The above may be generalized as follows:

1. Electricity is the stuff that matter is made of.
2. Any moving electron or aggregation of electrons constitutes an electric current.
3. An electron describing a closed orbit constitutes an elementary magnet.
4. An aggregation of electrons elastically bound in matter constitutes a dielectric.
5. The square of the average velocity of the electrons in a body is a measure of the temperature of the body.

There is no known magnetic insulator. Magnetic flux exists in space about moving electrons.

There is no known perfect electric conductor or dielectric. A good electric conductor is a very poor dielectric and a good dielectric is a very poor electric conductor.

The electric conducting ability, or the electric conductance, of such elementary metals as copper and aluminum is impaired as the temperature of the metal increases. The electric insulating properties of dielectrics are impaired or destroyed by too high temperatures, by too great mechanical pressures, or by too high voltages. The magnetic properties of magnetic materials are impaired or destroyed by relatively high temperatures.

Electric energy is converted into heat in electric conductors through the phenomenon of electric resistance (the reciprocal of electric conductance), which is comparable to friction between the moving electrons constituting the electric current and the atomic systems. Electric energy is also converted into heat in dielectrics, due to a phenomenon called dielectric hysteresis, which is analogous to friction, when the bound electrons are slightly displaced to form a displacement current. Magnetic energy is converted into heat when magnetic materials are magnetized or demagnetized through a phenomenon known as magnetic hysteresis, and which is analogous to friction between

the molecules as they turn like compass needles under the influence of the magnetizing force.

Electric energy is converted into magnetic energy, magnetic energy is converted into mechanical energy, and mechanical energy is expended in doing work. The electric conductor takes its toll of electric energy, the dielectric absorbs some of the electric energy, the magnetic material becomes magnetized through the expenditure of electric energy, and all of the mechanical energy is eventually converted into heat. In every case, some of the energy is converted into heat in the material energized, thus increasing its temperature and tending to impair its usefulness as a component part of an electromagnet. Undesirable conversions of energy into heat are called heat losses.

Good electric conductors also are good thermal conductors (conductors of heat), while dielectrics are poor thermal conductors. However, electricity and heat are conducted in different manners.

The above statements may be condensed as follows:

1. Electric energy is converted into heat in electric conductors through the phenomenon of electric resistance.

2. Electric energy is converted into heat in dielectrics through the phenomenon of dielectric hysteresis.

3. Magnetic energy is converted into heat in magnetic materials through the phenomenon of magnetic hysteresis.

4. Mechanical energy is converted into heat through the phenomenon of friction.

5. In every case, energy is converted into heat in the material acted upon by imparting some of the motion to the smallest particles of said material.

6. Electric Conductors and Electric Circuits.—An electric conductor contains electricity which, when set in motion by an electromotive force, or electric pressure, analogous to hydrostatic pressure, constitutes an electric current which can flow in a closed electric circuit. The practical unit of electric pressure is the volt. The term voltage always expresses the magnitude of the electric pressure in volts.

The practical unit of quantity of electricity, consisting of approximately 8,000,000,000,000,000,000 (8×10^{18}, or eight billion billion) electrons, is the coulomb. The practical unit of time-rate of flow of electricity, or the electric current strength or amperage, in an electric conductor is the ampere or coulomb

per second. The term amperage always expresses the strength of the electric current in amperes. When the current strength is one ampere, one coulomb of electricity flows across every plane in the cross-section of the conductor in one second.

A simple series electric circuit consists of one or more electric conductors electrically connected end to end, and may consist of a single turn. An electron describing a closed orbit is the simplest form of electric circuit. Every metallic circuit consists of at least one complete turn of electric conductor with the electricity already in it, regardless of what the actual shape of the turn may be. Hence, when a straight conductor in which an electric current flows is referred to, it is to be understood that said straight conductor is really a portion of a closed electric circuit.

The fundamental object of a commercial electric circuit is to transmit energy in the form of electric energy from a place where energy is available to a place where energy is required. Electricity is the medium of transfer of electric energy, much as a belt is the medium of transfer of mechanical energy from a driving pulley to a driven pulley, or as a nearly incompressible gas (like which the electricity in an electric conductor behaves) may be forced through a sort of porous structure constituting a circulating system by means of a pump. An electric generator is a pump which drives or tends to drive the electricity already in an electric conductor to form an electric current, and generates only an electric pressure or voltage.

Wherever there is electric resistance, a resistance reaction opposes the total impressed voltage when an electric current flows, and causes an active voltage drop which must be subtracted from the impressed or total voltage to determine what part of the total voltage remains for other purposes. In the cases of direct currents and voltages, algebraic subtraction is employed, but in the cases of effective alternating currents and voltages, geometric subtraction must be employed. The geometric difference between two quantities is equal to the square root of the difference in their squares.

The unit of electric resistance is the ohm or active volt per ampere. Electric conductance is the reciprocal of electric resistance. The unit is the mho or ampere per active volt.

The practical unit of electric energy is the joule, volt-coulomb, or watt-second. One joule of electric energy is expended when a

constant electric pressure or voltage of one volt forces one coulomb of electricity through any plane in the cross-section of an electric conductor. Hence, electric energy = voltage × quantity of electricity.

The conversion of one joule of electric energy into heat in one gram of copper, through the phenomenon of electric resistance, will cause an increase in the temperature of the gram of copper equal to 2.6 degrees Centigrade (C.), or 4.68 degrees Fahrenheit (F.).

7. Conversion Factors.—The principle units of length and mass (or weight) are expressed in the metric and English systems, respectively. Force is usually expressed in terms of weight. Mass is equal to weight divided by the gravitational constant. The components of any form of energy may be equated with the components of any other form of energy by making the two forms of energy equal to one another.

Electric energy = voltage × quantity of electricity.

Mechanical energy = force × length.

Hence, in a broad sense, when electric energy = mechanical energy, voltage × quantity of electricity = force × length.

The last equation may be definitely expressed as follows:

Joules = volts × coulombs = 0.098 06 × kilogram-centimeters.

But 1 joule of electric energy is equal to 10.198 kg.-cm. of mechanical energy. The number 10.198 is the conversion factor.[1]

In order to avoid confusion, statements like the above are to be interpreted as follows: When the number of joules is 1, the number of equivalent kilogram-centimeters is 10.198. Then

$$\frac{\text{kilogram-centimeters}}{\text{joules}} = 10.198;$$

the number of kilogram-centimeters is equal to 10.198 times the number of joules, and the number of joules is equal to 0.098 06 times the number of kilogram-centimeters. Hence, while

1 joule = 10.198 kg.-cm.

and 1 kg.-cm. = 0.098 06 joule,

joules = 0.098 06 × kilogram-centimeters

and kilogram-centimeters = 10.198 × joules.

[1] Tables of conversion factors are generally found in standard handbooks. A very complete work is "Conversion Factors," by Carl Hering.

Thus, to put any usual expression given for a conversion factor into the form of an equation for substitution into other equations, reverse the position of the conversion factor.

8. Coils.—Energy is distributed throughout an electric circuit during its flow. A coil is a portion of an electric circuit gathered together in as small a space as practicable, and so arranged and proportioned that the energy may be advantageously utilized. Were it not for the phenomenon of electric resistance, the thickness of the coil wall might be exceedingly small. The undesirable production of heat due to electric resistance makes it necessary so to design a coil that (1) the heat shall not be developed too rapidly, and (2) the heat may be disposed of at such a time-rate that the temperature of the coil shall not become sufficiently great to damage the insulation, or too greatly to increase the electric resistance of the coil conductor in the case of a coil designed for use on a constant continuous-voltage circuit.

Because of the phenomenon of electric resistance, the mass (or weight) of electric conductor required in a coil is proportional to its absolute temperature. The absolute zero temperature is −273° C. Ordinary "room temperature" is about 300° on the absolute Centigrade scale. If the temperature of a coil could be made 150° on the absolute Centigrade scale, the coil need be only one-half as thick as an otherwise similar coil designed for ordinary room temperature. Since the temperature of a coil is increased by an electric current flowing in its turns, the size of the coil must be determined from its final temperature, which may be about 50° C. greater than the temperature of the surrounding air. Hence, it must be about one-sixth larger for operation at the final temperature than for operation at normal room temperature.

Coils designed for strong electric currents often consist of a relatively few turns of a single bare round, square, or rectangular (usually wound on edge) soft-copper electric conductor in the form of a helix, with its turns so spaced that the intervening air is the electric insulation or dielectric between them. In such a case the turns are said to be in series. As the electric conductor becomes heated, due to the electric current flowing within its structure, the air between the turns, becoming heated, expands and rises away from the coil, thus carrying the heat away with it and tending to cool the coil. Heat also is radiated from a coil

and is conducted to cooler bodies through its terminals and any dielectric interposed between the coil and cooler bodies.

The great majority of commercial coils consist of superposed helices of alternate right and left pitch, forming even layers of turns of magnet wire electrically insulated with enamel, silk, cotton, or asbestos, either singly or in combination. All of these are dielectrics possessing such electrical and mechanical properties that neither short circuits nor conduction-current leakage can occur between the turns of copper wire due to electric pressures, or to mechanical pressures which might cause the turns of copper to come into electrical contact through the mechanical disruption of the dielectric under ordinary circumstances. Transient displacement currents occur in these dielectrics whenever the voltage between the turns varies, particularly in the cases of oscillating and alternating currents. The condensance (in ohms), or the insulating resistance to oscillating or alternating currents, decreases as the frequency of reversal of such currents increases. The insulation resistance to direct currents of a dielectric may be exceedingly great, and, simultaneously, the insulating resistance (condensance) to high-frequency currents may be very small.

These coils may be constructed by placing a form in a lathe or in a winding machine and then winding the magnet wire into the form as it revolves. Before its removal from the form, the turns of the coil may be securely bound together by means of fine strong twine. The inside and outside ends of the wire, which may or may not be reenforced, are the leads or terminals by means of which the coil is connected in the electric circuit. Such a coil has the general form of a hollow cylinder. The length of the cylinder is the coil length. The thickness of the wall is the coil-wall thickness. The average girth is the average length of conductor per turn. Coils of this general class are not so readily cooled as are those having open spaces between their turns, as the heat must flow through the coil wall to the inner, outer, and end surfaces before it can be disposed of. Whereas copper is an excellent thermal conductor, dielectrics are poor thermal conductors and heat cannot so readily flow within their structures. The time-rate of flow of heat from the interior of the coil wall to the cooling surfaces of the coil is directly proportional to the difference in temperature between the interior of the coil wall and any cooling surface, and is inversely pro-

portional to the coil-wall thickness. Obviously, the greater the amount of dielectric in the coil wall, the smaller will be the time-rate of flow of heat to the cooling surfaces.

9. Dielectrics and Electric Condensers.—A dielectric may be considered as an electrically elastic material. A "leaky" dielectric is a partial electric conductor. The equivalent is a perfect dielectric shunted by an electric conductor of exceedingly small cross-section, or of very great length, or both, in order that its resistance may be great. The dielectric strength of a dielectric is defined as the voltage per unit thickness required to puncture the dielectric. Units of voltage gradient are the volt per mil, volt per millimeter, and so forth.

The quantity of electricity displaced in a dielectric is dependent upon the voltage and the capacitance, which may be defined as the quantity of electricity displaced per dielectric volt; the quantity of electricity being that displaced through any plane in the dielectric at right angles to the direction of the voltage, and the dielectric voltage being the reactive voltage of the dielectric. The practical unit of capacitance is the farad or coulomb per dielectric volt. A more practicable unit of capacitance is the microfarad (millionth farad) or microcoulomb per dielectric volt. Elastance is the reciprocal of capacitance. A unit suggested by Prof. V. Karapetoff is the daraf, for the reciprocal of the farad, and the megadaraf for the reciprocal of the microfarad. With these definitions, the daraf is the volt per coulomb and the megadaraf is the volt per microcoulomb or the megavolt per coulomb.

Electric energy is stored in dielectrics due to electric pressures or voltages displacing the electricity elastically bound therein. Under the influence of an external voltage the bound electrons move slightly and a countervoltage, called the dielectric voltage, equally opposes the external voltage acting directly on the dielectric. The electric energy in joules thus stored is equal to one-half the product of the dielectric voltage and the quantity of electricity in coulombs displaced, since the dielectric voltage increases from zero to a voltage equal, but opposed, to the voltage (the e.m.f. of capacitance) causing the displacement of the elastically bound electricity. Hence, stored electric energy = one-half the dielectric voltage $\times$ quantity of electricity displaced.

Thin sheets of dielectric, as paper treated with paraffin or ceresin, are placed between sheets of metal foil (preferably aluminum foil) to form an electric condenser. The thinner the dielectric and the greater its extent the greater is the capacitance. The capacitance is also dependent upon the dielectric constant, which determines the relative capacitances of various dielectrics under similar conditions. The greater the dielectric constant, the greater is the relative capacitance.

An electric condenser (or a dielectric) can retain its charge of stored electric energy after the charging voltage has been effectively discontinued by opening the charging circuit, and the electric condenser then becomes an independent source of electric energy. An electric condenser may be charged with a definite amount of electric energy, which may be delivered to a conductor for purposes of heating or welding at such a time-rate that just enough energy will be delivered to accomplish the desired purpose. Electric condensers also are used to absorb the electric energy suddenly let loose when the electric circuit is broken, for producing oscillating currents, and for many other purposes.

A thin sheet of dielectric material between adjacent plates of metal foil may be thought of as an electrically elastic diaphragm, although the term "diaphragm" strictly refers to the elastically bound electricity within its structure. The displacement of the bound electrons in the structure of the dielectric causes the side towards which the electrons are displaced to become negatively charged, while the side away from which the electrons are displaced becomes positively charged.

The amount of electric energy in joules that can be stored for each microfarad of capacitance is equal to 0.000 000 5 times the square of the dielectric voltage, or joules per microfarad = 0.000 000 5 $\times$ dielectric voltage squared. Thus, in order to store 1 joule of electric energy in a 1-microfarad (or 1-mf.) condenser, the voltage must be 1,410 volts, which is too great for ordinary paper condensers.

10. Solenoids and Magnetic Circuits.—An electrically energized coil will attract to it pieces of magnetic material, will point north and south (if properly suspended), and will otherwise behave very much like a bar permanent magnet, or a straight bar of magnetized hardened steel. The above statement covers the technical definition of a solenoid.

An electric current produces a magnetic field of magnetic flux at right angles to it in the space surrounding it. The phenomenon is termed electromagnetism. One unit of magnetic flux is the maxwell. Another unit is the weber or volt-second (not officially adopted), which is equal to 100,000,000 maxwells. When an electric current is flowing in a straight electric conductor, the magnetic flux exists about it (as well as within the structure of the conductor) in space[1] in the form of a single closed loop, turn, or cylinder having a length equal to the length of the conductor and a limitless diameter. Magnetic flux always exists in closed loops (however distorted), since there is no known magnetic insulator. A closed loop of magnetic flux constitutes a magnetic circuit. There can be no continuous electric current in a cold metallic circuit unless the circuit is closed. Magnetic circuits are always closed. A more definite statement of the case is that a magnetic circuit can never be entirely opened.

Magnetic flux is always linked with the electric current producing it, and the electric and magnetic circuits are linked together. The product of the amperage and the number of turns in which the electric current is flowing is the number of ampere-turns. By winding a portion of the electric circuit into a coil, the magnetic flux surrounding the electric current in the structure of the conductor is gathered together, so to speak, to form a magnetic field whose strength or magnetic flux density, expressed in maxwells per unit cross-section and called the magnetizing force, is proportional to the number of ampere-turns per unit of coil length. The unit of magnetic flux density and of magnetizing force is the gauss or maxwell per square centimeter.

The magnetic flux exists in the core hole of the solenoid and extends indefinitely outside of it, thus forming a magnetic field linked with the solenoid. But not all of the magnetic flux is linked with all of the turns. The magnetic flux produced by the current in any given turn is linked with that turn and tends to link with other turns. However, in the usual commercial electromagnets, the flux linked with the outer turns does not completely link with the inner turns, and the flux linked with the turns at the middle of the solenoid does not all link with the turns at the ends of the solenoid.

While an electric current produces magnetic flux in the case just described, an electric current really produces a magnetomo-

[1] Or the ether.

tive force (m.m.f.), which, in turn, causes the production of magnetic flux. Two opposing magnetomotive forces can hamper or prevent the production of magnetic flux. The practical unit of magnetomotive force is the gilbert. It is equal to 1.257 amp.-turns. The magnetizing force in gausses is equal to the number of gilberts per centimeter length of magnetic circuit.

The product of the magnitude of magnetic flux and the number of turns of conductor linked with it is called the number of flux-turns. Units are the maxwell-turn and the weber-turn. One weber-turn is equal to 100,000,000 maxwell-turns.

Whenever there is a change in the number of linkages of magnetic flux with turns of conductor, an induced voltage is produced or induced in said conductor through the phenomenon of electromagnetic induction. When the time-rate of change in the number of maxwell-turns is 100,000,000 per second, the induced voltage is one volt. Likewise, the induced voltage is one volt when the time-rate of change in the linkage is one weber-turn per second. When one weber of magnetic flux uniformly links or unlinks with a single turn in one second, the constant induced voltage is one volt. If the time-rate of change in the linkage is not uniform, the average voltage will be one volt. Hence, the weber is also called the volt-second.

11. The Self-induced Voltage and the Storage of Magnetic Energy.—The induced voltage is called the self-induced voltage when the magnetic flux produced by an electric current reacts upon the electricity forming that current. The phenomenon is termed self-induction. The coefficient of self-induction is called the self-inductance. Practical units of self-inductance are the henry and the millihenry (thousandth henry). The self-inductance in henries is equal to the ratio of the number of weber-turns to the amperage, or henries = weber-turns per ampere.

Self-inductance is sometimes referred to as electromagnetic inertia or electromagnetic mass. The magnetic flux linked with the moving electricity constituting an electric current endows that electricity with the property of mass. Hence, when an impressed voltage tends to set electricity in motion and thus form a magnetic flux producing current, a self-inductance reaction, commonly called the self-induced voltage, opposes the impressed voltage. That component of the impressed voltage which is balanced by the self-induced voltage is termed the

e.m.f. of self-induction. After an electric current has produced magnetic flux, and the two are linked together, the electricity flows on its own "momentum" unless some countervoltage is established, as by the insertion in the electric circuit of electric resistance or capacitance. When any voltage reacts on the electricity to tend to stop its motion, the self-induced voltage acts in the direction in which the electricity is moving to tend to keep it moving in that direction. In this respect, electricity behaves very much like matter.

The phenomenon of self-induction is due to the effort of producing magnetic flux and storing magnetic energy in the magnetic circuit. The action is as though the magnetic flux is compelled to expand concentrically about the conductor when the impressed voltage starts the electricity moving and, being under tension, it tends to resist any change in the motion of electricity caused by any means other than its contraction. When any other voltage tends to retard the motion of electricity, the magnetic flux appears to contract upon the electricity and tend to keep it in motion in the direction in which it is then moving.

In absolutely cold elementary metals there is no electric resistance to tend to arrest the motion of electricity and thus cause the conversion of electric energy into heat. In such a case the self-induced voltage is exactly opposed to the impressed voltage and disappears when the impressed voltage is discontinued, but the current strength and the magnitude of the magnetic flux linked with the current remain constant for an indefinite period of time. The above belief appears to be satisfactorily confirmed by the experiments of Dr. Onnes. This shows conclusively that, while electric energy must be expended in establishing an electric current and producing magnetic flux, no voltage is required to keep the electricity in motion and no energy is expended in maintaining the magnetic field of magnetic flux. The magnetic flux appears to give the electricity its mass, and it is believed that all mass is electromagnetic in character.

The unit of magnetic energy is the joule. The magnetic energy in joules stored in a magnetic circuit, due to the motion of electricity in a linked electric circuit, is equal to one-half the number of ampere-weber-turns, or to one-half the product of the self-inductance in henries and the square of the amperage.

A self-induced voltage is in evidence under any of the following conditions:

1. When the current strength is increasing and magnetic energy is being stored.

2. When the current strength is decreasing and magnetic energy is being converted into electric energy, and thus returned to the electric circuit.

3. When the self-inductance is increasing and magnetic energy is being stored.

4. When the self-inductance is decreasing and magnetic energy is being converted into electric energy, and thus returned to the electric circuit.

5. When work is being performed by an electromagnetic system.

6. When work is being done on an electromagnetic system, as by separating its parts when the mechanical force due to the system is tending to pull said parts together, or by forcing the parts together when the mechanical force of the system tends to push them apart.

In (1), (3), and (5), the self-induced voltage opposes the continuous voltage. In (2), (4), and (6), the self-induced voltage reenforces the continuous voltage.

An electromagnetic device is a reactor containing self-inductance. An air-cored solenoid is an air-cored reactor. The greater the frequency of an alternating voltage impressed across the terminals of a solenoid or any reactor the greater is the inductive reactance in ohms and the smaller is the current strength for a given impressed alternating effective voltage. Owing to the self-inductance, time is required to set the electricity in motion and time is required to arrest its motion and to reverse its direction of motion. When the frequency is low, more time is available than when the frequency is relatively high, and a greater current strength may be obtained in the same reactor with the same alternating effective voltage.

12. Permeance.—The ratio of the magnitude of the magnetic flux to the m.m.f. producing it is the permeance of an electromagnetic system. No unit has been officially assigned. Permeance is closely related to self-inductance and can be substituted therefor in equations involving magnetic energy and self-induced voltages when the proper conversion factor is used. In a like manner, self-inductance may be substituted for permeance. The weber is the more convenient unit of magnetic flux for calculations involving magnetic energy and self-induced

voltages, while the maxwell is the more convenient unit in magnetic circuit calculations.

In this treatise, the two units of permeance are the weber per gilbert and the maxwell per gilbert. One weber per gilbert is equal to 100,000,000 maxwells per gilbert. The self-inductance in henries is equal to 1.257 times the product of the permeance in webers per gilbert and the square of the number of turns of conductor linked with the magnetic flux. As a general statement, the ratio of the self-inductance to the permeance of an electromagnetic system is proportional to the square of the number of turns. In an electromagnetic system containing but a single turn of electric conductor the ratio of the self-inductance in henries to the permeance in webers per gilbert is 1.257.

The magnetic energy in joules stored in the magnetic circuit of an electromagnetic system is equal to 0.628 32 times the product of the permeance in webers per gilbert and the square of the number of ampere-turns. The permeance in webers per gilbert is equal to 0.7958 times the ratio of the self-inductance in henries to the square of the number of turns, or to 1.59 times the ratio of the stored magnetic energy in joules to the square of the number of ampere-turns.

13. Effects of Opposing Magnetomotive Forces.—A so-called non-inductive coil may be made by doubling an insulated magnet wire upon itself at its middle and then simultaneously winding the two halves in turns and layers. When a voltage is impressed across the terminals of the coil, the current will flow simultaneously in opposite directions in adjacent turns, thus producing opposed m.m.fs., and the coil will possess an exceedingly small amount of self-inductance, since the opposing m.m.fs. tend to prevent each other from producing magnetic flux. The magnitude of the magnetic flux produced is equal to the product of the difference between the opposing m.m.fs. and the permeance of the coil. Such a coil is purposely made to contain only electric resistance (although it also contains capacitance between its turns), and is one form of resistor.

Any m.m.f. that acts in conjunction with a m.m.f. tending to produce or to maintain magnetic flux so that the total m.m.f. is greater than either of the individual m.m.fs. is a reenforcing m.m.f. Any m.m.f. that opposes a m.m.f. tending to produce or to maintain magnetic flux so that the total m.m.f. is less than the greatest individual m.m.f. is an opposing m.m.f. An opposing

m.m.f. may be so great as to reverse the direction of the magnetic flux. Opposing m.m.fs. are always productive of mechanical repulsion when the self-inductance of an electromagnetic system could be increased if the parts separated under the influence of that force.

14. Mechanical Forces in Magnetic Fields.—An electromagnet can perform work only through the storage of magnetic energy. The magnetic energy converted into mechanical energy in an electromagnetic system is equal to the magnetic energy stored in the magnetic circuit during the relative movement of the parts. Hence, one-half of the electric energy converted through the phenomenon of self-induction goes to perform work, while the other half is stored as magnetic energy in the magnetic circuit.

When no work is performed by an electromagnetic system while electric energy is being converted into magnetic energy, 1 joule of electric energy is converted into 1 joule of magnetic energy (barring losses) through the phenomenon of self-induction and is stored in the magnetic circuit. However, when 1 joule of electric energy is converted while work is being performed, only ½ joule (or 0.3688 ft.-lb.) may be expended in lifting 1 lb. vertically through a distance of about 4.4. in. or in imparting a horizontal velocity of about 4.9 ft. per second to a mass weighing 1 lb. (or one poundal), or partly lifting and partly accelerating the load, as well as overcoming friction, the other ½ joule will be stored in the magnetic circuit, provided there are no electric or magnetic energy losses during the conversion.

By the law of conservation of energy, the electric energy converted through the phenomenon of self-induction is equal to the sum of the mechanical energy expended and the increase in the stored magnetic energy. But the mechanical energy expended is equal to the increase in the stored magnetic energy. Consequently, twice as much electric energy is converted for a given storage of magnetic energy when work is performed as when magnetic energy is stored without the production of work. Conversely, only one-half of the electric energy expended is converted into force × length. Hence,

Mechanical energy = magnetic energy = mechanical force × length under these conditions, and the mechanical force developed between any two parts of an electromagnetic system, wherein only one of the parts is conveniently assumed to move, may be expressed as follows:

1. The mechanical force is directly proportional to one-half the rate at which the stored magnetic energy would increase with respect to a corresponding small change in the relative positions of the parts if either of the parts should move under the influence of that force.

But

Magnetic energy = ½ × self-inductance × amperage squared

or

Magnetic energy = ½ × the number of ampere-weber-turns.

The above rule (1) is based upon "What would happen if." Thus far, nothing has really happened. However, the rule may be practically applied by keeping the current strength or amperage constant and letting the self-inductance increase as the distance between the parts changes. Then the mechanical force will be the average of the forces in the first and second positions, and

$$\text{Average force} = \tfrac{1}{2} \times \text{amperage}^2 \times \frac{\text{difference in self-inductance}}{\text{difference in length}},$$

or

$$\text{Average force} = \tfrac{1}{2} \times \frac{\text{difference in ampere-weber-turns}}{\text{difference in length}}.$$

The last equation covers the general case. In any electromagnetic system where any relative movement of the parts would tend to change the number of ampere-weber-turns, there will be a mechanical force between the parts which will be proportional to one-half the rate of change. If the stored magnetic energy can be increased by the movement of a part into the system, the force will be of mutual attraction. If the stored magnetic energy can be increased by the movement of a part away from the system, the force will be of mutual repulsion. If no increase in the stored magnetic energy can occur, there will be no force and no motion.

The above is the basis of economical electromagnetic design, since a system can be so proportioned and arranged that the equivalent of the known amount of mechanical energy (average force × length) shall be stored in the magnetic circuit when one of its parts is moved into or out of the system for the desired distance, depending upon whether a mechanical force of attraction or one of repulsion is desired.

In any case, so arrange and proportion the electric and magnetic circuits that a minimum amount of stored magnetic energy

will cause the production of the desired or necessary initial mechanical force. Next find what actual length of travel in a straight line would be required to store another amount of magnetic energy numerically equal to initial force × length. Add this amount of stored magnetic energy to the original amount to obtain the amount of stored magnetic energy at the final position, from which the mechanical force at the final position can be determined. If the mechanical force in the final position is greater than is required, the movable part may be made to move at an angle with the straight line at first assumed. In this manner the average mechanical force may be adjusted so that the length of travel or range may be made greater for a given expenditure of mechanical energy and storage of magnetic energy than if the mechanical force is allowed to increase as magnetic energy is stored, or less magnetic energy need be stored for a given initial force × distance. The mechanical force can usually be increased by increasing the number of ampere-turns, but this is not always the case (as in a differential electromagnetic device).

15. Electromagnet with an Iron or Steel Core.—A piece of magnetic material, as wrought iron or hardened steel, may exhibit no indication or being magnetized in any manner. However, if it is placed or attracted into the core hole of an electrically energized solenoid it will become strongly magnetic through the phenomenon of magnetic induction. If the magnetic material is wrought iron, it will "lose" almost all of its magnetism when the electric current is discontinued. If hardened steel is used, it will retain much of its magnetism and become a permanent magnet. Hence, a solenoid with a wrought-iron core forms one kind of electromagnet, which will attract pieces of magnetic material to it when an electric current flows in its turns but will release the magnetic material when the current is discontinued. If hardened steel is used as a core, the electromagnet will not only be weaker but will not release the load when the current is discontinued. However, the steel may be demagnetized by first discontinuing the current and then again increasing the current to just the proper strength, either with the direction of the current reversed or with the position of the steel reversed.

16. Magnetic Induction.—When a magnetic material is placed in a magnetizing magnetic field, there is more magnetic flux per unit cross-section of magnetic field space than existed before the

magnetic material was placed therein. The reason is that the additional magnetic flux already is in the magnetic material. This may be understood when it is considered that a permanent magnet may be broken into any number of pieces, and each piece will be a separate magnet. Hence, it is natural to assume that each molecule of iron or steel is a magnet. Normally, the molecules of soft iron or steel swing around into various groups to form minute magnetic circuits, like triangles, squares, pentagons, and so forth, within the structure of the iron or steel, the exterior of the magnetic material exhibiting no magnetic properties in the absence of a magnet of some kind. However, when the steel is placed in a magnetic field, the molecules tend to swing around, like compass needles, so that the total magnetic field strength is increased. As the strength of the magnetizing field is increased the molecules swing further and further until, when they all lie in the direction of the magnetic flux of the magnetizing field, all of their magnetic flux is added to that of the magnetizing field and the steel is said to be saturated. This is the general principle of ferromagnetism.

17. Retentiveness.—After the magnetizing magnetic field is discontinued, or the magnetic material is removed from the magnetizing field, wrought iron will be very much more demagnetized than will hardened tungsten steel, for instance, because the molecules of the former can swing back to approximately their natural positions, while in the latter the molecules are held approximately in their new positions. The property whereby a magnetic material independently remains a magnet is called retentiveness.

18. Names for the Two Magnetic Fluxes.—The production of magnetic flux in space by an electric current occurs through the phenomenon of electromagnetism. The phenomenon of magnetic flux in magnetic materials is called ferromagnetism. In this treatise, the magnetic flux produced in space by an electric current is called the electromagnetic flux, while that already in the structure of a magnetic material is called the structural flux or the ferromagnetic flux.

19. Direct and Alternating Currents and Voltages.—A direct current (d.c.) flows continuously in a given direction in a closed electric circuit until the motion of the electricity is arrested by some countervoltage, as occurs when a dielectric is inserted in series in the circuit. All metallic circuits are opened in this

125909

manner by separating the metal contacting members or contacts and allowing air, oil, or some inert gas to fill the space between them. Additional barriers of solid dielectric are often placed between adjacent sets of contacts. One of the objects of the use of oil is to insert a dielectric of greater dielectric strength than that of air between the contacts, so as successfully to resist the relatively great self-induced voltage which results when an electric current linked with magnetic flux produced by it is suddenly stopped.

If the current is flowing under the action of a direct or continuous impressed voltage, the self-induced voltage may be many times greater than the impressed voltage and will act in the same direction as, and in conjunction with, the impressed voltage to tend to rupture the dielectric, and keep the electricity flowing, and thus produce a spark or arc where the dielectric is ruptured. Any voltage that acts in conjunction with a voltage tending to set or to keep electricity in motion, so that the total voltage is greater than either of the individual voltages, is a reenforcing voltage. Any voltage that opposes a voltage tending to set or to keep electricity in motion, so that the total voltage is less than the greater individual voltage, is a countervoltage or else an opposing voltage. An opposing voltage may be so great as to reverse the direction of an electric current.

Electric condensers often are placed across switch contacts to receive the electric energy in a thin layer of dielectric of relatively great extent, so that the voltage may readily produce a displacement current and thus take the strain from the dielectric (as air) between the relatively small contacts. When the capacitance is properly proportioned to the self-inductance and the resistance of the electric circuit, the magnetic energy converted into electric energy and discharged into the dielectric of the condenser may be transferred at such a time-rate that there will be very little sparking at the switch contacts. However, it is well to connect a suitable high-resistance resistor across the terminals of the condenser in order approximately to discharge the condenser and convert its stored electric energy into heat in the resistor, before the contacts are again closed, to prevent the possibility of the contacts being electrically welded together when they again touch.

In view of the foregoing, the electric energy of an electric circuit is discharged into a dielectric when the electric circuit

is opened. If only air is present, the capacitance of this dielectric usually is so small that the electric energy cannot be stored therein, and its dielectric strength (for a very thin layer) is so small that the dielectric is ruptured. There are other phenomena present, of course, but they are not so apt to occur when a condenser is suitably connected across the switch contacts.

When a charged electric condenser is connected to a reactor the dielectric voltage immediately acts to set in motion the electricity in the turns of electric conductor comprising the reactor, as well as the bound electricity in the dielectric. Assuming that there are no energy losses, the reactions are similar to those between the hair spring and balance wheel of a watch, wherein the velocity of the balance wheel is analogous to the current strength, the direction of the balance wheel is analogous to the direction of the current, and the inertia of the balance wheel is analogous to the self-inductance.

The voltages of the reactor and the condenser are equally opposed at all times, and are a maximum when the current strength is zero. When the current strength is a maximum, the two voltages are equal to zero. The electricity cannot move very far without rupturing the dielectric.

The current alternately flows in a given direction and then in the reverse direction. Each flow in either direction is an alternation. Therefore, two alternations constitute one cycle, or one complete to-and-fro movement. The number of cycles per second is the frequency of the current and of the alternating voltage associated with it. Since the current reverses twice per cycle, the current strength is equal to zero twice per cycle. Alternating currents produce alternating m.m.fs. alternating magnetic fluxes, and, therefore, alternating induced voltages. Hence, in alternating-current electromagnetic systems the mechanical force is equal to zero twice per cycle. In house, office, and many industrial lighting circuits the frequency is 60 cycles per second. Consequently, in electromagnetic systems connected in such circuits the mechanical force is equal to zero 120 times per second, and causes vibration of the parts unless special means are provided to prevent it.

The self-induced voltage is a maximum when the current strength is equal to zero, and is equal to zero when the current strength is a maximum.

While a direct current can flow but momentarily in a dielectric as a displacement current, an alternating current can flow indefinitely in a dielectric as an alternating displacement current, under the influence of an alternating impressed voltage from a source of electric energy capable of maintaining the alternating-current strength regardless of energy losses. However, when a charged electric condenser is the only available source of electric energy, and it is connected to an air-cored reactor as previously described, the energy of the system is "damped out" by its conversion into heat through the phenomena of electric resistance in the electric conductors and dielectric hysteresis in the condenser.

Under the influence of an impressed alternating voltage, as that produced by an electric generator, the frequency of the alternating current is the same as the frequency of the impressed alternating voltage. When a charged electric condenser is used as the only source of electric energy and is connected to a reactor, the resulting alternating current is called an oscillating current, which is said to oscillate or to alternate at the natural frequency of the circuit.

It is possible to open an alternating-current circuit at the instant when the current strength is equal to zero, but there is always an electric current in an electrically energized direct-current circuit when it is opened. In an alternating-current electromagnetic system, electric energy is converted into magnetic energy and magnetic energy is converted into electric energy twice during each cycle. In a direct-current electromagnetic system there may be no conversion of electric energy into magnetic energy, and *vice versa*, over long periods of time, as when the current strength is constant and there are no movements between the parts.

Alternating-current calculations are greatly simplified by using effective or root-mean-square (r.m.s.) values of current strength and voltage. Maximum magnitudes of magnetic flux are taken, since the average induced voltage is proportional to the total change in the linkage of magnetic flux with turns of conductor. One effective or r.m.s. ampere of alternating current will heat a conductor at the same time-rate that it will be heated by one ampere of direct current, and one effective or r.m.s. volt (alternating) has the same general value of one volt (continuous). Alternating-current ammeters, voltmeters, and watt-meters indi-

cate the effective or r.m.s. values. Hence, alternating-current measuring instruments of the above types also may be used for measuring direct currents and voltages (by taking the averages of reversed readings).

The effective or r.m.s. value is 0.707 times the maximum value of amperage or voltage when the wave is sinusoidal. Hence, the maximum values are 1.414 times the r.m.s. values, and this fact should be taken into consideration in connection with dielectric strength and the handling of live wires. Some maximum voltages corresponding to effective or r.m.s. voltages are as follows:

Rated or Effective Voltage	Maximum Voltage
110	155
220	310
440	620

20. Secondary Currents.—Commercial alternating currents are due to alternating voltages generated by electric generators through the phenomenon of mutual induction, by alternately and periodically linking and unlinking turns of electric conductor connected to the power transmission circuit (through transformers) with magnetic flux produced by an electric current in a separate electric circuit. The coefficient of mutual induction is termed mutual inductance. The unit is the henry. The mutual inductance between two electric circuits mutually linked with a common magnetic circuit is equal to the change in the number of magnetic linkages in either circuit per unit change in the current strength in the other circuit, or that quantity by which the time-rate of change in the primary current strength must be multiplied to give the secondary voltage; that is, it is the ratio of the secondary voltage to the time-rate of change in the primary current strength.

The circuit including the source of electric energy is the primary circuit. The mutually linked or inductively coupled circuit is the secondary circuit. The secondary voltage behaves as though the self-induced voltage is simply transferred to the other circuit (although its magnitude is usually different), and drives the electricity in the secondary circuit in a direction opposite to that in the primary circuit, thus tending to prevent the production of magnetic flux, that is, the secondary m.m.f. opposes the primary m.m.f. and is a counter m.m.f.

Voltages are induced in all forms of matter of whatever shape, and currents result when the electricity moves as the magnetic

flux links or unlinks with it. In electric conductors, such currents may be whirls of electricity called eddy currents. These are of little consequence from an energy standpoint, when the magnitude of the magnetic flux changes infrequently, and when quick mechanical response is not important, as in direct-current electromagnets. However, in alternating-current electromagnets, where the magnetic flux is constantly changing in both magnitude and direction, the energy loss may be considerable and the heat developed must be disposed of. This phenomenon is most troublesome in the cores of alternating-current electromagnets.

Secondary currents (through the m.m.fs.) tend to reduce the production of magnetic flux by the primary current, that is, they flow in a direction opposite to the primary current and have a demagnetizing effect. Therefore, when a given maximum magnitude of magnetic flux is to be maintained in the mutual magnetic circuit, more electric energy is abstracted from the primary circuit to supply any secondary energy output or losses than if such output or losses did not occur. In consequence, additional ampere-turns are required in a coil or solenoid when eddy currents or other secondary currents are present in the system, in order to supply the electric energy expended in such secondary circuits.

A solid iron or steel core of an electromagnet, a brass or other metal tube between the coil and the core, a closed metal tube surrounding the coil, or any metal placed in the same plane as the current in the coil, or at right angles to the direction of the magnetic flux, forms a closed secondary circuit with the coil acting as the primary.

21. General Forms of Energy.—The two general forms of energy are kinetic energy, or the energy of motion, and potential energy, or the energy of position. Kinetic energy is convertible into potential energy, and *vice versa*, as in the case of the hair spring and balance wheel of a watch. The kinetic energy stored in a moving body is equal to one-half the product of the mass and the square of the velocity of the body. Both forms may be expressed as force $\times$ length, that is, as the product of the applied force and the distance the body moves (regardless of its length or cross-section) while under the influence of that force.

All forms of energy are probably kinetic energy, potential energy, or combinations of kinetic energy and potential energy, but are given different names when all of the relations are not known or understood. For instance, an e.m.f. or voltage causes

electricity to be moved a given distance lengthwise of and within the structure of an electric conductor to form an electric current. The actual velocities of electric currents in metallic electric conductors are unknown because the number of coulombs of electricity per unit volume of conductor are unknown.

Voltage behaves like pressure $\left(\frac{\text{force}}{\text{cross-section}}\right)$, and current density $\left(\frac{\text{amperage}}{\text{cross-section}}\right)$ behaves like velocity. Hence, the amperage behaves like cross-section $\times$ velocity, and the electric power, equal to voltage $\times$ amperage, has the same value as mechanical power, that is, force $\times$ velocity, since cross-section cancels out. A current strength of one ampere in an electric circuit is a measure of the quantity of electricity per second to the same extent that one gallon per minute may express the time-rate of flow of water in a circulating system. It is known that electricity flows very slowly in electrolytes, and that voltages represent tremendous pressures, from actual observation. The quantity of electricity measured by means of an ammeter and a watch, that is, amperes $\times$ seconds, is the quantity of electricity that flows through any single plane in the cross-section of any conductor in a series circuit. It gives no basis on which to calculate the total quantity of electricity in coulombs in the entire electric circuit.

Kinetic energy = $\frac{1}{2} \times$ mass $\times$ velocity squared.

Magnetic energy = $\frac{1}{2} \times$ self-inductance $\times$ amperage squared.

Thermal energy = a number $\times$ mass $\times$ temperature.

Potential energy = force $\times$ length.

Electric energy = voltage $\times$ quantity of electricity.

There appears to be no doubt that the electric energy stored in the dielectric of a condenser is potential in nature. Likewise, magnetic energy has strong kinetic characteristics at the least. Thermal energy (heat) appears to be partly kinetic and partly potential in nature.

22. Power.—Time is required for all energy transferences, conversions, and storages. The greater the time-rate of flow, or conversion, or storage of energy the greater is the power. The unit of electric power is the watt or joule per second.

Under all circumstances, the electric power in watts at any instant is equal to the number of volt-amperes at that instant, when the magnitude and direction of each kind of power are separately considered. The time-rate of conversion of electric

energy into heat through the phenomenon of electric resistance is equal to the number of active volt-amperes; the time-rate of storage of electric energy in a dielectric through the phenomenon of capacitance is equal to the number of dielectric volt-amperes; and the time-rate of conversion of electric energy into magnetic energy through the phenomenon of self-inductance is equal to the number of self-induced volt-amperes.

23. Efficiency of an Electromagnetic System.—When an electromagnetic system accomplishes work through the conversion of electric energy into magnetic energy and thence into mechanical energy through the movement of one or more of the parts under the influence of the mechanical force thus produced, and if there are no energy losses, the magnetic circuit abstracts twice as much energy from the electric circuit as is converted into mechanical energy; one-half of the energy being converted into mechanical energy and the other half being stored as magnetic energy in the magnetic circuit. One joule = 0.7376 ft.-lb. = 10.198 kg.-cm., but only 0.3688 ft.-lb. or 5.099 kg.-cm. of mechanical energy can be obtained from 1 joule or one self-induced volt-coulomb of electric energy, since 0.5 joule of magnetic energy is stored in the magnetic circuit. When there are secondary-current and magnetic-hysteresis losses, an additional amount of electric energy is converted into magnetic energy to supply these losses.

For the foregoing reasons, wherever an electric current flows to produce magnetic energy through the phenomenon of self-induction, the maximum efficiency, defined as the ratio of the mechanical energy output to the electrical energy input, during the same time interval, cannot exceed 50 per cent unless the stored magnetic energy can again be reconverted into electric energy and returned to the electric circuit. Thus, while the maximum efficiency of a direct-current electromagnet is 50 per cent, that of an alternating-current electromagnet is 100 per cent, the maximum efficiency being the theoretical possibility.

During the conversion of electric energy into magnetic energy and thence into mechanical energy, the number of self-induced volt-amperes is double that required for the storage of magnetic energy alone, and the stored magnetic energy cannot be utilized in a direct-current electromagnet, that is, the stored magnetic energy is exhausted into dielectrics or converted into heat when the circuit is opened. In the case of the alternating-current

electromagnet, however, all of the magnetic energy stored during a quarter cycle could be returned to the electric circuit as electric energy during the succeeding quarter cycle if there were no energy losses due to resistance, secondary currents, and magnetic hysteresis. The same is true in the case of a unidirectional pulsating-current electromagnet, wherein the current strength increases from zero to a maximum and then decreases to zero, but does not reverse. Such pulsating currents are obtained by reversing the direction of the current during every other alternation of an alternating current.

24. Pressures and Forces.—Density of energy in space is the amount of energy per unit volume of said space and is identical with pressure, or force per unit surface or cross-section, when the cross-section is taken as any plane at right angles to the direction in which the pressure can act to tend to move a substance. Whenever there is an immediately available supply of energy, a mechanical force (pressure $\times$ cross-section or area) is in evidence. Thus, when the density of energy in an air gap between two magnetized bodies which mutually attract one another is 1 joule per cubic centimeter, the density of energy is also equal to 10.198 kg.-cm. per cubic centimeter, or 10.198 kg. per square centimeter. Hence, the pressure tending to move the two bodies together will be equal to 10.198 kg. for each square centimeter of cross-section of air gap. If this cross-section is, say, 2 sq. cm., the mechanical force of mutual attraction will be equal to $2 \times 10.198 = 20.4$ kg. $= 40.9$ lb.

Both e.m.f. and m.m.f. act more like pressures than forces. While a mechanical force can act to displace matter (thus causing the expenditure of mechanical energy) and an e.m.f. or voltage can act to displace electricity (thus causing the expenditure of electric energy), a m.m.f. cannot be said to displace magnetic flux in the general sense, but it does appear to cause magnetic flux to expand from the electricity in the structure of the conductor in the form of a continuous loop around (and within) the conductor.

25. Resistors, Reactors, and Condensers.—All commercial resistors contain small amounts of self-inductance and (sometimes much) capacitance, all coils and reactors contain some (often much) resistance and distributed capacitance, and all condensers contain some resistance and self-inductance. However, unless otherwise herein stated, a resistor will be assumed to contain only

resistance, a reactor only self-inductance, and a condenser only capacitance, meaning that other phenomena are present in such small percentages that they may be neglected, or else they are theoretically assumed to be absent. The distributed capacitance in high-voltage coils and in high-frequency coils is a most undesirable phenomenon.

In connection with alternating currents, a resistor is said to possess resistance, a reactor is said to possess inductive reactance, and a condenser is said to contain capacitance reactance, or condensance.

In general, a resistor limits the current strength because of the time required to convert electric energy into heat, and is independent of frequency. A reactor limits the time-rate of change in current strength because of the time required for the conversion of electric energy into magnetic energy, the inductive reactance (expressed in ohms) increasing as the frequency increases. A condenser can be instantly charged from a source of direct voltage in the absence of resistance and self-inductance, that is, the electric impulse will travel at the velocity of light, the condensance (expressed in ohms) decreasing as the frequency increases when alternating voltage is applied.

CHAPTER II

FORMS OF ELECTROMAGNETS

Specific conditions for producing mechanical forces by electromagnetic means are described and the general forms and types of electromagnets discussed. Then follows the general classification and a discussion of specific forms and their general characteristics and phenomena. A number of specific types are illustrated.

26. Possible Forms.—An electromagnet consists of any combination of electric conductor or conductors, with or without magnetic material present, so arranged that when electricity flows in one or more turns of conductor a mechanical force is established between at least two of its parts.

When two parallel conductors have electric currents flowing in the same general direction within their structures the resulting m.m.fs. join to produce magnetic flux; at least some of which shall mutually loop both conductors and thus produce a mechanical force tending to pull the two conductors together, and thereby increase the stored magnetic energy of the system by increasing the self-inductance. This force really tends to pull the two currents together but, since the currents are confined to the conductors, they also are moved when the currents are moved toward each other under the action of the mechanical force.

The turns of a loosely wound helix of flexible conductor can be seen to move together when a strong current flows in them in a common direction because, by so doing, the coil length and, therefore, the magnetic circuit, are shortened and the permeance is increased. More magnetic energy is then stored in the system, and the system is condensed into a smaller space, more magnetic flux being linked with the turns for a given current strength. Therefore, the permeance and the self-inductance are increased in the same proportion, and the density of energy (pressure) is increased.

When two parallel conductors, as a conductor and return conductor, have electric currents flowing lengthwise in their

structures in opposite directions, the m.m.fs. oppose one another. If the conductors can be separated, each m.m.f. can produce more magnetic flux around each respective conductor and thereby increase the self-inductance of the system. Consequently, there is a mechanical force of mutual repulsion between the currents in the conductors, which is directly proportional to the length-rate of increase in self-inductance that would result if the conductors should move apart, as they certainly would if free to do so. Hence, power cables are thrown from their supports when short-circuits occur.

In one of his papers,[1] Dr. Steinmetz has shown that, in the case where a quarter-phase alternator feeds through single-conductor cables spaced 6 in. apart with a normal full-load current of 910 amp. per phase, a momentary short-circuit current of 12 times the normal full-load current, or 10,900 amp., produces an average mechanical force of 10.8 lb. for each foot of length of cable, and this force pulsates between 0 and 21.6 lb. per foot of cable at double frequency. Hence, the cable is lifted from its supports and thrown aside.

Mechanical forces of this character are constant in direction, since the square of the amperage is positive whether or not the amperage is positive or negative, that is, the mechanical force does not alternate in direction.

Two coils placed end to end are mutually attracted when the current or currents in the turns flow in a common direction, but they are mutually repelled when the currents flow in opposite directions in the turns of the two coils, as when one coil is connected in an alternating-current circuit and the other is short-circuited by connecting the inside and outside terminals together, or else the latter consists of a hollow cylinder or a ring of solid electric conductor, such as copper.

A helix of small diameter is drawn into the core hole of a larger helix when the current or currents flow in a common direction in the turns, but mutual repulsion results when the currents flow in dissimilar directions in the turns of the helices or coils.

Steel cores are drawn into the core holes of solenoids because, by their moving into said core holes, the permeance and, therefore, the self-inductance of the system are increased.

When a freely suspended permanent magnet is placed near one end of a solenoid in such a relative position that the permeance

[1] Steinmetz, C. P., "Mechanical Forces in Magnetic Fields," *Trans.* A.I.E.E., vol. 30, p. 367, 1911.

and the self-inductance of the system will be decreased by its presence, the permanent magnet will first move away from the solenoid and then will swing around and move into the core hole of the solenoid.

In every conceivable case, mechanical forces due to electric currents linked with magnetic flux result only when the permeance or the self-inductance would change if the relative motion between any of the parts of an electromagnetic system should take place. In cases where the current strength is constant and the permeability (ratio of the magnetic induction to the magnetizing force) does not change during the movement, and no magnetic energy is expended in supplying energy losses, the average mechanical force in kilograms is equal to 5.1 times the product of (1) the square of the amperage and (2) the ratio of the increase in the self-inductance in henries to the length in centimeters traversed.

For instance, if the self-inductance of an electromagnetic system is 0.04 henry before the movement takes place and is 0.045 henry after the movable part has moved through a distance of 1 cm., the increase in the self-inductance is equal to 0.005 henry (5 millihenries). If the constant current strength is 10 amp., the average mechanical force during the movement is equal to $5.1 \times 100 \times 0.005 = 2.55$ kg. = 5.62 lb. The same rule holds for a decrease in self-inductance, but the mechanical force will then be one of mutual repulsion. When the unit of force is the pound and the unit of length is the inch, the conversion factor is 4.426 instead of 5.1. In the above example, 2.55 kg.-cm. = 0.25 joule of magnetic energy will be stored in the magnetic circuit due to the increase of 0.005 henry in the self-inductance, since $\frac{1}{2} \times 0.005 \times 100 = 0.25$ joule = increase in magnetic energy = 0.25×10.198 kg.-cm. of work performed = 2.55 kg.-cm.

A great variety of forms of electromagnets is possible. Imagine a pile of mixed bolts, nuts, nails, and washers made of copper, brass, iron, and steel. If a solenoid with a sufficiently strong continuous current flowing in its turns is placed endwise upon or near this pile, all of the iron and steel pieces will endeavor to get inside of the core hole, because there the magnetic field is the strongest. The materials having the greatest magnetic permeability will be the more strongly attracted, and all will tend to move towards the strongest portion of the magnetic field. Those unable to do so will cling to their fellows in such manners

as to form the shortest and broadest ferromagnetic circuit all around, as well as within, the outside of the solenoid. The non-magnetic pieces, as those of brass and copper, will remain inert and will take no part in the phenomenon, except that eddy currents may be induced in them when the magnetic flux is changing in magnitude. Eddy currents tend to cause the brass and copper to be repelled from the rest of the parts, particularly when the current alternates.

The tendencies of magnetic materials to get inside of the core holes of solenoids is taken advantage of when it is desired to pull a load through a distance of several inches, more or less, by electromagnetic means. Obviously, when it is desired merely to hold a large mass of magnetic material, the solenoid can be completely surrounded by magnetic material. For separating magnetic materials from non-magnetic materials, some intermediate form of electromagnet may be employed. In fact, all classes of electromagnets fall between these two extreme forms.

27. General Types.—There are two general types of electromagnet. An electromagnet designed for merely holding or

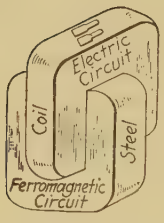


FIG. 1.—Linkage of electric and ferromagnetic circuits.

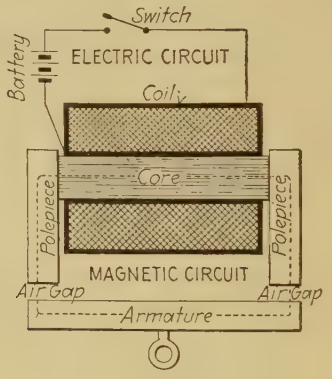


FIG. 2.—Electromagnet.

gripping some magnetic material is called a portative electromagnet, which usually must be operated with direct current, while one designed to move its armature through a distance and thus perform work is known as a tractive electromagnet.

Figure 1 illustrates a square-cored coil linked with a ferromagnetic circuit. This is one form of reactor. The electromagnet in Fig. 2 is developed from Fig. 1 by using four steel parts

instead of one. The heavy black lines surrounding the coil (shown in cross-section) represent electric insulation. With all of the pieces widely separated, the electromagnet may be assembled as follows: First, close the switch. Next, place the steel core near one end of the solenoid so that it will be drawn into the core hole. Then place one end of one of the polepieces near one end of the core, so that it will occupy approximately the position shown in Fig. 2. The polepiece will fly to the core end and will be firmly held thereto. A similar result will be obtained when the remaining polepiece is placed near the opposite end of the core. Each operation causes an increase in the linkage of magnetic flux with ampere-turns and, therefore, causes an increase in the permeance, the self-inductance, and the stored magnetic energy. But the electromagnet can do even more work.

If the armature is now placed near the polepieces, as in Fig. 2, it will be attracted to the polepieces and, unless too greatly opposed by some opposing mechanical force, as that due to friction, gravity, or a spring, it will jump to the polepieces, thereby striking them a sharp hammer blow, and will stick to them so firmly that it can be removed only with considerable effort. The armature is attracted to the polepieces because by changing its position relative to the polepieces it eliminates the air gaps still further, thereby increasing the permeance, the self-inductance, and the stored magnetic energy of the electromagnet. The discharge of the stored magnetic energy, through its conversion into electric energy when an electric circuit containing an electromagnet is opened without the current strength having first been reduced to zero, may be called the exhaust of the electromagnet.

After the electric circuit has been opened and the current has ceased to flow, there will be absolutely no attraction between the coil and the core, but there will be a general tendency on the parts of the core, polepieces, and the armature to stick together on account of the remanent magnetism which, while sometimes very feeble, persists in all commercial magnetic materials once they have been magnetized, particularly when they form a closed ferromagnetic circuit. Hence, this effect is most noticeable when the pieces are first pulled apart. If they are again placed together without turning on the current, it will be found that the tendency to stick together will be much reduced. Jarring, heating, and a reversed m.m.f. will cause further or complete demag-

netization. While the four pieces tend to remain magnetized after the electric circuit has been opened, but before they are pulled apart, they become demagnetized to a considerable extent when separated, since the continuity of the previously closed ferromagnetic circuit has been broken, and the molecules in each piece tend to approach their normally neutral positions or so to adjust their positions as to form the best magnetic circuit under the circumstances. Owing to the presence of hysteresis, the molecules do not swing entirely back to the positions required for complete demagnetization, and the magnetic flux leaks back from one side of the middle of each piece to the opposite side of the middle of the same piece, with the result that so much magnetic flux does not pass through the ends of the pieces as exists at their middles. The longer the piece, the greater is the remanence.

If an alternating current was employed in the above assembly, the core would be pulled into the core hole as described; but the pulls on the steel parts would be equal to zero at twice the frequency, and the resulting vibration would prevent the firm assembly of the parts and would cause a humming or chattering sound and the production of heat in the steel parts due to eddy currents and magnetic hysteresis. Furthermore, with constant r.m.s. impressed voltage, the r.m.s amperage would decrease with each movement of magnetic material into the system. If the r.m.s amperage was kept constant, the r.m.s. self-induced voltage would increase with each increase in the permeance and the self-inductance. There is little or no humming when the core is pulled into the core hole of the solenoid because the core strikes nothing, and its momentum gained during any half-cycle keeps it moving when the amperage is equal to zero. When the core is forcibly and rigidly restrained from entering the core hole, the usual humming and chattering may occur between the core and its support, or if the core is loosely attached to a mass to be moved with it, there may be much humming and chattering.

Referring again to Fig. 2, in which all of the parts except the armature are now assumed to be rigidly fastened together mechanically, the armature might be a magnetic load too hot to be lifted by the hand but not so hot as to cause it to lose its magnetic properties. Then the electromagnet might conveniently be lifted by hand and its polepieces placed on the piece of iron or steel to be moved. After closing the switch,

the combination could be moved to any desired place and the switch then opened, when the magnetic load would be released if sufficiently heavy to overcome the attraction due to remanent magnetism. Otherwise, the direction of the current must be reversed and the current made just strong enough to produce a m.m.f. capable of reducing the magnitude of the remanent magnetism to zero, or to a degree where the force of gravity will overcome the force produced by it.

In the above case, only the mechanical force of the electromagnet would be utilized. The electromagnet would perform no work except for the slight displacement which, naturally, would occur to bring the polepieces and the armature, or the magnetic load, into as close a mechanical contact as possible; but the work of moving the electromagnet and its load would be manual. Work is accomplished by an electromagnet only when motion results due to the action of a mechanical force between its parts.

When the electromagnet in Fig. 2 is to be used as a tractive electromagnet, the armature is normally at a distance from the polepieces, so that air gaps are introduced. Hence, the armature may be moved by the force of the electromagnet through a distance equal to the length of one air gap. The armature may be attached to a lever or to a mass of material which is to be moved with the armature. Examples of tractive electromagnets are those used in ordinary electric bells and buzzers, wherein the armatures are normally held away from the polepieces or core ends by means of suitable springs. The tractive electromagnet is a reciprocating device. Levers, toggles, and equalizers may be used in connection with electromagnets to adjust the mechanical-force characteristics, and the parts of the electromagnet may be so shaped and proportioned, or may be made to travel at such relative angles, that the stroke, range, or travel of the armature or other equivalent part or parts may be increased for a given uniform mechanical force, or for obtaining almost any mechanical-force characteristic relative to armature positions. However, alternating-current electromagnets are apt to transmit vibration through toggles and the like.

Tractive electromagnets of great range all operate either on the principle of oblique approach, that is, approach at an angle with the direction in which the maximum mechanical force acts, on the movement of a core or armature into the core hole of a

solenoid or between the polepieces of an electromagnet, or else upon some explosive or throwing action. On authority of *Bulletin No.* 1 *of the Naval Consulting Board of the United States,* "a magnet which can lift 20 tons, when placed in contact with an iron plate of that weight, will not lift a 2 in, cube of iron or steel if separated from it at a distance of 2 ft." Other forms of long-range electromagnets operate as straight-line motors, that is, they operate on the principles of electric motors but the parts move in a straight line instead of revolving.

Any electromagnet may be used as a portative electromagnet or as a tractive electromagnet but, in any case, the best results are obtained when the electromagnet is designed to perform the specific duty required of it. Some types of electromagnets are designed to move a load through a distance as a tractive electromagnet and then to hold the load for an indefinite period of time with the minimum expenditure of electric energy, thus acting as a portative electromagnet, without unduly heating while performing this function. Such an electromagnet may be called a tractive-portative electromagnet.

28. Classification of Electromagnets.—Electromagnets may be classified according to the following characteristics:

1. *The General Type of the Electromagnet.*—Whether the electromagnet is merely to grip or hold the load, or is to perform work by attracting or repelling a load through a distance.

Examples: Portative electromagnet.
Tractive electromagnet.

2. *The Nature of the Source of Electric Energy.*

Examples: Direct-current electromagnet.
Alternating-current electromagnet.

3. *The Manner in Which They Are Connected in Circuits.*—Coils of electromagnets may be connected in series with other apparatus requiring strong currents for their operation, or they may be connected directly across, that is, in shunt with, a line of relatively great voltage. However, the current and voltage characteristics of alternating-current electromagnets may be quite different from those of direct-current electromagnets. Series coils and shunt coils may be so arranged that their m.m.fs. will either partly or wholly neutralize or else reenforce one another, as desired; or currents developed from two or more

separate sources of electric energy may flow in as many coils in the same electromagnet, as in telegraph and other relays.

Examples: Series electromagnet.
Shunt electromagnet.
Current electromagnet.
Voltage electromagnet.
Balanced electromagnet.
Neutral electromagnet.
Differential electromagnet.

4. *Their Physical Forms or Shapes.*
Examples: Annular electromagnet.
Ring electromagnet.
Bar electromagnet.
Horseshoe electromagnet.

5. *Their Principal Structural Characteristics*
Examples: Laminated electromagnet.
Steel-clad electromagnet.
Plunger electromagnet.
Electromagnet with Z-armature.

6. *The Number of Phases of an Alternating-current Circuit.*
Examples: Single-phase electromagnet.
Two-phase electromagnet.
Three-phase electromagnet.

7. *The Constancy of Voltage or of Current Strength.*—In series electromagnets, the current strength may be constant and the voltage variable as the armature moves. The reverse is the case in shunt electromagnets.
Examples: Constant-current electromagnet.
Constant-voltage electromagnet.

8. *The Duty Which an Electromagnet Is Expected to Perform.*
Examples: Heavy-duty electromagnet.
Light-duty electromagnet.
Continuous-duty electromagnet.
Intermittent-duty electromagnet.
Momentary-duty electromagnet.

9. *The Range, or Length of Stroke.*
Examples: Long-range electromagnet.
Short-range electromagnet.

10. *Whether the Circuit is Normally Open or Closed.*—Such electromagnets are for either continuous, intermittent, or momentary duty.

Examples: Closed-circuit electromagnet.
Open-circuit electromagnet.

11. *The Direction of the Current.*—The addition of a separately excited electromagnet or a permanent magnet to an electromagnet causes the latter to respond to reversals in the direction of the current, unless the armature is biased, as by a spring.

Example: Polarized electromagnet.

12. *The Relative Polarities of the Parts.*—By properly arranging two or more coils with respect to the magnetic materials, either attraction or repulsion, or both, alternately, may be obtained. Hence, either direct or alternating currents will produce repulsion as well as attraction between the parts.

Examples: Repulsion electromagnet.
Push-pull electromagnet.

13. *Brief Duration of Energy Supply.*—Electromagnets for this purpose must operate on impulse, as on the energy discharged from the dielectric of a condenser, from the magnetic field of a reactor, or while two coils, one of which is energized are passing each other. They may be of the polarized type, or they may be of the tractive type with auxiliary contacts to close another circuit containing the same, or another, coil to hold the load after the initial impulse has ceased. In some cases an electromagnet is made to release its load under the influence of an electrical impulse.

Examples: Impulse electromagnet.
Tractive-portative electromagnet.
Quick-response electromagnet.
Quick-release electromagnet.

14. *Purposely Delayed Action.*—Slow action may be obtained by dash pot, oil pot, or bellows. Slow release may be obtained as above in some cases, but it is preferably obtained by placing a copper tube, or a short-circuited coil, around each core of a direct-current electromagnet. The latter should never be applied to an alternating-current electromagnet for this purpose. Such time-element features are easily obtained in alternating-

current devices through rotating disks, especially in control and protective apparatus.

15. *Combined Quick and Slow Response and Release.*—These may be generally classified as follows:

Examples: Quick-response, quick-release.
Quick-response, slow-release.
Slow-response, quick-release.
Slow-response, slow-release.

16. *The Specific Purpose for Which an Electromagnet Is to Be Used.*—The term *magnet* is often used where the term *electromagnet* is meant.

Examples: Track magnet.
Switch magnet.
Valve magnet.
Relay magnet.

No attempt is here made to classify electromagnets definitely, but rather to indicate the great variety of names which are given them.

29. Brief Classification of the General Types.—This is given in the chart in Fig. 3.

30. Portative Electromagnets.—This general type of electromagnet usually holds some kind of magnetic material as an armature. Therefore, the term *electromagnet*, or simply *magnet*, is given the electromagnet minus the armature.

Portative electromagnets are made in a variety of forms. In the accompanying illustrations the term *magnetic load*, meaning a load of magnetic material, is substituted for the word *armature*, although it is obvious that a non-magnetic load can be attached to an armature and moved in the manner previously described. In the accompanying illustrations the ends of the coil conductors are not always shown, as it is desired to show only the general forms of portative electromagnets as used for specific purposes.

The electromagnet in Fig. 4 is similar to that shown in Fig. 2. The polepieces are fastened to the ends of the core, although the core might be made longer and the ends bent over to form polepieces if the method was commercially practicable. Figure 5 shows a common application of this electromagnet to the low-voltage release on a direct-current motorstarter. To start the motor, the lever is gradually pushed over towards the electromagnet against the counterforce of a spring, in order to cut out

resistance as the armature of the motor accelerates, until the armature attached to the lever strikes the polepieces of the electromagnet when the mutual attraction between them is sufficient to overcome the counterforce of the spring, so long as the volt-

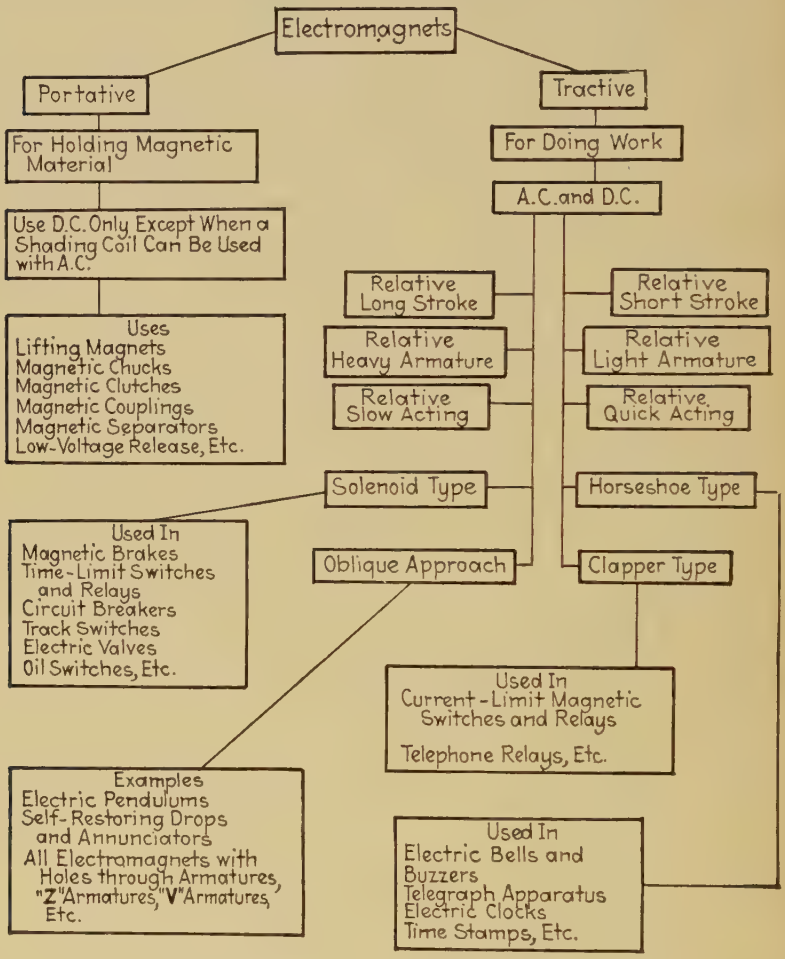


FIG. 3.—Brief classification of the general types of electromagnets.

age of the circuit is great enough to maintain the number of ampere-turns required to produce the necessary mechanical force. When the voltage is sufficiently reduced, due to any cause, the electromagnet can no longer hold the armature and lever against the counterforce of the spring, and they are forced back to their

original, or "off," position. Unless a thin piece of non-magnetic metal or other substance is placed between the ends of the pole-pieces and the armature, the remanent magnetism may prevent the release of the armature and lever when the voltage is

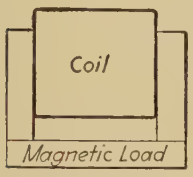


FIG. 4.—Portative electromagnet.

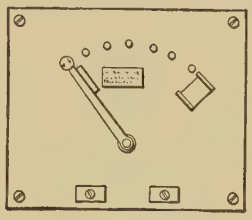


FIG. 5.—Use of a portative electromagnet on a motorstarter.

decreased, as when the line switch is pulled or the line voltage fails. This may be accomplished by heavily copper-plating the polepieces, the armature, or both.

Instead of placing the coil in the position shown in Fig. 4, it can be placed as in Fig. 6. The core on which the coil is wound, or upon which it is placed after being wound, usually is round (for direct currents), while the other parts of the electromagnet may be rectangular or round in cross-section, or of many other

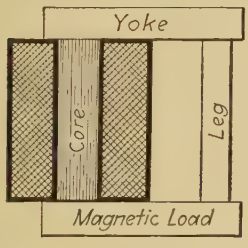


FIG. 6.—Modification of the electromagnet in Fig. 4.

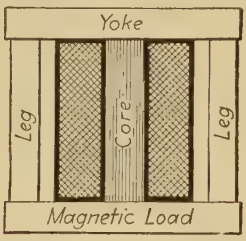


FIG. 7.—Portative electromagnet with two branches in its ferromagnetic circuit.

convenient shapes. Figure 7 shows a modification of the electromagnet in Fig. 6.

Obviously, if the yoke or "back iron" was in the form of a disk, the coil might be completely surrounded by "legs," or by a continuous shell of iron or steel. Figure 8 shows such an electromagnet in cross-section, wherein the core, yoke, and shell are cast integral (usually of cast steel), after which the insulated coil is placed in position and mechanically protected at the end

nearest the magnetic load by some non-magnetic material, or else by a very hard steel, as manganese steel, of very low permeability. Brass may be used for this purpose on small electromagnets. In the larger types, the space surrounding the coil usually is filled with some melting compound having good moisture-repelling and insulating properties.

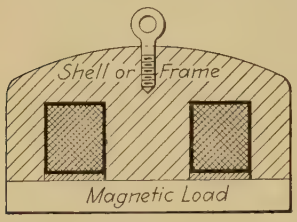


Fig. 8.—Portative electromagnet with a shell of magnetic material completely enclosing the coil.

The proportions of an electromagnet of the portative type are such that the mechanical force developed between the electromagnet and its armature or magnetic load shall be a maximum for a minimum current strength in the coil at constant direct voltage. The form illustrated in Fig. 8 is the basis of one form of lifting magnet.

There are two fundamental types of circular portative electromagnets. One may be described as a steel ring with a coil wound

Fig. 9.

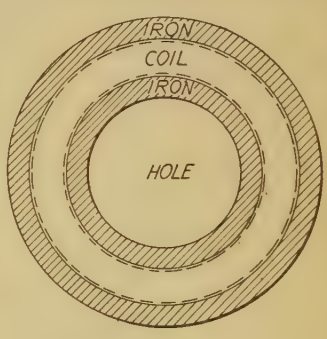


Fig. 10.

Fig. 9.—Toroidal electromagnet. Steel ring surrounded by coil. Magnetic circuit inside. Electric circuit outside. Small attracting surface. Long ferromagnetic circuit of small cross-section. Short length of wire per turn. Many ampere-turns required. High m.m.f. Low magnitude of magnetic flux.

Fig. 10.—Annular electromagnet. Ring coil surrounded by steel. Electric circuit inside. Magnetic circuit outside. Great attracting surface. Short ferromagnetic circuit. Great length of wire per turn. Few ampere-turns required. Low m.m.f. High magnitude of magnetic flux.

completely around it, while the other consists of a ring coil with magnetic material completely encompassing it. The former may be called a toroidal electromagnet or a ring electromagnet, and the latter an annular electromagnet. That shown in Fig. 8 is of the annular type. These two types are respectively shown

in Figs. 9 and 10. When the steel parts of these two types are made solid, that is, when there are no joints in their ferromagnetic circuits, practically no evidence of magnetic flux can be detected in the region surrounding the toroidal electromagnet, since the entire magnetic circuit is completely enclosed by the electric circuit, but there is still a magnetic field around the annular type which is entirely due to the electromagnetic flux, or that produced by the ampere-turns in the coil, as distinguished from the structural flux of the steel. These also form the bases of certain tractive electromagnets.

31. Tractive Electromagnets.—A tractive electromagnet is one designed to move its armature, with or without an attached load, through a distance or length and thus perform work.

In general, any form of movable-core electromagnet is commercially called a *solenoid*, but there are cases where the term plunger electromagnet is better applied to distinguish the forms

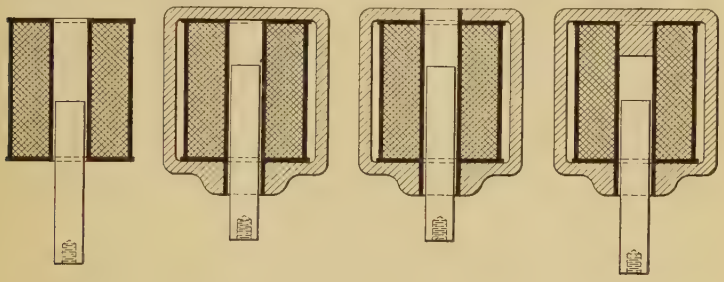


FIG. 11. FIG. 12. FIG. 13. FIG. 14.

FIG. 11.—Simple solenoid-and-plunger.
FIG. 12.—Iron-clad solenoid. Plunger travel limited by frame.
FIG. 13.—Iron-clad solenoid. Magnetic-cushion type.
FIG. 14.—Plunger electromagnet. Strong sealing pull.

from one another. The term *solenoid* is already applied to a coil, an electrically energized coil, and to any form of coil with a movable core regardless of the construction of the electromagnet.

When a solenoid, as in Fig. 11, is either completely or partially surrounded by a magnetic material to form a frame therefor to increase the permeance, or for mechanical protection, or both, as in Fig. 12, it is called an iron-clad solenoid, although the term *steel-clad* might better be applied when steel castings are employed. The iron or steel surrounding the coil may be in the form of a tube with washers of similar material forced into the ends of the tube, or it may be in the form of one or more U-shaped pieces.

The type shown in Fig. 13 is called the magnetic-cushion type, because the inner end of the plunger, while attracted into the coil due to the increase in permeance and self-inductance by its movement therein, is also attracted to the inner end of the steel frame at the end of the travel of the plunger without any mechanical hammer blow resulting. In the cases of alternating-current electromagnets, the parts must be laminated or slotted, according to the duty rating, in order to avoid eddy currents as much as possible.

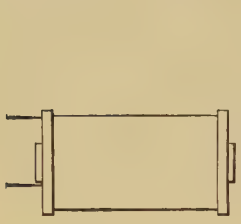


FIG. 15.

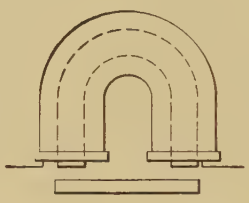


FIG. 16.

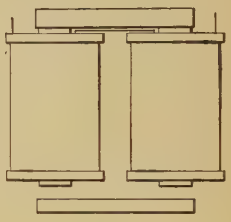


FIG. 17.

FIG. 15.—Bar electromagnet. Coil and core fastened together.
FIG. 16.—Semi-toroidal type of horseshoe electromagnet. Impracticable.
FIG. 17.—Common horseshoe electromagnet.

When a steel plug or stop is placed in the core hole of the solenoid, as in Fig. 14, the combination is called a steel-clad stopped solenoid or a plunger electromagnet. This type has a long range of travel of the plunger with a very strong pull at and near the end of the stroke. It is particularly adapted for first moving a load and then holding it against the counterforce of a spring or gravity. In other words, it is a good tractive-portative electromagnet. The pull is so much greater at and near the end of the stroke, for a given number of ampere-turns, that a load pulled by it from the initial position may be held at the final position with but a small proportion of the number of ampere-turns required to start and to move the load.

A solenoid with a core rigidly fastened therein, as in Fig. 15, is a bar electromagnet which may be thought of as a ring or toroidal electromagnet straightened out. This type is useful in surgery, as it will attract to either of its core ends bits of magnetic material. Commercial bar electromagnets form units which are combined with yokes, legs, cores, angle pieces, frames, shells, and the like of steel or iron to form various types of stationary-core electromagnets. For instance, in Fig. 16 the bar electromagnet in Fig. 15 is bent into U-form in order to form

a horseshoe electromagnet, but this form is impracticable from a manufacturing standpoint. Consequently, the form shown in Fig. 17 is the one usually employed. The employment of two coils on various electromagnets makes it possible to connect them in two different manners, that is, in series like the links of a chain, or in parallel, or in shunt, like the strands of a cable, so far as their circuits are concerned. The two-coil electromagnet in Fig. 17 is of the same general type as that shown in Fig. 6, except that only one coil is employed in the latter case.

When a good sealing pull is required electromagnets must have nearly closed ferromagnetic circuits. Good tractive electromagnets do not need to have nearly closed ferromagnetic circuits, especially in the cases of long-range electromagnets, unless it is also desired to have a good sealing pull at the end of the stroke when electric resistance is sometimes inserted in series with the coil of a direct-current electromagnet to reduce the current strength and thus prevent undue heating of the coil while the electromagnet is holding its load. The initial pull may require so many ampere-turns that the coil would burn out if left in circuit very long. The sealing pull of a tractive electromagnet is usually much greater than the initial pull, although, of course, the reverse is often the case, an example being the simple solenoid-and-plunger, wherein the pull at the final position of the plunger is equal to zero, since each half of the solenoid is mutually attracted by each opposite half of the plunger, with the result that the opposing mechanical forces are balanced and no available force is in evidence.

32. Direct-current and Alternating-current Tractive Electromagnets.—The abbreviations *d.c.* and *a.c.* are generally used for *direct current* and *alternating current*, respectively. The term *magnet* is generally used. Alternating-current portative electromagnets are not practicable except in special cases, because a pulsating mechanical force moves or holds the load by a series of jerks, 120 times per second when connected in a 60-cycle circuit. Hence, only tractive electromagnets are herein discussed, wherein an effective or r.m.s. unit may refer to a direct-current as well as to an alternating-current phenomenon, which is assumed to be of single-phase.

An alternating-current magnet will operate better in a direct-current circuit than in an alternating-current circuit, and is usually better constructed than a corresponding direct-current

magnet, because the conditions imposed for alternating current demand it. Theoretically, direct-current magnets should follow alternating-current magnet design. Practically, alternating-current magnets should follow direct-current design when they are to be energized only for brief and infrequent time intervals, and where cheapness is an important factor.

The effective mechanical force characteristics of any given alternating-current magnet are approximately the same whether the current is alternating or direct, provided the r.m.s. current strength is made somewhat greater (often about 20 per cent) than the direct or continuous current strength, as indicated by an alternating-current ammeter, to allow for the demagnetizing effects of eddy currents and hysteresis.

One r.m.s. alternating ampere-turn produces magnetic flux of the same r.m.s. magnitude as that produced by one continuous ampere-turn under otherwise similar conditions when saturation is absent. Consequently, in an alternating-current magnet having a constant number of turns, the same number of r.m.s. ampere-turns would produce an effective mechanical force approximately equivalent to that due to an equal number of continuous ampere-turns were it not for the fact that some of the electric energy is converted (through magnetic energy) into heat in the core, frame, and other metallic parts through eddy currents and hysteresis, the latter being confined to the iron or other magnetic parts. Therefore, these components of the total number of r.m.s. alternating ampere-turns are not available for producing useful mechanical force. In consequence, more r.m.s. ampere-turns are required for a given useful mechanical force in a magnet connected in an alternating-current circuit than are required for an equivalent constant, non-pulsating mechanical force when the magnet is connected in a direct-current circuit.

Eddy currents can be eliminated to a considerable extent by laminating the core (in particular), frame, and other metal parts in such a manner that, wherever there is a tendency for eddy currents to be produced, the paths which the eddy currents would naturally take shall be cut across so as to prevent their flow as much as practicable. Figure 18 is an end view of the magnet in Fig. 4, with the core, polepieces, and armature laminated to reduce the production of eddy currents as much as practicable. These steel laminations have high permeabilities but, since the eddy currents tend to flow at right angles to the magnetic flux

paths, the eddy currents are greatly reduced. The steel parts of an alternating-current magnet are usually stamped from thin sheets, called electrical sheets and, after being varnished, are riveted together, or else are welded together under pressure in such a manner that the laminations shall be firmly fastened together along one edge only of the part, thus producing a solid structure without forming a closed circuit for the eddy currents, except in each individual lamination.

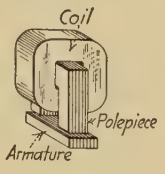


FIG. 18.—Method of laminating the ferromagnetic circuit.

While direct-current magnets can be built in almost any size, and will operate successfully with currents which can easily be controlled, except when the voltage is extremely low and the current strength is proportionately great, the sizes in which alternating-current magnets can be constructed are limited in many ways. Armatures or plungers of alternating-current magnets are heated from the instant the electric circuit is closed until it is opened, whether or not the armature or plunger is in motion, whereas eddy-current and hysteresis losses take place in direct-current magnets only when there is a change in the current strength or in the self-inductance or permeance, although they tend to retard the quickness of response and release of the magnet. Usually, the eddy-current and hysteresis losses of alternating-current magnets are smaller than the heat losses in the coils due to resistance, when the armatures, cores, and frames are laminated. However, when the air gap is closed, the reverse may be the case. Laminated cores or plungers occupy more space (have larger cross-sections) than the equivalent amount of magnetic material in a solid core or plunger, thus causing the average girth of the coil to be greater under otherwise similar conditions, as where square cores are used, and laminating mechanically weakens the structures of the parts.

Solid steel cores in alternating-current magnets of the movable-coil type are apt to become very hot. In some clapper and horseshoe types the cores may be of large cross-section and made of cast iron, in which case the flux density may be relatively low and the resistance of the cast iron will be relatively high, thus tending to reduce the strengths of the eddy currents, and ample cooling surface can be provided. Solid cores and metal tubes should always be slotted.

For a given r.m.s. current strength in an alternating-current magnet, the higher the frequency the greater will be the eddy-current and hysteresis losses, because the energy transformed into heat per second due to hysteresis is proportional to the frequency, and that due to eddy currents is proportional to the square of the frequency, in cycles per second. The self-induced voltage also is directly proportional to the frequency. Therefore, while a 110-volt, 30-cycle alternating current magnet will operate quite well on a 220-volt, 60-cycle alternating-current circuit, the eddy-current and hysteresis losses will be greater when the frequency is 60 cycles per second than when it is 30 cycles per second. For a given r.m.s. current strength, the r.m.s. self-induced voltage at 60 cycles will be double that at 30 cycles. If the resistance of the coil is relatively low, the same magnet can be operated in either circuit, since the r.m.s. self-induced voltage will nearly balance the r.m.s. impressed voltage in both cases.

33. Duty Ratings.—A continuous-duty electromagnet is one designed to operate at the full volt-ampere rating for an indefinite period of time. The rating in volt-amperes of an intermittent-duty electromagnet is such that its insulation would be destroyed by the temperature produced due to heat losses if it should operate at its full volt-ampere rating for, say, more than 10 min. during any half hour. A momentary-duty electromagnet may be operated at its full volt-ampere rating for the time specified, which may be 1 or 2 min. during any half hour. The ratings of the last two electromagnets are arbitrary, and different time intervals may be specified.

34. Current-limiting Phenomena.—The inductive reactance of a circuit, coil, electromagnet, or a reactor is equal to 6.2832 times the product of the frequency in cycles per second and the self-inductance in henries. When the frequency is 60 cycles per second the inductive reactance is equal to 377 times the self-inductance in henries. When the frequency is 25 cycles per second the inductive reactance is equal to 157 times the self-inductance in henries. Also, 6.2832 times the frequency in cycles per second is the angular velocity, so the inductive reactance in ohms is equal to the product of the angular velocity in radians per second and the self-inductance in henries.

All coils contain some resistance. The square root of the sum of the squares, that is, the geometric sum, of the inductive reactance and the resistance (both in ohms) is the impedance of

a circuit, coil, electromagnet, or a reactor when the capacitance may be neglected, as it usually can in connection with electromagnets unless condensers are purposely introduced or the frequency is abnormally high.

The ohm or r.m.s. ampere per r.m.s. impressed volt is the practical unit of impedance.

When no resistance or capacitance is present (as may be assumed when they are relatively small), the r.m.s. alternating amperage is equal to the ratio of the r.m.s. impressed alternating voltage to the inductive reactance in ohms, and the r.m.s. alternating self-induced voltage (almost) exactly opposes the r.m.s. alternating impressed (or total) voltage.

When both self-inductance and resistance are present, and the capacitance may be neglected, the r.m.s. alternating amperage is equal to the ratio of the r.m.s. alternating impressed (or total) voltage to the impedance in ohms.

When only resistance is present, as when there is no voltage due either to inductive reactance or condensive reactance, the r.m.s. amperage (either alternating or direct) is equal to the ratio of the r.m.s. impressed (or total) voltage (either alternating or direct) to the resistance in ohms.

35. Series Tractive Electromagnets.—Current electromagnets are normally connected in circuits in series with some other device, as a motor, and may be controlled through the switch controlling the device or devices with which they are in series, or they may be normally short-circuited and cut into a circuit by opening a switch. An example of an application of a series magnet is a magnet brake with the coil of the magnet connected in series in the circuit containing the motor with the brake attached. When the circuit is closed, the magnet pulls back the brake shoes and holds them until the circuit is opened, when the brake is released and brakes the motor through the pull of gravity on a mass or by means of a spring. In what follows, alternating currents are assumed to be of single phase.

Alternating-current series magnets designed for use in constant r.m.s. current alternating-current circuits are provided with about 20 per cent more turns of conductor than would be used if they were to be connected in constant-current, direct-current circuits of equivalent amperage and for equivalent duties, in order to provide for eddy-current and hysteresis losses. Therefore, aside from the mechanical construction, there is not

much difference between alternating-current and direct-current series magnets designed for similar duties. They are wound with approximately the same number of turns of conductor and their self-inductances, permeances, and resistances are approximately equal. Consequently, the effective mechanical-force characteristics of alternating current magnets are nearly the same whether obtained with r.m.s. alternating current or with direct current of nearly equal strengths.

Under equivalent conditions, direct-current series magnets are as quick-acting (when laminated) as alternating-current series magnets, but there is the important difference that, while the voltage of a constant-current direct-current magnet is constant regardless of the actual magnitude of the self-inductance, so long as it is constant, the self-induced voltage of a constant r.m.s. current, alternating-current magnet increases with the magnitude of the self-inductance and the permeance, because the instantaneous current strength is continually changing. The self-induced voltage is equal to the product of the self-inductance and the time-rate of change in the current strength. Consequently, the self-induced voltage of a given constant r.m.s. current, 60-cycle magnet is double that which would result if the same magnet was connected in a 30-cycle circuit and the same r.m.s. current strength flowed in its coil, and also is greater when the armature is in its final position than when it is in its initial position.

Long-range alternating-current series magnets must be well-insulated to withstand the increased voltage when the armature or plunger is in its final position where the self-inductance becomes a maximum.

36. Shunt Tractive Electromagnets.—Alternating-current phenomena are herein assumed to be of single phase. Voltage electromagnets are connected in shunt with the supply circuit. When the resistance of the coil is small and the capacitance effects need not be considered, the r.m.s. self-induced voltage of an alternating-current shunt magnet is always approximately equal, but opposed, to the r.m.s. impressed or total voltage. This is because the r.m.s. current strength in the turns of such an alternating-current shunt magnet is equal to the ratio of the r.m.s. self-induced voltage to the inductive reactance in ohms, and the r.m.s. self-induced voltage is equal to the product of the r.m.s. amperage and the inductive reactance in ohms. Consequently,

when the self-inductance changes, due to a change in the position of the armature towards its final position, the inductive reactance increases and the r.m.s. amperage decreases; the product of the self-inductance and the amperage tending to remain constant and equal to the ratio of the r.m.s. impressed voltage to the angular velocity in radians per second. For instance, in a 60-cycle circuit the angular velocity is 377 radians per second. Consequently, the maximum product of the self-inductance in henries and the r.m.s. current strength in amperes in a 110-volt, 60-cycle shunt electromagnet is equal to $\frac{110}{377} = 0.292$ henry-amp.

Alternating-current shunt magnets are designed to hold their loads at the final position without overheating. However, when the armature is in the initial position the current strength may be relatively great, and in case the load does not move when the current is switched on, the coil may burn out, due to excessive current. The heating of the coil varies directly as the product of the resistance and the square of the current strength.

Nevertheless, alternating-current shunt magnets are automatic in their actions, producing strong mechanical forces in both their initial and final positions if properly designed, and the current strength is reduced as the load moves, under the action of the mechanical force, towards the final position where the r.m.s. current strength is sufficient to hold the load but is not so great as to cause overheating. Under similar conditions, a protective switch must be employed to insert a resistor in the circuit of a direct-current shunt magnet when the armature arrives at the final position, so as to limit the current strength.

Large, long-range alternating-current shunt magnets permit the flow of alternating currents of such great r.m.s. strengths when their armatures or plungers are in the initial positions that their use is often prohibitive. On the other hand, the strength of the current in a corresponding direct-current shunt magnet is limited only by the resistance of the coil and not by the position of the armature or plunger when not in motion.

It is necessary to use a conductor of relatively great length and small cross-section in order that the coil of a direct-current shunt magnet shall contain sufficient resistance to limit the current to such a strength that the coil shall not heat too rapidly, since resistance limits the current strength by converting electric energy into heat in the coil conductor. Hence, the coil of a direct-

current shunt magnet contains many more turns than does an alternating-current shunt magnet designed for similar duty and r.m.s. voltage, with the result that the self-inductance of a direct-current shunt magnet is very much greater than that of an otherwise similar alternating-current shunt magnet. In a specific case, it was found that an alternating-current shunt magnet designed for use in a 110-volt, 60-cycle circuit produced a similar effective (but non-pulsating) mechanical force under similar conditions when a voltage of 12 volts (continuous) was impressed upon the coil.

The continually changing magnetic flux in an alternating-current shunt magnet links and unlinks with the turns of conductor, thus producing a r.m.s. self-induced voltage which almost equally opposes the r.m.s. impressed voltage. The number of r.m.s. weber-turns is equal to the number of r.m.s. henry-amperes (0.292 for a 110-volt, 60-cycle circuit), but the maximum number of weber-turns is equal to 1.41 times the number of r.m.s. henry-turns, being 0.413 for a 110-volt, 60-cycle circuit. As the self-inductance increases the r.m.s. amperage decreases in the same proportion, so that their product is nearly constant, the active voltage having but little influence in most cases. Hence, the number of flux-turns (but not necessarily the magnitude of the magnetic flux, as in the case of a solenoid or plunger electromagnet) is always approximately constant in an alternating-current shunt magnet containing very little resistance, for a constant impressed voltage, regardless of the relative positions of the parts of the magnet; whereas in direct-current shunt magnets there are self-induced voltages only when their armatures are in motion or when the current strength is otherwise varied, and the magnitude of the magnetic flux varies directly with the self-inductance and the permeance when the current strength is constant, increasing as the armatures approach their final positions.

The self-inductance of an alternating-current shunt magnet must be much less than that of a direct-current shunt magnet for similar r.m.s. impressed voltage and duty, because, while a direct-current shunt magnet is allowed time for the continuous voltage to accelerate the electricity in the coil and to establish the number of ampere-weber-turns necessary to develop the required mechanical force through the storage of magnetic energy, an alternating-current shunt magnet must store sufficient magnetic energy to develop the required mechanical force

and pick up its load many times more quickly than must the direct-current shunt magnet, unless both are required to perform their functions in equal time intervals. This is because the actual instantaneous current must increase from zero to a maximum strength at a rate equivalent to four times the frequency of the alternating voltage, and also must develop the maximum mechanical force corresponding to a maximum number of ampere-weber-turns at a rate equivalent to quadruple frequency, or at the rate of 240 times per second in the case of a 60-cycle magnet, since the current and flux increase only during the first half of any alternation, decreasing during the other half. Again, the effective mechanical force characteristics of alternating-current shunt magnets, for a short time after the circuit is closed, depend upon whether the circuit is closed when the instantaneous impressed voltage is equal to zero, a maximum, or between the two extremes.

Consequently, in the case of a 60-cycle magnet, the electricity in the coil must be accelerated in $\frac{1}{240}$ sec. to produce the required mechanical force. When the self-inductance increases, as when the armature moves towards its final position, the electricity cannot be accelerated so quickly with the same value of the alternating voltage. Hence, the maximum (and r.m.s.) current strength is decreased. A direct-current magnet designed for otherwise similar effective voltage and duty produces very little mechanical force when connected in an alternating-current circuit of that effective voltage, because there is not sufficient time for the electricity to be accelerated by the impressed voltage to form a current strength sufficient to store the necessary magnetic energy, or the necessary number of ampere-weber-turns, in the magnetic circuit on account of the excessive self-inductance of the direct-current magnet. Conversely, the coil of an otherwise similar alternating-current shunt magnet would develop an abnormal current strength which would cause an abnormal mechanical force and cause the coil soon to burn out.

When an alternating-current shunt magnet has approximately the same number of ampere-weber-turns as a similar direct-current shunt magnet designed for similar effective voltage and mechanical force, the number of amperes is greater and the number of turns is smaller in the former than in the latter. In the alternating-current shunt magnet the number of amperes decreases as the permeance and self-inductance increase and,

therefore, the magnitude of the magnetic flux tends to remain constant; whereas, in the direct-current shunt magnet, the number of ampere-turns is constant when the impressed voltage is constant and the armature is not moving, regardless of the magnitudes of the permeance and the self-inductance.

Due to the above reasons, the mechanical-force characteristics of alternating-current shunt magnets are often very different from those of direct-current shunt magnets. The alternating-current shunt magnet is inherently a quick-acting magnet. Owing to the rapid accelerations of armatures due to the mechanical forces, the sudden arresting of the motions of armatures, as when they strike core ends, produces a strong counterforce or hammer blow at the point of impact. This not only causes noise, but also hardens the steel and thus tends to cause sticking unless precautions are taken to prevent it.

37. Electromagnets with Movable Cores.—The cores or plungers are assumed to move in brass tubes in the following illustra-

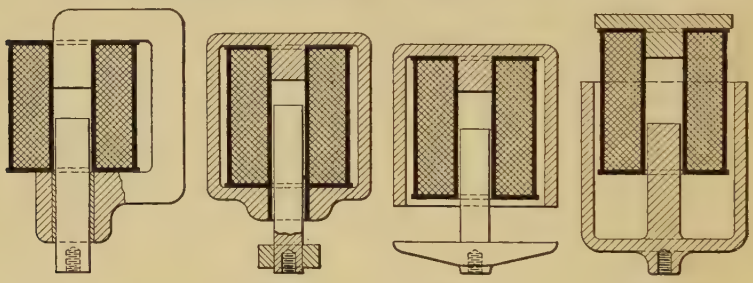


FIG. 19. FIG. 20. FIG. 21. FIG. 22.

FIG. 19.—Plunger electromagnet with open frame.
FIG. 20.—Plunger electromagnet with collar on plunger.
FIG. 21.—Plunger electromagnet with movable end.
FIG. 22.—Plunger electromagnet with movable shell or frame.

tions (Figs. 19 to 30), which are intended to show the evolution of the various types. Figures 11 to 14 belong to this group.

38. Electromagnets with Stationary Cores.—Electromagnets of this general class are used where strong pulls through short distances or where relatively weak but rapid actions, are required. They may be divided into single-core electromagnets and poly-core electromagnets. Where the cores extend throughout the entire lengths of the coils and are rigidly fixed thereto, there is no available pull between any coil and core in electromagnets of this general class. All types of electromagnets with external

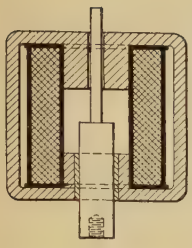


Fig. 23.

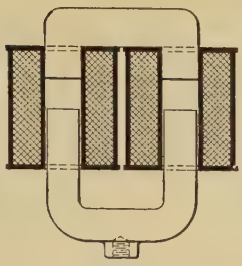


Fig. 24.

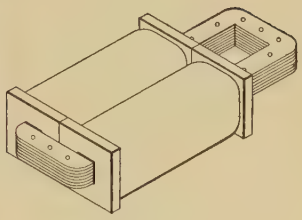


Fig. 25.

Fig. 23.—Tubular iron-clad electromagnet.
Fig. 24.—Two-coil plunger electromagnet.
Fig. 25.—Two-coil plunger electromagnet with laminated U-shaped cores. Alternating-current type.

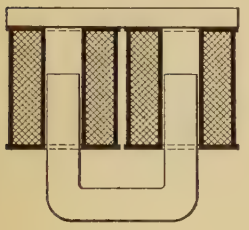


Fig. 26.

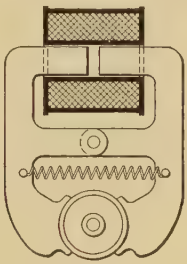


Fig. 27.

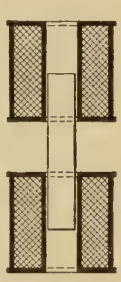


Fig. 28.

Fig. 26.—Two-coil plunger type with yoke.
Fig. 27.—Double-acting solenoid for brakes.
Fig. 28.—Differential solenoid.

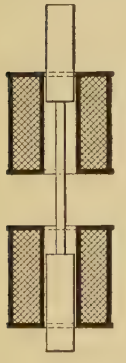


Fig. 29.—Differential solenoid.

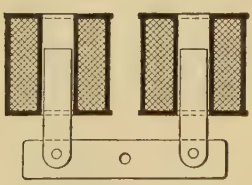


Fig. 30.—Differential solenoid. Balanced plungers.

armatures may be said to have stationary plungers and movable portions of frame. Single-core types are illustrated in Figs. 31 to 41, while Figs. 42 to 46 show some polycore types in the schematic form. Many polycore electromagnets have coil spaces,

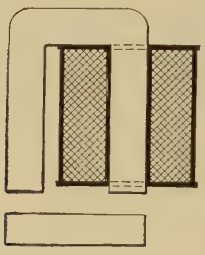


FIG. 31.—One-coil horseshoe electromagnet.

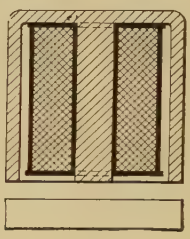


FIG. 32.—Iron-clad type.

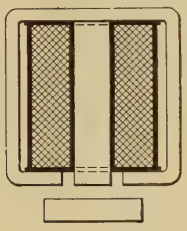


FIG. 33.—Open-frame type.

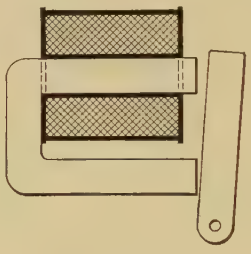


FIG. 34.—Clapper type.

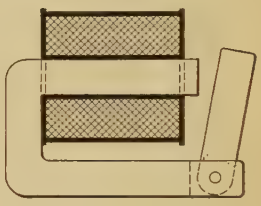


FIG. 35.—Clapper type.

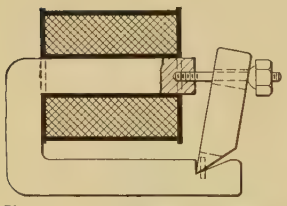


FIG. 36.—Modified-clapper type.

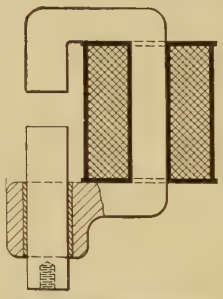


FIG. 37.—Slidable-armature type.

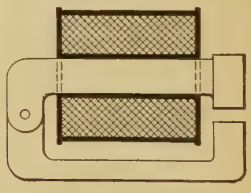


FIG. 38.—Hinged type.

commonly called slots, but the magnetic material between said slots really forms cores in the general sense. These also are called legs or limbs. All electric generating and motor equipment consists of various forms of electromagnets. Polycore

Fig. 39.—Telephone-relay type.

Fig. 40.—Telephone-relay type.

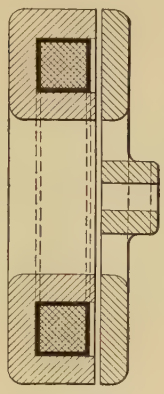


Fig. 41.—Clutch type.

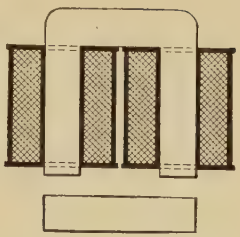


Fig. 42.—Two-coil horseshoe electromagnet.

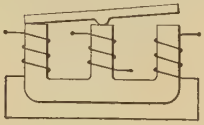


Fig. 43.—Polarized type.

Fig. 44.—Magnetic-chuck type.

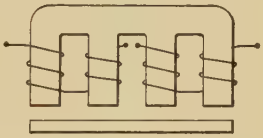


Fig. 45.—Two-phase type.

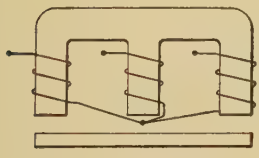


Fig. 46.—Three-phase type.

Fig. 47.—Rotary solenoid.

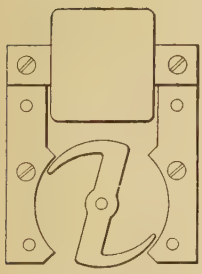


Fig. 48.—Z-armature type.

Fig. 49.—Rotary core and armature.

Fig. 50.—Spiral-frame type.

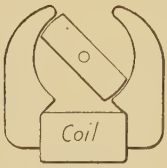


Fig. 51.—Rotary type.

electromagnets are used as both portative and tractive electromagnets. Polyphase electromagnets depend upon the principle.

FIG. 52.—Rotary armature.

FIG. 53.—Z-armature type.

FIG. 53.—Pivoted-armature type.

FIG. 55.—Phonic wheel. The magnets alternately pull on the wheel.

39. Rotary Electromagnets.—These belong to the rotary motor class, but they do not rotate more than 180 deg. Figures 47 to 55 show a number of forms. Figure 43 belongs to this class.

CHAPTER III

MECHANICAL FORCES OF ELECTROMAGNETS

The general pull characteristics of common forms of constant-current electromagnets, as well as methods of varying these characteristics, are shown and described. The hammer blow, methods of its prevention, and some special phenomena are also described and explained.

40. Constant-current Electromagnets.—The mechanical-force characteristics of both alternating-current and direct-current electromagnets are about the same when the number of effective ampere-turns is constant in any particular case and the effective

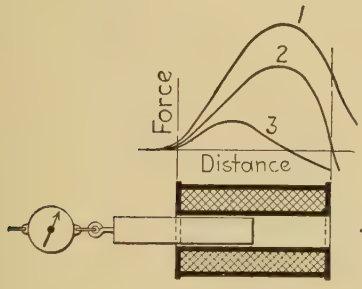


Fig. 56.—Solenoid-and-plunger.

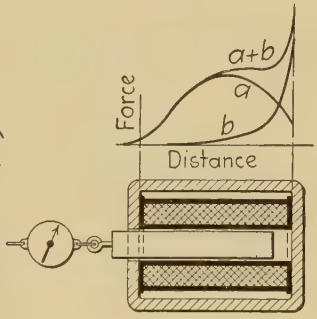


Fig. 57.—Iron-clad solenoid.

pull, as measured with a spring scale, is taken. In what follows, only the armature is assumed to be movable. These characteristics also hold for direct-current, constant-voltage electromagnets when the armature is at rest. The mechanical force under such circumstances is called the static pull.

In Fig. 56, curve 1 is due to a plunger longer than the solenoid; curve 2 results when the plunger is of the same length as the solenoid; while curve 3 is due to a plunger shorter than the solenoid.

In Fig. 57, curve *a* is the pull due to the solenoid and plunger, and curve *b* is the attraction between the entering end of the plunger and the inner surface of the steel frame towards which it

moves. The sum of the two pulls ($a + b$) is the total pull due to an iron-clad solenoid.

The mechanical-force characteristics of the plunger electromagnet in Fig. 58 are similar to those of the iron-clad solenoid, except that the presence of the stop or plug shown at the right in Fig. 58 somewhat increases the air-gap pull and, since the air-gap pull is added to the solenoid-and-plunger pull at a different place than in the case of the iron-clad solenoid, the total-pull characteristics are changed.

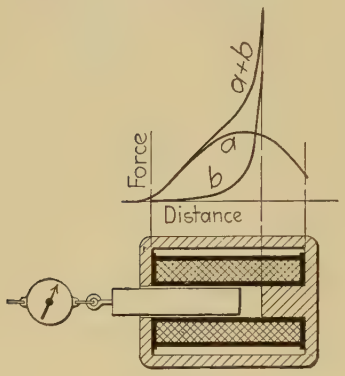


FIG. 58.

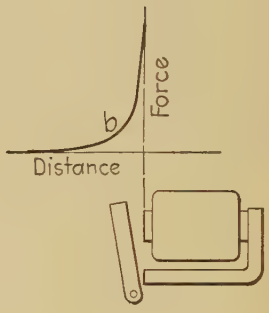


FIG. 59.

FIG. 58.—Plunger electromagnet.

FIG. 59.—Clapper-type electromagnet. Here there is very little attraction between the coil and armature, and the force distance curve is similar to that of curve *b*, Fig. 58.

41. The Air-gap Pull.—Theoretically, the pull through an air gap varies directly as its cross-section and inversely as the square of its length. In this it is assumed that the entire m.m.f. acts to produce magnetic flux in the air gap. Practically, the pull varies directly as the cross-section of the air gap, and inversely as the square of the following quantity—the length of the air gap plus an additional length—to allow for the reluctance of the ferromagnetic material and any useless air gaps, as well as the leakage of ferromagnetic flux around the air gap or that which does not pass through it. If the actual pull followed the theoretical law, the product force $\times$ length of air gap would be directly proportional to the reciprocal of the length of the air gap. However, by allowing additional ampere-turns to overcome the additional reluctance and leakage effects, what is here called the theoretical law holds for the air gap only.

The pull between the coil and the armature, when the latter is outside of the coil, is very small, as may be understood by referring to curve *b*, Fig. 58. Furthermore, the farther the core end projects from the coil, the greater is the magnetic leakage from the core end to various parts of the frame and core of opposite polarity. Hence, strong pulls can only be obtained through relatively short distances with electromagnets having external armatures, and the pull increases very rapidly with respect to slight decreases in the length of the air gap. A typical force-distance curve for an air gap between a flat-faced core and a flat-faced armature is shown in Fig. 59. The pull varies directly as the cross-section of the air gap. When the ends of the air gap have different cross-sections or surfaces, the average may be taken.

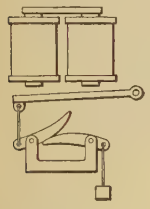


FIG. 60.—Equalizer.

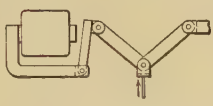


FIG. 61.—Toggle equalizer.

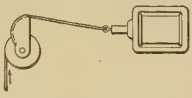


FIG. 62.—Cam-and-cable equalizer.

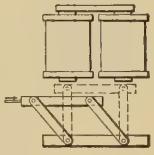


FIG. 63.—Electromagnet with linkage.

42. Varying the Force-distance Characteristics.—In some cases toggles, cams, or bell cranks are employed in connection with electromagnets in order to obtain force-distance characteristics different from those exhibited by the electromagnet alone. Such mechanical devices either increase the applied mechanical force but correspondingly decrease the range of travel; increase the range but correspondingly decrease the mechanical force; or else they are employed to make the force approximately uniform throughout the entire range of motion. A few examples are shown in Figs. 60 and 63. Simple levers are also employed in some cases. However, it is undesirable to use any auxiliary device unless absolutely necessary.

Other methods involve the shaping and proportioning of coils, plungers or armatures, or air gaps. Figure 64 shows four methods of making the mechanical-force characteristics of solenoids with plungers more uniform. These were patented by W. R. Michl in 1894. The Patent Office records show many methods of shaping and proportioning the parts for various purposes. The method (*d*) of winding the magnet wire far away from the core is

not recommended. The general object is to increase the number of ampere-turns at the places where the natural pull is relatively small.

In the simple solenoid-and-plunger, tapering the ends of the plungers causes the maximum pull to occur beyond the farther end of the solenoid instead of within its limits. Combinations of tapered plungers and asymmetric solenoids, patented by I. A. and E. W. Timmis, are shown in Fig. 65. Asymmetric coils also are shown in the Patent Office records for use with plunger electromagnets of the single-coil and two-coil types.

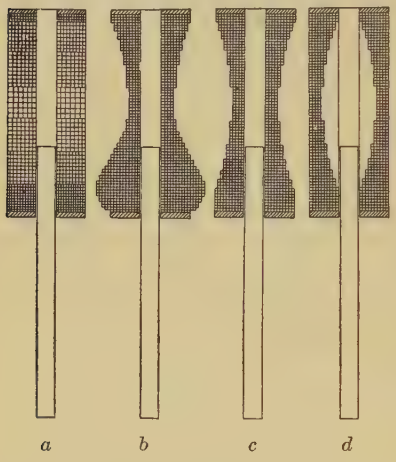


FIG. 64.

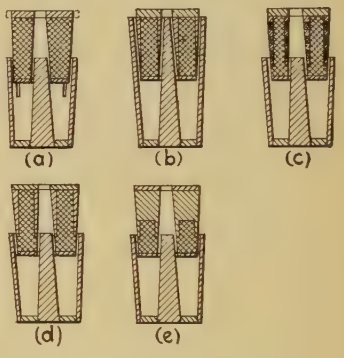


FIG. 65.

FIG. 64.—Four methods of changing the force-distance characteristics of solenoid-and-plungers.

FIG. 65.—Five methods of changing the pull characteristics of the electromagnet in Fig. 22.

Asymmetric coils heat more rapidly where the current density (amperes per unit cross-section of conductor) is purposely increased in order to increase the number of ampere-turns in a given space for the purpose of increasing the pull at that place. Consequently, the various means of changing the pull characteristics are generally mechanical in nature, to the extent that the iron or steel parts are so arranged and proportioned that the desired results may be obtained with uniformly wound coils. Varying the position of the air gap with respect to the coil in a plunger electromagnet or an iron-clad solenoid makes possible a wide variation in the pull characteristics, particularly in constant-voltage alternating-current electromagnets.

The pull through the air gap of a constant-voltage alternating-current electromagnet was at one time said to be constant. This was based on the fact that the number of flux-turns in a constant-voltage alternating-current electromagnet is constant. Practically, the solenoid pull and the reluctance and magnetic leakage so alter the former theoretical conditions that it was necessary to abandon that theory altogether and adhere to the actual results obtained. However, the discrepancy is explained in Chap. XXII. The trouble was that the theorists did not separate the flux due to the current in the coil from that originating in the steel.

Since the mechanical force is directly proportional to the rate at which the self-inductance and the permeance change with respect to corresponding changes in position of the armature, any force-distance curve also represents the length-rate of change in the self-inductance and the permeance of an electromagnet when the proper factors are introduced.

In solenoids, it is desirable to make the leakage a maximum, as by enclosing the solenoid within a steel shell, so that there shall be the maximum linkage of the magnetic flux that leaks from the plunger with the ampere-turns in the coil.

For electromagnets with external armatures, it is desirable to make the magnetic leakage as small as possible in order that the maximum amount of magnetic flux shall pass through the air gaps.

In plunger electromagnets an average of the two conditions is desired, the maximum air-gap pull being obtained when the middle of the air gap is at the middle of the solenoid. But the maximum pull will be obtained when the air gap is between the middle of the solenoid and the farther end of the solenoid.

43. The Hammer Blow.—In general, quick-acting electromagnets produce hammer blows when the movements of their armatures and loads are suddenly arrested. Hammer blows result when heavy armatures and loads are released and fall to their initial positions or are returned by means of springs. Thus relatively great mechanical forces may be obtained through short ranges by means of the hammer blow, which also is useful in overcoming starting friction or to accelerate a load which may be then handled by the electromagnet. In the latter cases, the plunger or armature is allowed to travel a short distance before it picks up the attached load. However, this constant hammering in electromagnets is detrimental to the apparatus and, unless

such apparatus is especially designed to withstand it, means must be provided to reduce it or to prevent it altogether.

There are three general methods of overcoming the hammer blow, which is proportional to the time-rate of reduction in the velocity of the armature or plunger in any given case. One method is to reduce the velocity of the plunger, as in time-limit devices. Another is to provide a spring or else a floating stop to absorb the kinetic energy; or it may be damped out by friction. The third general method is so to arrange the electromagnet that the armature or plunger can strike nothing at the end of the

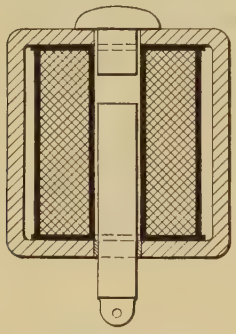


FIG. 66.—Plunger electromagnet with floating stop.

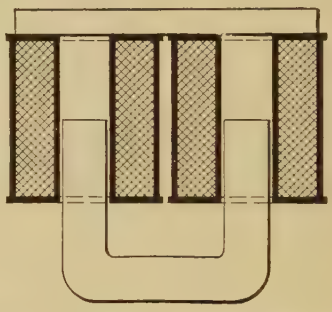


FIG. 67.—Two-coil plunger electromagnet with common floating stop.

stroke. Such an electromagnet is said to be of the magnetic-cushion type. Springs also are employed to absorb the shocks on the return stroke. Various forms of shock absorbers may be applied.

44. Electromagnets with Floating Stops.—The upper pieces of magnetic material in Figs. 66 and 67 are floating stops. When the plungers strike these stops they move upward with the plungers, thus absorbing the kinetic energy, after which they drop by gravity to the positions shown but with the plungers in magnetic contact with them.

45. Increasing the Initial Pull.—Several cases are here illustrated. All operate on the principle of oblique approach or else on the principle of the magnetic shunt. Where a flat-faced armature is attracted by a flat core end, the reluctance of the air gap decreases as the armature moves towards the core end. Since permeance is the reciprocal of reluctance, the permeance increases at a very high rate as the armature approaches the core end, so that the pull for a given number of ampere turns

(squared) varies approximately as the inverse square of the distance between the core end and the armature.

In cases where it is desired to obtain a more uniform pull throughout the range of travel of the armature, portions of the armature or core are so arranged that the distance between the coacting parts shall be small at the initial position of the armature in order that a strong initial pull may be obtained, and then a portion of the magnetic flux is increasingly shunted at an angle to the direction of travel of the armature as the latter moves towards the core end. In this manner, the magnetic energy is expended throughout a greater distance, and the resulting average pull is correspondingly less. The principle is equivalent to a magnetic lever.

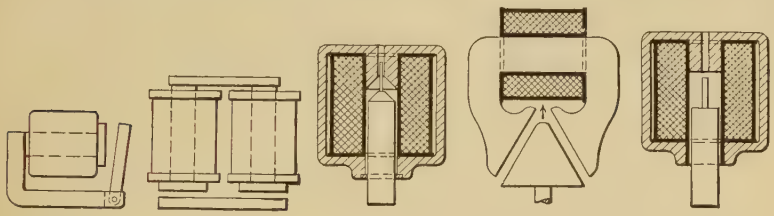


FIG. 68. FIG. 69. FIG. 70. FIG. 71. FIG. 72.

FIG. 68.—Enlarged core end. A steel washer increases the cross-section of the air gap, thereby increasing the initial pull.
FIG. 69.—Horseshoe electromagnet with enlarged core ends.
FIG. 70.—Electromagnet with coned plunger and stop.
FIG. 71.—Tapered armature and polepieces.
FIG. 72.—Plunger electromagnet with magnetic shunt.

In general, the initial pull through an air gap is increased by increasing the cross-section and decreasing the length of the air gap. However, unless the total permeance of the electromagnet is increased when the above method is applied, there will be no gain. For instance, in the electromagnets in Figs. 98 and 99 the reverse is the case, that is, apparent repulsion ensues. Methods of increasing the cross-sections of air gaps with flat ends are illustrated in Figs. 68 and 69. Cone-shaped air gaps are shown in Figs. 70 and 71.

Various methods of increasing the initial pull by the magnetic-shunt principle are illustrated in Figs. 72 to 80, which also show some specific electromagnetic devices.

46. Magnetic Cushions.—Figure 81 illustrates a form of electromagnet long used on interrupters for large induction coils. The armature is free to travel between the polepieces of the

electromagnet, but is prevented from moving sidewise by suitable mechanical means. When this type is used with interrupters, it is desirable that the armature shall oscillate to and fro

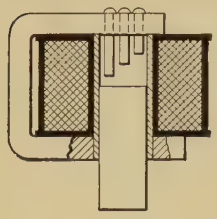


FIG. 73.

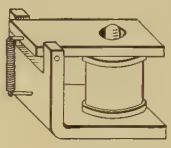


FIG. 74.

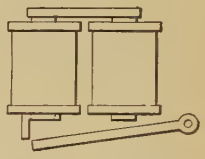


FIG. 75.

FIG. 73.—Plunger electromagnet with a series of slidable magnetic shunts.
FIG. 74.—Magnetic shunting by means of a hole through the armature.
FIG. 75.—Increasing the initial pull of a horseshoe electromagnet.

between the positions indicated by full and dotted lines. The mass of the armature and its lever or mounting bears such a

FIG. 76.—Signal mechanism operating on the magnetic-shunt principle.

relation to the tension or compression of the spring or springs attached to it that it will oscillate at a predetermined frequency based on the time required for properly operating the induction

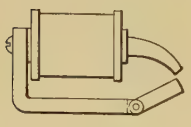


FIG. 77.—Indicator mechanism.

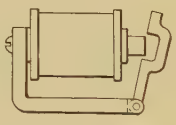


FIG. 78.—Principle of the self-restoring drop.

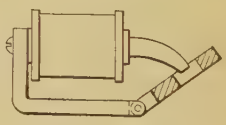


FIG. 79.—Principle of the horn drop.

coil due to the relation between its inductance and resistance, that is, its time constant.

In its operation the armature is attracted by the electromagnet until it is midway between the polepieces, where its velocity is a maximum. The electric circuit is then automatically opened and the momentum of the armature and its mounting is so great that it travels against the counterforce of a spring to the dotted

position, when its motion ceases and the spring returns it to the initial position.

In this general type of electromagnet, if the electric circuit is closed and then is not opened for some time, the armature will at first tend to oscillate between the polepieces, but the surplus mechanical energy is soon damped out by friction. While in

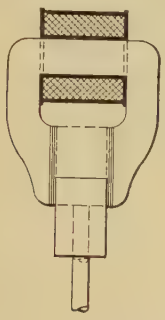


FIG. 80.—Combination of magnetic shunt and magnetic cushion.

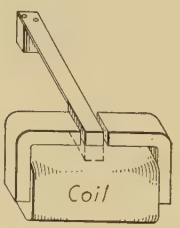


FIG. 81.—Vibrator mechanism.

this position, the armature is said to float in the magnetic field and, if the armature is equally spaced on both sides from the polepieces, it is equally attracted in opposite directions and there is no side pull, and it can be operated with alternating current without producing any noise, as the only tendency is for it to vibrate slightly up and down.

Advantage is taken of this principle in one form of alternating-current telegraph sounder. When the electric circuit is closed, the armature is attracted to the position of maximum permeance between the polepieces, but the momentum of the armature and lever carries it beyond this position, so that the usual stop screw which passes through the lever strikes the usual stop beneath (thereby producing the desired sound), from which it rebounds and is held floating noiselessly in the magnetic field between the polepieces until the electric circuit is opened, when the armature and lever are forced upward against the opposite stop, where the complementary sound is produced in the usual manner.

This general principle has many applications, of which are the electric pendulum (Fig. 82), the electric balance wheel (Fig. 83), and the electric tuning fork, all of which operate on the principle of harmonic motion and wherein the electric circuit is to be interrupted when the permeance of the system becomes a

maximum. In a general way, this principle is the basis of alternating-current hair clippers and vibro-massage apparatus, in which latter cases it is unnecessary to interrupt the electric circuit.

The principle of the magnetic cushion has been applied to a motor for operating phonographs and other devices. In this instance, the turntable contained a number of laminated armatures

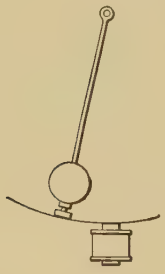


Fig. 82.—Electric pendulum.

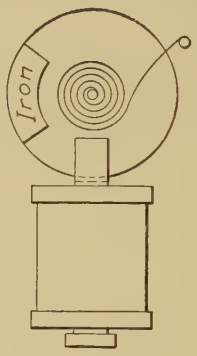


Fig. 83.—Electric balance wheel.

on its under side which successively passed between the polepieces of the electromagnet, the latter attracting each armature in succession to the position of maximum permeance and then releasing it, as the electric circuit was interrupted, until the next armature came within its field, due to the angular momentum of the turntable, when the operation was repeated. In a general way, all electric motors operate on this principle, except that repulsion as well as attraction takes place and the parts are radially arranged. The stored magnetic energy is not wasted in the latter cases, however. Some simple rotary types are shown in the preceding illustrations.

An application of the magnetic cushion to the iron-clad solenoid is shown in Fig. 13. While this form is a good tractive electromagnet with a fairly uniform pull, it is not a very good portative electromagnet.

47. Methods of Increasing the Sealing Pull.—The term *sealing pull* refers to the portative properties of a tractive electromagnet after the armature has completed its stroke and it is to hold the load against the counterforce of a spring, gravity, compressed air, and so forth. In general, this pull is made a maximum by reducing the lengths of the parts and the air gaps and making the cross-

sections of the parts and all joints a maximum, so that the electromagnet shall contain the maximum ratio of flux density squared to the area of contact.

While the attracting areas of cores, armatures, and plungers should be made large for tractive electromagnets, the holding force of a portative electromagnet is increased by decreasing at least one portion of the attracting area in many cases. This principle is illustrated in Fig. 84. About the same amount of magnetic flux passes through both core ends, but the area of contact at the rounded end is less than that at the flat end. With the square of the flux density the same in both cases, the pull at the rounded end is the greater, since the pull is proportional to $\frac{\text{flux density}^2}{\text{area of contact}}$.

The equivalent of a strong sealing pull is obtained by the auxiliary use of toggles, as in Fig. 85. There are many other ways of accomplishing similar results, as by the use of cams.

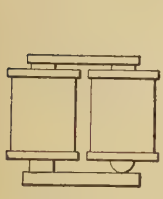


FIG. 84.

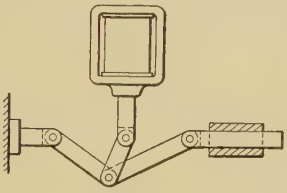


FIG. 85.

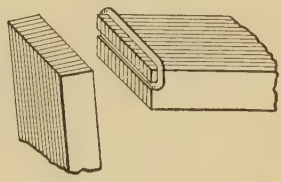


FIG. 86.

FIG. 84.—The greater pull is at the rounded core end.

FIG. 85.—Application of a toggle for producing the equivalent of a strong sealing pull.

FIG. 86.—Principle of applying a shading coil.

In alternating-current electromagnets, especially of the single-phase class, shading coils are used to prevent the armature from chattering against polepieces or coacting stops or plugs after it has completed its stroke. Figure 86 shows one method of applying a shading coil to one of the coacting members of an electromagnet. The object of the shading coil is to split the phase so that there shall be the equivalent of a two-phase effect at the place where the parts meet, thus assuring some pull between the parts at all times, even when the current strength in the main coil is equal to zero. Toggles may be used in connection with alternating-current electromagnets provided with shading coils. In all cases, the coacting faces of the electro-

magnet should be smooth and flat with respect to each other, and the load should be kept below the full pulling capacity.

48. Pushing Electromagnets.—By extending some non-magnetic material from the armature or plunger of an electromagnet, a pushing effect may be obtained. The general principle is illustrated in Figs. 87 and 88. In any case, the mechanical construction should be such that there will be no binding of the parts.

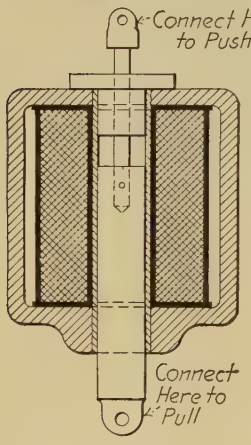


FIG. 87.—Pushing electromagnet.

49. Reversing Mechanisms.—These usually are employed for

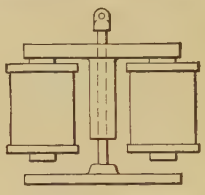


FIG. 88.—Pushing electromagnet.

operating double-throw switches, but sometimes are used to operate small mechanical clutches. In the case of the single-coil reversing mechanism in Fig. 89, gravity acts on a mass to pull the lever beyond center, so that it will fall over in such a position that the plunger will project from the opposite end of the solenoid at each successive throwover.

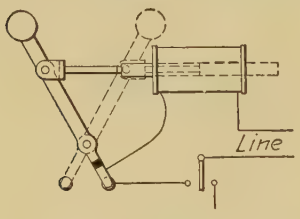


FIG. 89.—Gravity reversing mechanism.

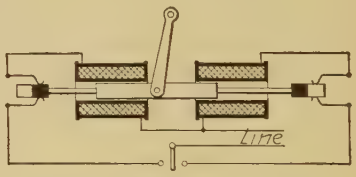


FIG. 90.—Two-coil reversing mechanism.

In the case of the two-coil reversing mechanism (Fig. 90) the solenoids alternately act to pull a common plunger in opposite directions. In all such cases, the movement of the armature causes the controlling circuit to be opened. Reversing mechanisms also may be of the polarized type.

50. Long-range Electromagnets.—The solenoid-and-plunger is the representative type. In the Marcel Deprez electromagnet, a series of short solenoids are placed end to end and a relatively short plunger is attracted within them and carried through them by successively energizing groups of solenoids by means of a commutator-like switch, as in Fig. 91. The shorter the individual coils and the greater their number, with a corresponding increase in the number of segments, the smoother is the action of the device and the more closely may the plunger be positioned with respect to any of the coils. The plunger may be moved to any desired position and its direction of motion under the influence of the coils may be reversed. One series of the coils may be used to move the plunger quickly to a stop at one end, where

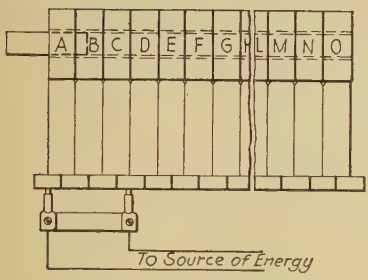


Fig. 91.

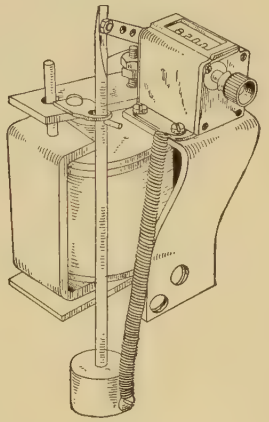


Fig. 92.

Fig. 91.—Connections of Marcel Deprez electromagnet.
Fig. 92.—Counter operated by kinetic energy stored in a mass by means of an electromagnet.

another group of coils containing more turns and resistance may hold the plunger in position. An objection to this form of direct-current electromagnet is the arcing or sparking at the commutator-like switch. Electromagnetic cannons, operating on the above general principle, have been suggested. This type of electromagnet has also been suggested for operating drop hammers and pile drivers. Since its efficiency cannot exceed 50 per cent, it has never proved of much value except in small devices where efficiency is not of much importance. Economy and convenience often are the prime requisites. The electromagnet

may be made in the semi-rotary form. Straight-line alternating-current motors will probably supplant electromagnets of this general type.

Any form of electromagnet which stores kinetic energy in its moving member, to tend to keep it in motion, is a long-range electromagnet. The kinetic energy thus stored may be gradually expended over a relatively great distance, something after the manner of the expenditure of magnetic energy over a much smaller range. Electromagnets of this general class are of the quick-acting type.

The electromagnet in Fig. 92 could not possibly operate the counter if the armature was directly connected to the latter. However, by storing kinetic energy in the mass at the lower end of the vertical rod, this energy is expended over a greater distance as the mass continues to move after the armature has completed its stroke. The mass is returned to its initial position by the potential energy stored in the spring.

51. Repulsion Electromagnets.—These exert the maximum mechanical force at the beginning of the stroke and may be made to move loads over relatively great distances. Mechanical forces exist wherever the self-inductance and the permeance of

FIG. 93.—Repulsion electromagnet.

the system could be improved if any of the parts could move and, therefore, more magnetic energy could be stored in the magnetic circuit. The coils of a horseshoe electromagnet, for instance, are acted upon by a mechanical force to move them further back against the yoke in the absence of an armature. However, if the polarity of one of the coils is reversed, they will tend to move towards the core ends. Referring to Fig. 93, the two coils are at the middle of the bar (which may have a laminated structure) and the self-inductance of the system cannot be much improved by any further movement of the coils towards each other. However, if the coils are so connected as to have opposed polarities, the self-inductance and the permeance will be greatly increased if the coils are widely separated. This is due to the fact that the m.m.fs. oppose each other when the coils are adjacent to one another, and thus tend to prevent the production of magnet flux. Consequently, when the two coils in Fig. 93 are

energized, the current attains its maximum strength very quickly, because there is then very little self-inductance in the system. Each coil then explosively moves to its respective end of the bar, where each coil and core end constitute separate electromagnetic systems with the maximum self-inductance and permeance obtainable under the circumstances.

This arrangement is particularly suitable for alternating currents, since the current strength is reduced as the self-inductance (and the inductive reactance) increases as the coils separate. When a direct current is employed, means must be provided to introduce a resistor in the circuit after the electromagnet has performed its function, or else the circuit must be opened to prevent the coils from burning out. Neither coil will move beyond the end of the bar when a current is flowing. This electromagnet produces a strong mechanical force, and the range may be made great by adding mass to the moving parts so that the quickly stored kinetic energy may be expended over a relatively long range of travel; but it is not suitable for continuous duty unless the duty is relatively light. Increasing the mass of the moving part increases the time of storage for the greater amount of kinetic energy required.

In the case of an alternating-current device of this kind, a wound coil may be rigidly fastened to the bar and a hollow cylinder of solid copper employed as the moving member, since the secondary currents induced in it will flow in a direction opposite to that in the primary or stationary coil. Resistance need not be inserted at the end of the stroke in cases where alternating currents are employed, because the self-inductance (and the inductive reactance) reduces the current strength as the former increases when constant r.m.s. voltage is impressed upon the primary coil.

The above phenomena take place in the static transformer when short circuits occur between some of the turns. The induced current of enormous strength flows in a direction opposite to that in the adjacent turns which act as a primary coil, with the result that the transformer simply explodes unless ample mechanical protection is provided to prevent it. This statement refers particularly to those large transformers which are connected in systems of practically unlimited power. According to Dr. Steinmetz, the nearest approach to the appearance of a large transformer which has been wrecked due to the above cause is

that due to a head-on collision between two locomotives. Short-circuited turns should be avoided in all forms of alternating-current electromagnets, except for specific purposes, and they may also be detrimental to the performance of direct-current electromagnets in some cases.

If stationary coils are placed at each end of the bar in Fig. 93 and a movable coil is placed between them, the movable coil (solid copper for alternating current) may be made to travel to and fro between them by reversing the current. The coils at the respective ends of the bar should have opposed polarities, so that one will tend to attract (as the other repels) the movable coil. Constant-current transformers operate on this general principle.

Electromagnets of this general class are useful for throwing objects upward, said objects performing some function over a relatively long range of action on their downward travel. Such actions may be referred to as pile-driver action, drop-hammer action, and so forth, or as time-limit action when a ball is thrown upward by an electromagnet and caused to travel over a given distance on its descent on an inclined plane, which may be spirally arranged.

In rotary electromagnetic apparatus, the combined repulsion and attraction produce rotation. A copper ring placed on the core end of a vertical alternating-current bar electromagnet is hurled upwards or else it slides sidewise, depending upon the mechanical force developed. If pivoted above the core end, it will turn so that its axis is at right angles to the direction of the magnetic flux, the term *axis* referring to the usual axis of a turn or coil. If the bar electromagnet lies in a horizontal position and the ring is placed near, but not concentrically with, one of the core ends, the ring will tend to slide sidewise towards the neutral-polarity plane (middle) of the electromagnet. In all such cases, the self-inductance and the permeance are increased by the change in the position of the ring. It is impossible to strike the polepiece of a large direct-current electromagnet a sharp blow with a copper disk about $\frac{1}{16}$ in. thick and about 10 in. in diameter, due to the secondary current induced therein by the movement. Likewise, when an attempt is made to remove the disk suddenly an inertia is noticeable. These effects were described by Dr. Elihu Thomson.

Various rotary and other effects are produced by the mutual repulsion of ferromagnetic parts placed inside of core holes of coils. Examples are shown in Figs. 94 and 95. A bundle of soft-iron wires loosely placed within the core hole of a coil will spread apart when the coil is electrically energized. Unless laminations are securely riveted, welded (on one edge), or otherwise made firm, the mutual repulsion between these parts when an alternating current flows in the coil, regardless of the instantaneous direction of the current, will cause vibration and produce

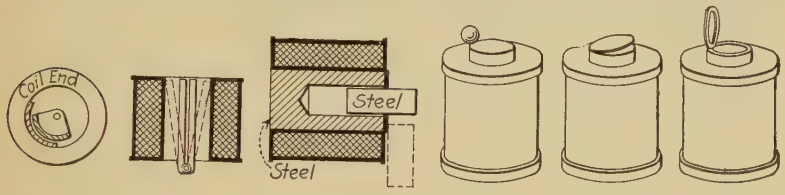


FIG. 94. FIG. 95. FIG. 96. FIG. 97. FIG. 98. FIG. 99.

FIG. 94.—Rotation due to repulsion of parts of similar polarity.
FIG. 95.—Repulsion mechanism.
FIG. 96.—Repulsion due to increase in permeance.
FIG. 97.—The ball tends to remain at the edge of the core.
FIG. 98.—The steel disk will not lie flat on the core end.
FIG. 99.—Position assumed by steel disk or ring.

humming, and may cause wear on adjacent insulating materials. The energy expended in causing loose laminations or other parts to vibrate is similar to the magnetic hysteresis loss, the latter causing a structural vibration (of the molecules) while the former causes a bodily vibration.

The following phenomena are due to magnetic circuit conditions only, that is, they are not due to any changes in the current strength. A plunger is drawn within the core hole of a solenoid because, by its change in position, the number of ampere-weber-turns is increased. However, if the solenoid is rigidly mounted on a thick soft-steel tube and a soft-steel plunger is loosely placed therein with a tube of non-magnetic material between the steel tube and the plunger, as in Fig. 96, the plunger will move out of the steel tubing and will hang as shown while the solenoid is energized. By so moving, the plunger forms a better path for the magnetic flux than would be the case if it remained within the steel tube, that is, the permeance is increased.

The flux density is greatest at and near the edges of a free core end, because less flux leaks from the center than from any other point on the surface of the core end, and also because the

flux density is greatest on sharp edges. This may be proved by suspending a small soft-steel or iron cube or ball from a spring scale and measuring the force required to detach it at different points on the surface of the core end.

Interesting experiments are easily made by placing a steel ball on the flat surface of a core end, as in Fig. 97, in which case the ball will always be attracted to the edge, or by attempting to place a soft-steel disk on the core end, as in Fig. 98. In the latter case, the disk will assume the position shown in Fig. 99, which illustrates the same experiment tried with a hollow steel core and a steel ring.

CHAPTER IV

TIME AND POLARITY CHARACTERISTICS

In the present chapter methods of making electromagnets quick-acting, sluggish, or reversible are discussed. Means for preventing sticking and sparking also are considered.

52. Single-phase and Polyphase Electromagnets.—A single-phase electromagnet is the common single-circuit form of alternating-current electromagnet which may be connected in an ordinary two-wire electric lighting circuit. Single-phase electromagnets are exceedingly noisy in their actions, particularly when attempts are made to use them as portative electromagnets or when the air gap of an electromagnet of this class is approximately closed so that the armature can stroke the polepieces, stop, or other parts. When the air gap is very short, this noise can be overcome to some extent by introducing a shading coil in one of the coacting parts, as in the armature or in the ends of the polepieces, or stop, which causes the single-phase electromagnet to establish a local magnetic flux between the ends of the abutting steel pieces when the main-coil current and the main magnetic flux are passing through zero. Thus, a pull always exists between such parts when the main electric circuit is closed. The shading coil is said to split the phase.

In a general way, a phase may be thought of as a wave. Alternating currents, voltages, and magnetic fluxes have periodic magnitudes, varying from zero to positive and negative maximums in each case. When any of these waves do not pass through zero at the same instant, there is said to be a difference in phase between them. Thus the voltage wave and the current wave are always in phase when there is only resistance in the electric circuit, and always are out of phase, that is, there is a phase difference between them, when there is only self-inductance in the electric circuit. Two men may be walking at the same velocity and both may be taking exactly the same number of steps per minute. However, they may be out of step, in which case there is a difference in phase between the movements of their feet.

In a similar manner, the current wave may lag behind the voltage wave when there is self-inductance in the circuit, although the frequency of the voltage is the same as that of the current. This is due to the momentum of the electricity, which tends to cause it to continue in motion in the direction in which it is moving when the impressed voltage is equal to zero. Consequently, time must elapse after the impressed voltage is reversed before the impressed voltage can attain a sufficient magnitude in the reversed direction to stop the motion of the electricity, and then cause it to move in the direction in which the impressed voltage is then acting. When only self-inductance is present, the impressed voltage is a maximum when the current strength is equal to zero, and *vice versa.*

A polyphase electromagnet is one designed for use in an alternating-current circuit having more than one phase. In general, the term *polyphase* applies to successive phases of the voltage, or of the current strength, or of the magnetic flux, rather than to the phase relations between voltage, current strength, and magnetic flux. At least three line conductors and at least two separate coils must be used in polyphase systems.

A two-phase electromagnet is one designed to be connected in a two-phase (or else a quarter-phase) alternating-current power circuit. In principle, a two-phase electromagnet consists of two single-phase magnets combined in one. When the instantaneous current strength and the magnitude of the magnetic flux are equal to zero in one portion of a two-phase magnet, the current and flux are at or near their maximums in the other portion of the same magnet. Therefore, one or the other portion of a two-phase magnet always exerts a pull on the common armature, and most of the time they are pulling together.

A shading coil causes a single-phase magnet to behave like a two-phase magnet, but the voltage and current of the shading coil are induced therein by the constantly changing magnetic flux in the steel linked with the single turn of the shading coil through the phenomenon of mutual induction. The current in the shading coil is a secondary current, and is really an eddy current in the short-circuited turn constituting the shading coil.

A three-phase electromagnet is the equivalent of three single-phase electromagnets combined in one, all acting on a common armature. If three single-phase magnets were placed in a circle (they would really form a triangle) and they were properly con-

nected in a three-phase, three-wire circuit, they would operate in rotation. The maximum instantaneous pulls would follow one another in rapid succession, so that there may be said to be a rotating magnetic field in a polyphase electromagnet. In three-phase magnets, two of the portions are always pulling on the common armature, and most of the time all three are pulling together. The number of phases is comparable to the number of cylinders of an engine acting on a common shaft.

The use of shading coils on polyphase magnets increases the effective pulls and reduces the humming which is apt to occur when the loads are relatively great.

53. Quick-acting Electromagnets.—An alternating-current electromagnet is inherently a quick-acting electromagnet. The strokes of quick-acting electromagnets are usually short. A quick-acting electromagnet is characterized by its relatively small self-inductance and moving mass, great maximum current strength, and relatively few turns, for a given mechanical force. It is easily shown that the current strength in a direct-current quick-acting shunt magnet is proportional to the velocity of the armature when the ratio of the mechanical force to the self-induced voltage is constant. In constant-current direct-current series magnets the self-induced voltage is proportional to the velocity of the armature when the ratio of the mechanical force to the current strength is constant.

54. Slow-acting Electromagnets.—Unless purposely made slow-acting, this term also applies to direct-current electromagnets of the heavy-duty type, or to those in which the time required for the electromagnet to complete its stroke is relatively unimportant, particularly in cases where the electromagnet is to remain electrically energized after the work has been performed without the introduction of resistance into the circuit. Electromagnets of the plunger or solenoid types are generally slow-acting, since the strokes are usually quite long and the self-induced voltage is proportional to the velocity of the plunger in the case of a constant-voltage, direct-current electromagnet. When the number of turns is relatively great and the cross-section of the plunger is correspondingly large, the self-induced voltage may nearly balance the impressed voltage when the velocity of the plunger is relatively small. The velocity of the plunger or armature of a direct-current shunt magnet is limited by the self-induced voltage due to its motion.

In general, slow-acting direct-current electromagnets do not have the refinements found in the quick-acting types. The brass or other metal tubes surrounding the cores or plungers are not slotted, and the latter are solid. However, this general type of electromagnet is often wound with relatively few turns in order to obtain a strong pull with a given fixed voltage, until the armature completes its stroke, when a resistor is cut in, the quickness of action being obtained through excessive mechanical force despite the energy losses.

55. Quick-acting and Slow-acting Electromagnets in the Same Circuit.—Whether they are connected in series or in parallel with one another, a quick-response electromagnet will respond to an impulse, but a slow-response electromagnet will not pull its load through its range because (1) there is not sufficient time for the current to increase to a strength great enough to produce the pull necessary to move the load, or (2) there is not sufficient time for the electromagnet to complete its stroke. In such cases, the slow-response electromagnet only operates when the circuit remains closed for a time interval sufficient for it to perform its function. When the circuit is opened with the two electromagnets connected in parallel, the electromagnet having the greater time constant may discharge or exhaust into the other electromagnet, thus delaying the time of release. Lamps connected across the circuit may have a retarding effect, provided the voltage also is switched from them.

When two electromagnets are connected in series, there is no such effect when the circuit is interrupted. In the stock ticker a quick-response electromagnet operates the escapement and causes the type wheel to turn at each impulse. When the desired letter or other character is selected, the circuit is momentarily held closed, and this gives the printing magnet an opportunity to pick up its armature and thus impress the type upon the paper.

Slow-acting electromagnets are low-frequency alternating-current magnets without the refinements necessary for continuous operation in alternating-current circuits. Quick-acting direct-current electromagnets are relatively high-frequency alternating-current magnets, and should have the same refinements as continuous-duty alternating-current magnets in order that the best results may be obtained.

56. Time Constant.—The ratio of the self-inductance in henries to the resistance in ohms of an electromagnet is the time constant in seconds of that electromagnet, and is the time required for the current strength to increase from zero to 63.2 per cent of its maximum strength for a given constant impressed direct voltage, the resistance limiting the maximum current strength. If there was no self-inductance present, the current would instantly attain its maximum strength. If there was no resistance, the current strength would increase from zero at a constant time-rate, in amperes per second, equal to the ratio of the constant direct voltage to the self-inductance in henries. Hence, when the resistance and the impressed voltage are constant, the current will always increase from zero to a definite final strength, but the time required for it to increase to any given strength will vary directly with the amount of self-inductance present. When the self-inductance is constant, the current strength will increase from zero at a time-rate inversely proportional to the resistance, or directly proportional to the conductance, and its final value also will be inversely proportional to the resistance or directly proportional to the conductance.

In terms of conductance (the reciprocal of resistance), the time constant is equal to the product of the self-inductance in henries and the conductance in mhos. So long as their product is a constant number, the time constant will not vary regardless of how much either the self-inductance, the conductance, or both may be varied.

In any given case, decreasing the time constant by decreasing the self-inductance causes a decrease in the pull for a given current strength, because less magnetic energy is stored. Likewise, increasing the resistance (or decreasing the conductance) to decrease the time constant causes a decrease in the pull, because the current strength is thereby reduced. Consequently, quick-response electromagnets, as a general class, do not have such strong pulls as do slow-response electromagnets of equal sizes and for the same impressed voltages. Stated in another manner, for equal pulls and voltages, quick-response electromagnets require greater current strengths.

In one form of telegraph polechanger, two horseshoe electromagnets oppose one another, the *S* pole of one being opposite that of the other. The same is true of the *N* poles. Between the two electromagnets is an armature which is mutually acted on by both

when they are energized. However, one of the electromagnets is made quick-acting by carefully laminating the cores, while the other is made slow-acting by leaving the cores solid and surrounding them with copper sleeves. Normally, the armature is biased towards the slow-acting electromagnet by a light retractile spring. When the circuit is closed, the quick-acting magnet attracts the armature before the slow-acting electromagnet can act upon it. When the circuit is opened, the quick-acting electromagnet at once releases the armature while the slow-acting electromagnet quickly attracts it, due to the secondary currents induced in the copper sleeve and the solid core.

57. Slow-response Direct-current Electromagnets.—A shunt electromagnet may be made slow-acting in response by placing a reactor in series with it so as to increase the time constant and thus cause a delay between the instant that the circuit is closed and the instant that the current strength has increased to such a value as to produce a mechanical force sufficient to set the armature in motion. If the mass of the armature or its load is relatively great, more time is required to move it through a given distance than when the mass is small. Both of these methods are based on increasing the inertia, one being electrical, and the other mechanical, in nature. The methods may be combined.

Another method, which may be combined with the above, is based on the conversion of a portion of the electric energy into heat in a secondary circuit. This may be accomplished by (1) short-circuiting some of the turns of the coil; (2) short-circuiting a secondary coil arranged coaxially with the magnetizing coil; (3) placing a single tube of solid copper over the core between the latter and the magnetizing coil; or (4) placing a single heavy turn of solid copper at the end of the core adjacent to the air gap. In all of these methods, secondary currents are induced in the secondary circuits due to the induced voltage which exists while the current strength in the magnetizing coil is changing, and each secondary current produces a m.m.f. which opposes the m.m.f. due to the primary current and thus tends to prevent the storage of magnetic energy in the magnetic circuit. Much of the electric energy is converted into heat in the secondary circuit, however, and this causes the secondary current soon to die out. A very similar effect is produced by placing a copper disk over the end of each polepiece or in the air gap. In all cases, secondary currents delay the production of magnet flux so

that the current strength in the magnetizing coil increases at a greater time-rate than the magnetic flux. The eddy currents in solid cores have exactly the same effect, although less marked.

58. Slow-release Direct-current Electromagnets.—By permanently connecting a resistor across the terminals of the magnetizing coil the voltage induced in the latter when the electric circuit is broken persists as long as there is a change in the magnitude of the magnetic flux, thus causing the stored magnetic energy to be converted into heat in the resistor and the magnetizing coil, as the resulting current flows therein, through resistance reaction. The time constant in this case is the time required for the current to decrease from its original maximum strength to 36.8 per cent of its original strength, and is equal to the ratio of the self-inductance to the resistance, as in Art. 56.

A short-circuited secondary tends to maintain the magnetic flux after the circuit in which the magnetizing coil is connected is broken, the conditions being identical with the above except that no external resistance is present. Both the self-inductance and the resistance of a coil vary as the square of the number of turns, so one turn in the secondary is really better than a large number of turns, because in the latter case the insulation on the wire occupies the space which should be occupied by copper. Eddy currents in disks on core ends and in the cores themselves have the same kind of retarding effect.

Slow release is also effected mechanically by using massive armatures and light retractile springs, and placing the electromagnet in such a position that the movement shall be horizontal.

59. Quick-response Direct-current Electromagnets.—Quick response is obtained by eliminating the slow-response features as much as practicable. Laminating the parts reduces the eddy currents, and slotting the cores and other parts in the same directions as the magnetic flux paths also is beneficial. The moving parts should be light and the time constant small. The latter may be adjusted by connecting a resistor in series with the coil, or by varying the self-inductance by varying the number of turns, the cross-section of the core, or both. The time of response is unaffected by the presence of a resistor connected in shunt with the coil for slow-release purposes.

60. Quick-release Direct-current Electromagnets.—Quick release may be accomplished by permanently connecting a reactor

across the terminals of the magnetizing coil, said reactor containing considerably more self-inductance than the magnetizing coil. When the electric circuit is broken, the self-induced voltage of the reactor will overcome that of the electromagnet, thus tending not only quickly to reduce the current in the magnetizing coil to zero, but also to reverse its direction and thereby tend to overcome the effects of remanent magnetism in the ferromagnetic circuit. In all cases, the time constant should be made as small as practicable, and the moving parts should be made light when the direction of motion is horizontal. It should be remembered that all objects tend to fall with the same acceleration regardless of their masses or weights.

61. Quick Release through Reversed Current.—Repulsion is always due to reversed polarity. Sticking is due to remanent magnetism. Hence, if a piece of steel of high retentiveness is attracted to the core ends of an electromagnet and the current is then reversed, but limited in strength, the electromagnet tends to repel the piece of steel. A hardened steel ball held by an electromagnet becomes a permanent magnet with poles where the ball touches the polepieces of the electromagnet. The spherical shape of such an armature makes the area of magnetic contact small and produces a good holding force, even when the current strength is reduced to zero. When the current strength is quickly increased to a suitable limiting value in the reverse direction, however, the ball is repelled from the polepieces of the electromagnet and drops therefrom, as in the case of the skull-cracker ball.

62. Quick Response, Quick Release.—Endeavor to eliminate all secondary currents, massive parts, and friction. Keep the self-inductance as small as practicable. Avoid connecting the electromagnet permanently in parallel with another electromagnet having a smaller time constant, as the latter may act as a receiver for a part of the magnetic energy stored in the first electromagnet and thereby make the action sluggish. If the second electromagnet contained very little self-inductance as compared with its resistance, it would function as a resistor, as in Art. 58.

63. Quick Response, Slow Release.—Follow the instructions in Art. 59, but permanently shunt the electromagnet with a suitable resistor. The lower the resistance, the more slowly will the electromagnet release.

64. Slow Release, Quick Response.—Permanently connect a reactor of low resistance in series with the electromagnet and follow the instructions in Art. 60.

65. Slow Response, Slow Release.—Follow the instructions in Arts. 57 and 58.

66. Direct-current and Alternating-current Electromagnets.—Dash pots and bellows are commonly used to control the time of action of alternating-current electromagnets, which are inherently quick-acting. Short-circuited turns should never be allowed in alternating-current electromagnets, except for specific purposes, as in the case of a shading coil. While the coils of an alternating-current electromagnet may be shunted with a resistor, the circuit may be broken when the current strength is equal to zero. Series reactors and resistors tend to limit the current strength and to affect the time constant.

An electric condenser may be no detriment when placed in series with an alternating-current electromagnet, but a direct-current electromagnet cannot be made to operate (except for a very brief instant, or on impulse) under such circumstances. On the other hand, a reactor may be no detriment to the operation of a direct-current electromagnet, while it may prevent the operation of an alternating-current electromagnet, when connected in series in a constant-voltage circuit.

Large copper collars are placed on the ends of the cores in certain telephone relays so that they will not respond to alternating currents but will respond to direct currents. Alternating-current electromagnets cannot pick up "dead weights" that may be lifted by direct-current electromagnets, because the former are not given time to overcome the inertia before the current again falls to zero.

67. Quick Action Obtained through Quick Release.—This reversed operation is important in certain cases. The electromagnet stores potential energy in a spring of small mass by either placing the spring under tension or compression. When the electric circuit is opened, the energy stored in the spring is quickly released to accomplish the desired work.

68. Heavy-duty Electromagnet with Spring-releasing Electromagnet.—In this case, the large electromagnet stores potential energy in a spring and operates a latch at the end of the stroke, thus opening its circuit as the latch engages the armature or plunger to hold the spring under tension or compression. A small

quick-acting electromagnet connected in another branch of the circuit trips the latch and releases the energy stored in the spring. By this method, the general effect of a heavy-duty, quick-acting electromagnet is obtained. While additional switches always are objectionable, current flows only when energy is being stored in the spring and while the spring is being released. The general principle is that of a steel trap or the action of a rifle. A similar device may be made to deliver a strong push at the beginning of the stroke.

69. Methods of Reducing Sparking at Switch Contacts.—In all of the following methods, the magnetic energy converted into electric energy in the coil is converted into heat through resistance reaction. A condenser shunted by a resistor may be connected across the contacts, so that when the electric circuit is broken at that place the energy of the magnetic circuit will be quickly transferred to the dielectric of the condenser and thence more slowly converted into heat in the resistor. Another method is to shunt the coil with a resistor or else a condenser, or with both. Any electrical method which tends to cause slow release is effective for reducing the sparking. Hence, copper tubes over cores and short-circuited turns may be employed. A modification of the latter principle is to wind two parallel wires so as to form two circuits through the same coil. After the armature completes its stroke, the secondary coil may be short-circuited before the main electric circuit is broken. Still another method consists in winding the coil in six or more concentric sections or hollow cylinders. The first coil is wound in several layers. Insulation is wrapped around the first coil. The next coil is then wound and insulated. After the coil is completed, the inside ends are connected together and the outside ends also are connected together. The inner turns have low resistances but high self-inductances, while the reverse is the case in the outer turns. Hence, the time constants are different and the energy of the inner turns is expended in the outer turns. Series electromagnets are short-circuited when they are cut out of circuit. Breaking the circuit in a magnetic field or under oil causes the spark or arc to be repelled in the former case and quickly suppressed in the latter case.

70. Sticking.—This is due to remanent magnetism. It may be overcome by reversing the direction of the magnetizing current and limiting the current to such a strength that it shall only reduce the magnitude of the magnetic flux to approximately zero,

but shall not increase its magnitude in the reversed direction. This may be accomplished by limiting the reversed current strength by means of a resistor, or by making the time of contact such that the current strength shall increase only to the desired strength by the time the circuit is again opened.

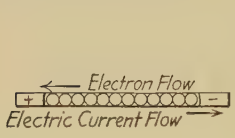
The remanent magnetism may be avoided by maintaining an air gap in the ferromagnetic circuit and by using magnetic materials of low retentiveness. It is customary to place a non-magnetic stop between the coacting members of electromagnets so that the air gap can never be entirely closed. Another method is to open some other part of the ferromagnetic circuit, as by placing an air gap or some non-magnetic material in the yoke of a horseshoe electromagnet. Such air gaps may be made adjustable. In the former method, brass pins or rivets are driven into small holes in the core ends, or the core ends are heavily copper plated. In applying such devices it is often important not to remove much of the magnetic material. A penny dropped between the magnetic plug and the plunger of a brake magnet once relieved an awkward situation. Lengthening the air gap by any such method decreases the self-inductance and tends to make its response more rapid with a stronger current, but copper sheets on the ends of cores and stops tend to make the release more sluggish.

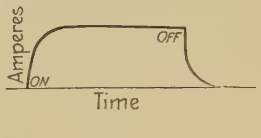
71. Polarity.—Electric polarities are respectively indicated by the terms positive (+) and negative (−). Magnetic polarities are represented by the terms north-seeking (*N*) and south-seeking (*S*), respectively. In a dry cell, the carbon is the positive pole and the zinc is the negative pole. When the positive and negative poles are connected by means of a wire the direction of flow of the electrons in the wire is from the negative pole towards the positive pole (since the electrons are of negative polarity and negative repels negative, while positive attracts negative).

Electrons flowing from the negative side towards the positive side of a source of electric energy produce the same general effect as positive electricity flowing from the positive side towards the negative side. Before the discovery of electrons it was assumed that the equivalent of a current of positive electricity flowed from the positive side towards the negative side. Because of conventional usage, positive electric currents are generally assumed, that is, the electricity is thought of as flowing according to the old assumption. The term electron current, however,

always expressly indicates that the current flows from negative to positive.

The end of a magnetic compass needle which naturally points towards the north pole of the earth is the north-seeking pole. The opposite pole is the south-seeking pole. The north-seeking





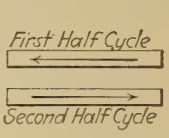
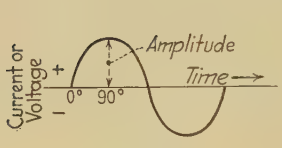


Fig. 100. Fig. 101. Fig. 102.

Fig. 100.—Electron flow and direct current.
Fig. 101.—Direct current.
Fig. 102.—Alternating current.



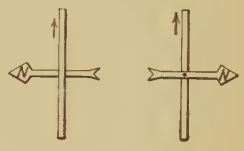
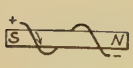
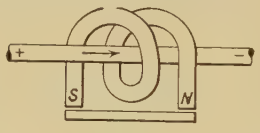


Fig. 103. Fig. 104. Fig. 105.

Fig. 103.—Alternating current.
Fig. 104.—Relative directions of electric current and magnetic field.
Fig. 105.—Effect of electric currents on permanent magnets.





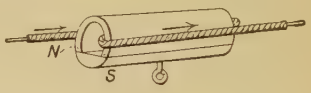
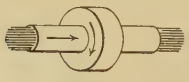
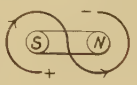


Fig. 106. Fig. 107. Fig. 108.

Fig. 106.—Bar electromagnet.
Fig. 107.—Helical core.
Fig. 108.—Joule's electromagnet.





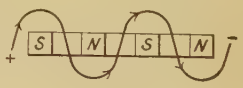


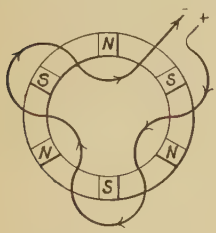
Fig. 109. Fig. 110. Fig. 111. Fig. 112.

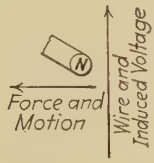
Fig. 109.—Coil surrounding core.
Fig. 110.—Steel surrounding cable.
Fig. 111.—Horseshoe electromagnet.
Fig. 112.—Magnetic chuck.

pole of a compass needle naturally points towards the south-seeking pole of any magnet, and *vice versa*.

It is convenient to express the directions of m.m.f. and magnetic flux in terms of the polarities of the parts of the magnets where the magnetic flux is said to leave or to enter the magnets.

Hence, outside the magnets the direction of the m.m.f. and the magnetic flux is conventionally assumed to be from the N-pole towards the S-pole, while within the magnets their directions are from the S-pole towards the N-pole.





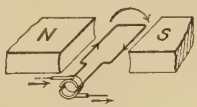
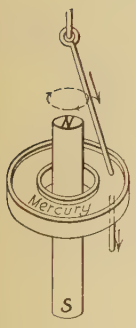
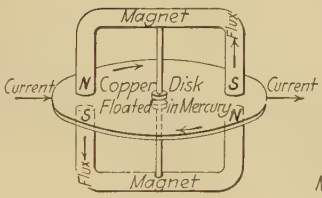


Fig. 113. Fig. 114. Fig. 115. Fig. 116.

Fig. 113.—Magnetic chuck
Fig. 114.—Induced voltage due to relative motion.
Fig. 115.—Principle of the dynamo.
Fig. 116.—Motion of current-carrying wire in magnetic field.





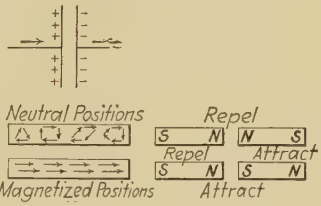


Fig. 117. Fig. 118. Figs. 119 and 120. Fig. 121.

Fig. 117.—Rotation of current-carrying wire around a magnet.
Fig. 118.—Principle of the mercury motor.
Fig. 119.—Charged condenser.
Fig. 120.—Ferromagnetism.
Fig. 121.—Permanent magnets.

Figures 100 to 121 illustrate the relative polarities and the resulting directions of motion, or the polarities resulting from relative motions of the parts.

72. The Effect of an Influencing M.m.f.—The use of an additional electromagnet in connection with the operating electromagnet makes it possible to reduce the initial self-inductance to a very small value. For a given number of turns in the coil, the self-inductance is directly proportional to the ratio of the magnitude of the magnetic flux to the current strength. Consequently, if two properly proportioned electromagnets are so

arranged that their m.m.fs. shall approximately neutralize one another when a voltage is impressed upon the operating electromagnet, the magnitude of the resulting magnetic flux will be very small and the current strength will increase from zero to a relatively great value during a very brief time interval if the electric resistance of the circuit is small. The result is a relatively great mechanical force of repulsion between the two electromagnets, since, as they separate, magnetic flux will link with the ampere-turns at a relatively great length-rate of change. Under ideal conditions the initial self-inductance would be equal to zero, so that the current would instantly attain its maximum strength. Then, as the mechanical movement took place the self-inductance would increase at an enormous rate. In this case, the initial impulse is relatively great and overcomes the inertia of the movable parts during a very brief time interval. The maximum mechanical force is developed when the length-rate of change in the self-inductance is a maximum, and this occurs when the self-inductance changes from zero to a finite quantity (with constant current strength), due to a relative movement of the parts. Advantage is taken of this principle to obtain usable mechanical forces with exceedingly weak currents.

In Fig. 122 is shown a horseshoe electromagnet with a third coil and core between the usual coils and cores. Only the middle coil is energized. While this electromagnet may be advantageously employed in reversing mechanisms and the like, it is more extensively used in polarized relays, wherein a permanent magnet is substituted for the middle electromagnet.

73. Polarized Electromagnets.—These may be generally classified and described as below. The term *magnet* as used in the schematic illustrations, indicates a permanent magnet. The general term *pole* refers to polepieces or core ends. Limiting stops and other usual mechanical parts are not generally shown herein.

By the use of a polarizing magnet an armature or other movable part may be normally held in either a neutral position or in a biased position, and may be made to move in a direction corresponding to the direction of the current in the operating coil. In other cases, a polarizing magnet is placed in series in the magnetic circuit of the operating electromagnet, as in the case of the polarized telephone receiver.

Polarized electromagnets are invariably of the quick-acting type, and they may be made very sensitive.

1. *An Electromagnet Influenced by a Permanent Magnet.*—In the polarized electromagnet in Fig. 123 the free end of the armature is always of *N* polarity. When no current flows in the coils each core end is of *S* polarity. If an electromagnet was substituted for the permanent magnet, the self-inductance of the polarizing electromagnet would be greatest when the free end of the armature was in contact with either core end, as in Fig. 122. Theoretically, the permanent magnet possesses

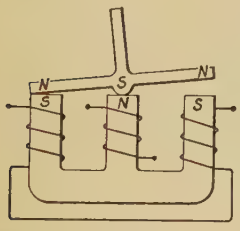


Fig. 122.

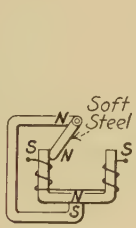


Fig. 123.

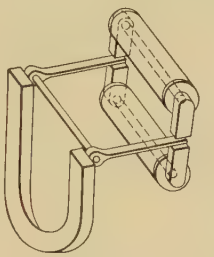


Fig. 124.

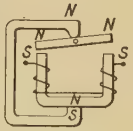


Fig. 125.

Fig. 122.—Polarized electromagnet. Only the middle coil is electrically energized.

Fig. 123.—Polarized electromagnet. No current in any coil.

Fig. 124.—Principle of British P. O. pattern relay.

Fig. 125.—Striker magnet.

self-inductance due to its electron currents and the magnetic flux linked with them. The energy of the ferromagnetic circuit, including the magnet, the armature, and one of the cores, is increased when the free end of the armature touches either of the cores. Hence, the free end of the armature moves to the core end nearest to it.

When a current flows in the turns of the coils, the polarity of one of the core ends becomes *N* and the other remains *S*. If the free end of the armature is nearest the *S* end, it will remain as in Fig. 123. If the core end nearest the free end of the armature becomes of *N* polarity, however, the armature will move to the core end of *S* polarity, because, by so moving, the stored magnetic energy of the system will be increased. In either case, when a current flows in the coils, the direction of the magnetic flux in one of the cores is reversed, the m.m.f. of the operating electromagnet reversing the direction of the magnetic flux in one of the cores and increasing the magnitude of the magnetic flux

in the other core. This particular arrangement is employed in telegraph and signal apparatus. A modification is shown in Fig. 124, being the principle of the British P. O. Pattern Relay. In this, as in many other cases, the permanent magnet is bent so that the relay shall occupy as small a space as practicable.

Figure 125 illustrates a device somewhat similar to that in Fig. 123, but with the armature pivoted at its middle. This is an example of the striker magnet of telephone ringing equipment, and is the nearest approach to the polarized electromagnet in Fig. 122.

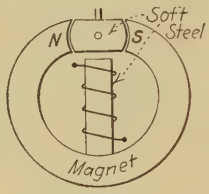


Fig. 126.

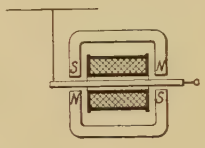


Fig. 127.

Fig. 126.—Rotary-armature polarized electromagnet.
Fig. 127.—Loud speaker.

Figure 126 shows a polarized electromagnet wherein the armature normally lies in a neutral position in the magnetic field of a permanent magnet. When a current flows in the coil in a given direction the armature rotates through an angle in a direction corresponding to the direction of the current. With the polarities shown in Fig. 126, if the polarity of the core end was made *N*, the armature would rotate through an angle in a clockwise direction, while, if the polarity was made *S*, the direction of rotation would be counterclockwise. This principle is employed in the remote control of searchlights, and for other purposes.

Figure 127 shows a form of polarized electromagnet used in loud speakers. When a current flows in the coil, the soft-steel spring-suspended armature is attracted by one magnet more than it is repelled by the other, due to the direct-current component of the current, unless an audio-frequency transformer is interposed between the vacuum tube and the coil. A single magnet is usually so constructed as to perform the function of the two magnets in the schematic illustration.

2. *An Electromagnet with a Permanent Magnet for an Armature.* Figures 128 to 131 illustrate some possible forms. Where the permanent magnets are small, they are apt to become demagnet-

ized, and even have their polarities reversed if the m.m.fs. of the electromagnets become relatively great. In all cases, the ferromagnetic circuits should be as nearly closed as practicable. Permanent magnets are not recommended for armatures, as the constant jarring due to hammer blow tends to demagnetize them.

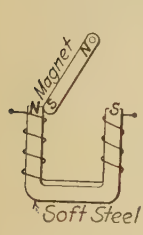


Fig. 128.

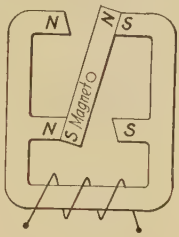


Fig. 129.

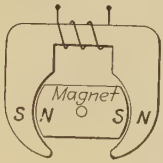


Fig. 130.

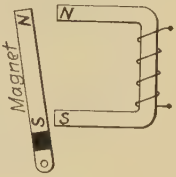


Fig. 131.

Fig. 128.—Impracticable type.
Fig. 129.—Pivoted-magnet type.
Fig. 130.—Rotary-magnet type. Impracticable
Fig. 131.—Push-pull type.

3. *A Permanent Magnet with an Electromagnet for an Armature.* The loud speaker in Fig. 132 is operated through an amplifying transformer. Therefore, the armature is balanced in the magtic field of the permanent magnet, and the device is very sensitive to changes in the current strength.

One form of galvanometer is illustrated in Fig. 133. This is commonly used in direct-current measuring instruments.

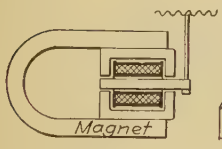


Fig. 132.

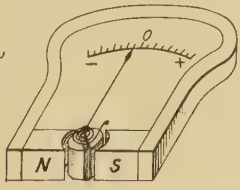


Fig. 133.

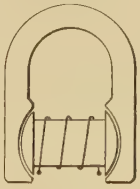


Fig. 134.

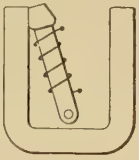


Fig. 135.

Fig. 132.—Loud speaker.
Fig. 133.—Galvanometer.
Fig. 134.—Synchronous-motor or magneto type.
Fig. 135.—Ringer movement.

When a high-resistance resistor is connected in series with its coil, it becomes a voltmeter. When a low-resistance (or high-conductance) resistor is connected in parallel or in shunt with its coil, it becomes an ammeter. The piece of circular soft iron surrounded by the coil is stationary and only the coil moves.

Small synchronous motors operate on a similar principle, but with alternating currents, as indicated in Fig. 134, wherein the coil is wound directly on the soft-steel armature. Magnetos operate on the reverse principle, that is, alternating voltages are produced when the armatures are mechanically rotated.

The device in Fig. 135 illustrates a form suitable for alternating current ringing devices. In the device in Fig. 136 the coil and permanent magnet are stationary.

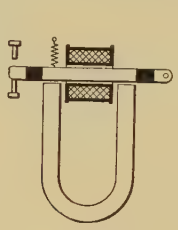


Fig. 136.

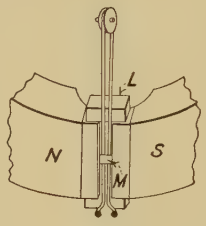


Fig. 137.

Fig. 136.—Direct-current polarity-directional relay.
Fig. 137.—Principle of the oscillograph.

Figure 137 illustrates the principle of the Duddell Oscillograph. The poles are made sharp to secure a dense magnetic field between them. The current flowing upward in one branch of the fine metal ribbon and downward in the other causes a rotating movement of the mirror attached to the ribbon through a very small angle. The stop L prevents too great a movement. The mirror M reflects a beam of light to a piece of ground glass or a photographic film.

4. *An Electromagnet with a Second Electromagnet for an Armature.*—Obviously, any of the above permanent magnets might be replaced by electromagnets for more powerful or for more sensitive actions. In the cases cited, however, it is preferable to employ permanent magnets because no energy is constantly being expended in maintaining the magnetic field of a permanent magnet. The electron currents associated with the molecules of magnetic material move in a frictionless medium as a part of the natural structure of matter. When employing permanent magnets, their dimensions should be made liberal, because they do not generally possess so great intensities of magnetization as do electromagnets.

Figure 138 illustrates a method of controlling the direction of motion of the armature in the case of a large and powerful

electromagnet. Figure 139 shows a type which produces quick and strong repulsion. Figure 140 illustrates, in principle, an electric motor or generator which may be constructed in any of the usual forms. The multiplicity of the poles for a given

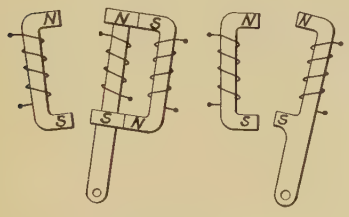


FIG. 138. FIG. 139.

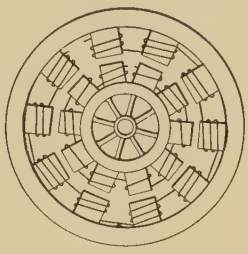


FIG. 140.

FIG. 138.—Push-pull electromagnet.
FIG. 139.—Repulsion electromagnet.
FIG. 140.—Rotary-motor type.

diameter or radius increases the mechanical force (or torque) but decreases the distance of travel from pole to pole.

In electrodynamometers, one coil rotates through an angle with respect to the other. Wattmeters are constructed on this

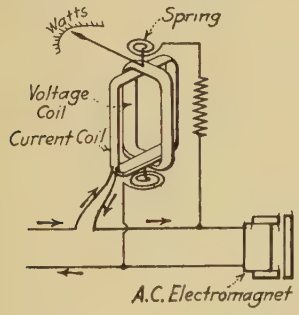


FIG. 141.—Principle of the wattmeter.

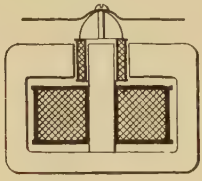


FIG. 142.—Loud speaker.

principle, as indicated in Fig. 141. The loud-speaker mechanism in Fig. 142 has a coil attached to the diaphragm which moves up and down over the core end on the general principle of the coil-and-plunger but with the coil moving, with variations in the current strength within it, in the strong magnetic field of the stationary electromagnet.

5. *Polarized Electromagnets with Biased Armatures.*—Two polarized electromagnets may be made to operate independently

in series or in parallel by biasing their armatures, as by means of springs, so that when an armature is moved from its normal position the spring will return it to that position when the current has ceased to flow. Thus, if two polarized devices are connected in the same circuit, only one of them will respond when the current flows in a given direction, while the other will respond only when the current flows in the reverse direction in the same circuit. The principle is adopted in party-line telephone equipment where either positive or negative impulses are transmitted over a line to call a subscriber. The principle is illustrated in Fig. 143. Armatures are sometimes biased by means of small permanent magnets.

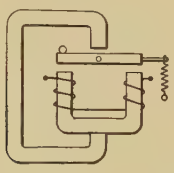


Fig. 143.—Party-line ringer.

Fig. 144.—Bipolar telephone receiver.

6. *Polarized Electromagnets with Armatures in Normally Neutral Positions.*—Those in Figs. 126 and 133 belong to this general class.

7. *Attraction Polarized Electromagnet.*—The bipolar telephone receiver in Fig. 144 is an example of this general type. It may also be produced by removing the yoke or back iron from an ordinary telegraph line relay and replacing it with a U-shaped permanent magnet. The author has found this relay very sensitive and quick-acting. In both cases, the armature (diaphragm, in the case of the telephone receiver) and cores are of soft iron. The permanent magnet is connected in series in the ferromagnetic circuit. The remarkable sensitiveness of the telephone receiver is well known. The reason for its sensitiveness is described below.

For a given pull between the permanent magnet and the diaphragm, through the air gaps, the effect is exactly as though an electromagnet was substituted for the permanent magnet. However, a slight but sudden difference in the pull on the diaphragm will cause it to emit a sound. Telephonic currents may be very feeble. The permanent magnet produces a relatively strong magnetic field between the poles of the operating electromagnet

and the diaphragm, through the intervening air gaps. Only a mechanical force of mutual attraction exists between them. When a current flows in one direction in the coils of the electromagnet, the magnitude of the magnetic flux in the air gaps is increased. When the current strength is decreased or the direction of the current is reversed, the magnitude of the magnetic flux in the air gaps is decreased. Consequently, there is a difference in the pull on the diaphragm proportional to the difference in the length-rates of change in the number of ampere-maxwell-turns for any two positions of the diaphragm with respect to the poles of the electromagnet.

For normally fixed air-gap and coil conditions in the telephone receiver, the pull on the diaphragm is directly proportional to the square of the magnetic flux density in the air gaps and, therefore, to the square of the magnetic flux, since the cross-sections of the air gaps are constant.

Let it be assumed that the magnetic flux density in one air gap is 1,000 gausses when no current is flowing in the coils, and let the following example be worked out on the basis of one air gap, for convenience. Now let a feeble current flow in the coils, and let this current produce 5 additional gausses in the air gap. If this current acted alone, without the permanent magnet, the initial pull would be equal to 0 and the 5 gausses would produce a pull equivalent to $5^2 = 25$ units of force. However, with the permanent magnet already producing 1,000 gausses, and the electromagnet producing 5 more gausses, the total magnetic flux density is 1,005 gausses and its square gives the equivalent of 1,010,025 units of force. But the permanent magnet alone produces a force equivalent to $1{,}000^2 = 1{,}000{,}000$ units of force. Hence, the gain in force due to the electromagnet acting with the permanent magnet is equal to $1{,}010{,}205 - 1{,}000{,}000 = 10{,}025$ units of force, a gain of 40,000 per cent over that which could be produced by the electromagnet alone. The above forces are relative only and are not reduced to the common units.

8. *Repulsion Polarized Electromagnets.*—All polarized electromagnets not belonging to class 7.

9. *Push-pull Polarized Electromagnets.*—This title refers to any type of electromagnet in which the armature is simultaneously attracted by one set of poles and repelled by another set. The action always is reversible unless restraining means are employed.

10. *Reverse-phase Electromagnets.*—Figure 145 shows the two conditions for a particular type.

74. Reversible Solenoid.—The double-acting solenoid in Fig. 146,[1] which also has the property of attaining a neutral position,

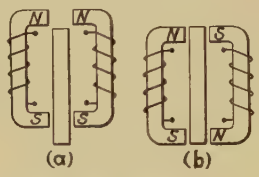


Fig. 145.—Phase-rotation relay type.

was developed to operate a duplex friction clutch on a head frame. Grooves are turned in the surface of the hollow plunger to receive the turns of wire. When the plunger tends to overtravel, the reverse winding establishes a counterforce as it passes

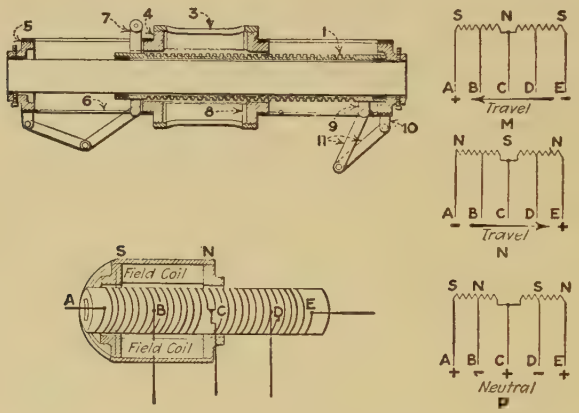


Fig. 146.—Reversible solenoid.

into the stationary coil. "On a test of the solenoid of this type with a 3-in. (7.5-cm.) plunger, ⅞ in. (2.25 cm.) thick and 15 in. (37.5 cm.) long with a consumption of 500 to 600 watts, a pull of 40 lb. (18 kg.) developed with a 6-in. (15-cm.) stroke." The stationary or field coil is entirely surrounded by an iron-enclosing cylinder and the end pieces of the cylinder also are of iron.

[1] Murray, J. W., "Double-acting Solenoid Which Has a Strong Pull," *Elec. World*, p. 435, Aug. 28, 1920.

CHAPTER V

PORTATIVE ELECTROMAGNETS

The enormous holding capacity and great field strengths of large electromagnets make them adaptable wherever materials having magnetic properties are to be held, sorted, or otherwise handled. The flexibility of the invisible magnetic field makes possible some very desirable clutching and safety features.

75. Magnetic Chucks.—The function of the magnetic chuck is to hold magnetic materials, usually iron or steel, while they are being worked on. Such materials are herein referred to as work. While magnetic chucks are particularly adapted for use in grinding machines, they are extensively used on vertical milling machines, iron planers, and lathes. By the use of magnetic jigs, or extensions from the polepieces of the chuck, irregular pieces may be held. There are many types of magnetic chucks, but all depend upon the extension of the magnetic edge for increased holding capacity. Consequently, magnetic chucks are provided with sinuous, zigzag, and similar forms of polepieces in order to produce air gaps of small length but of great cross-section over as great a surface of the chuck as practicable. The face plate usually consists of a grid casting. The air gaps are filled with a non-magnetic filler which may consist of nine parts lead, two parts antimony, and one part of bismuth, across which the work to be held is placed, and which has the property of expanding upon cooling from the molten state.

Magnetic chucks are made in two general shapes; namely, the rectangular magnetic chuck and the rotary magnetic chuck. General types of magnetic chucks are the single-pole, multiple-pole, neutral-pole, independent, and modifications thereof. Figures 147 to 150 are from the article by O. A. Kenyon referred to on page 105.

Magnetic chucks containing more than one coil may be connected in direct-current circuits of various voltages by changing the connections. The voltage seldom exceeds 230 volts.

Various claims are made for the different types. An independent magnetic chuck does not depend upon its casing to form a

part of the ferromagnetic circuit. Coils are so designed that they will not overheat on continuous duty and are impregnated and protected from oil and water, the interior of the

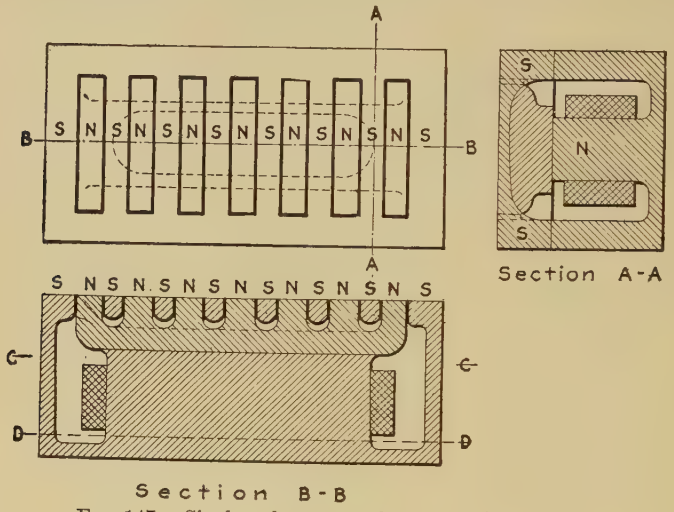


FIG. 147.—Single-pole rectangular magnetic chuck.

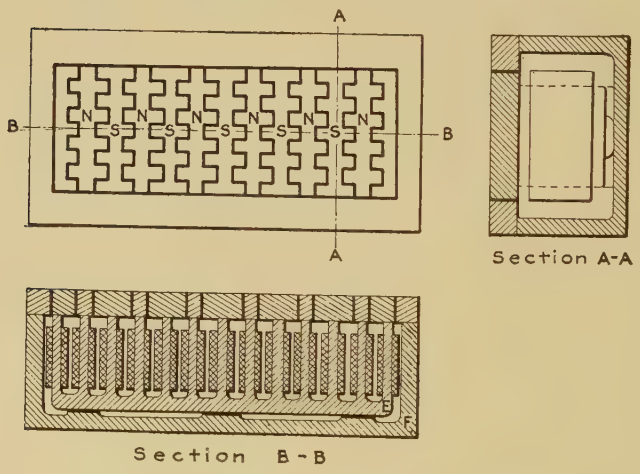


FIG. 148.—Multiple-pole, or unit-pole, rectangular magnetic chuck. Independent type.

chuck sometimes being filled with an insulating and waterproofing compound. Magnetic screens are employed in some cases to prevent the machines from becoming magnetized. A

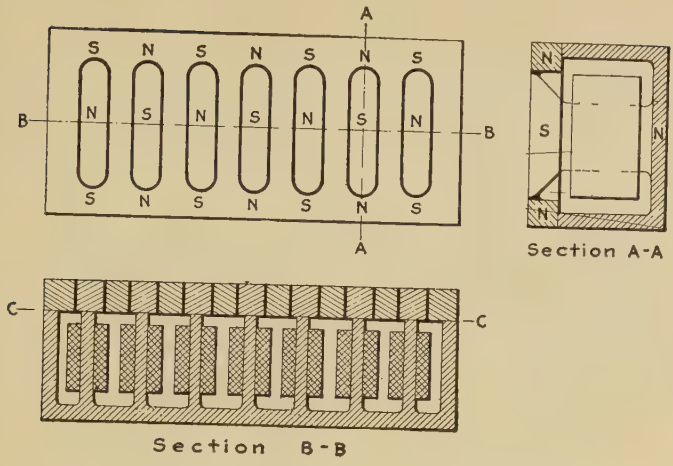


FIG. 149.—Neutral-pole rectangular magnetic chuck.

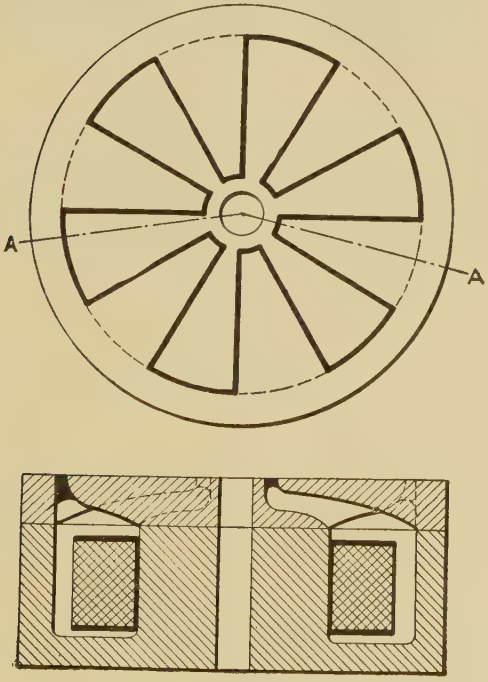


FIG. 150.—Exterior and cross-section of rotating single-pole magnetic chuck.

magnetic screen is a good magnetic conductor which shunts the ferromagnetic flux away from the machine. Rotary magnetic chucks are provided with slip rings and brushes for connecting the coils in the supply circuit.

While the magnetic material used in the construction of magnetic chucks usually is of high-grade soft steel, there is a tendency for certain kinds of work, such as high-carbon or tool steel, to stick to the chuck after the electric circuit has been opened, due to remanent magnetism in such steels, and then it is necessary to demagnetize the chuck and its work by momentarily reversing the direction of the current and again opening

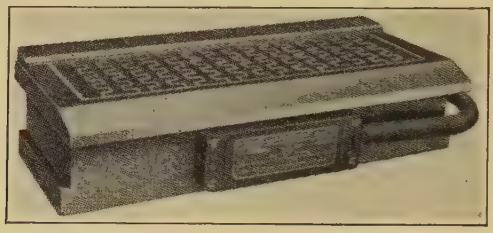


Fig. 151. Taft-Peirce unit-pole rectangular magnetic chuck.

the circuit. This is accomplished by means of a double-pole, double-throw switch which the operator soon learns to manipulate in the proper time relation for demagnetizing the chuck. A resistor may be included in the reverse side of the switch to limit the strength of the demagnetizing current. Then the time element need not be considered.

Demagnetizers are employed further to demagnetize certain classes of work when desired. The effect is that of placing the work in an alternating magnetic field and then gradually withdrawing it from the field. In the two general forms of demagnetizers commonly used for this purpose there are two steel plates upon which the work is placed and from which it is withdrawn. In the direct-current demagnetizer the effect is equivalent to rotating an electromagnet between the plates.

Magnetic chucks require roughly from 0.5 to 1 watt per square inch of working surface. It is claimed that they will hold loads at maximum pressures of more than 100 lb. per square inch; depending upon the characters and thicknesses of the magnetic materials which they hold. Typical forms are shown in Figs. 151 and 152.

For full information regarding magnetic chucks see the reference literature in the footnote.[1]

76. Lifting Magnets.—The little hand magnet in Fig. 153 is an example of a lifting magnet of small size. Connected to a direct-current circuit by means of its cord and plug, it can be

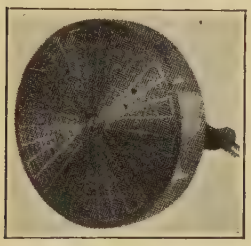


FIG. 152.—Taft-Peirce single-pole rotary magnetic chuck.

FIG 153.—Hand magnet.

energized or deenergized by means of the small push switch on its handle. It is used in machine shops for clearing chips and borings out of the machinery or for removing them from parts of the work not easily accessible. Dropped tools, bolts, boring bars, and so forth are easily recovered with the aid of the magnet from places where it would be difficult to fish them by ordinary means. In foundries, this magnet may be used to pick up warm or oddly shaped castings, and smooth plates, which sometimes are difficult to lift from a flat surface.

The above principle is carried out on a large scale by means of huge lifting magnets, some of which are 62 in. in diameter and capable of lifting thousands of pounds of magnetic material per load. It is claimed that a 62-in. magnet will lift 50,000 lb. of billets and slabs, or 60,000 lb. of solid iron or steel affording a good magnetic contact. However, the lifts of these magnets may not be greater than 1,000 lb. under adverse conditions, as when the material to be lifted consists of small parts. These types of lifting magnets also are made in smaller sizes.

[1] THOMPSON, H. L., "The Inside of the Magnetic Chuck," *Machinery*, pp. 667 and 763, 1914; WALKER, O. S., "Types of Rotary Magnetic Chucks," *Machinery*, p. 843, 1914; "New Heald Magnetic Chuck," *Iron Age*, p. 7, July, 1914; CLEWELL, C. E., "Notes on Magnetic Chucks," *Am. Machinist*, vol. 42, p. 595, 1915; "Persons-Arter Chuck," *Machinery*, p. 467, Jan., 1918; KENYON, O. A., "Relative Characteristics of Magnetic Chucks," *Elec. World*, vol. 74, p. 11, 1919; SHELDON, E., "Magnetic Chucks," *Am. Machinist*, 1921 (series of articles).

Figure 154 shows the construction of one make of lifting magnet. The coils are generally wound on spools with strap copper for the conductor, insulated with asbestos, or mica and asbestos, and are impregnated and filled with a compound. Therefore, the magnets are claimed to be waterproof and fireproof. Sometimes they are accidentally left in circuit overnight, becoming red hot, and they are exposed to all kinds of

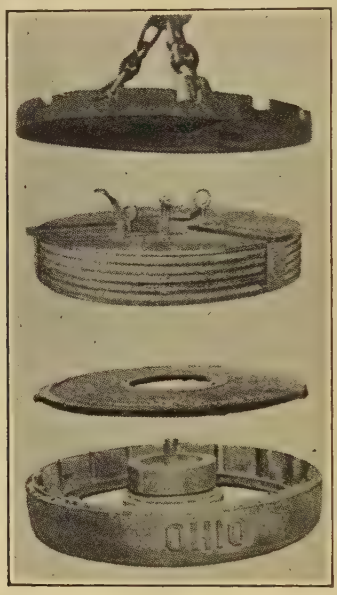


Fig. 154.—Relation of parts of lifting magnet.

weather, even being under water for days, without injury. Some years ago, a lifting magnet was employed for salvaging nails from the bottom of the Mississippi River, at a depth of 70 ft. There is a fine motion picture of a magnet recovering pig iron from a river bottom.

Lifting magnets are very rugged and withstand exceedingly rough usage. The wearing bottom plate of the annular type, under the coil and between the pole shoes, is of heavy manganese steel (12 per cent) which is very hard and a very poor magnetic material, so that it withstands the wear and very little magnetic flux is shunted through it. Types of lifting magnets, other than of the annular type, also are employed. Two types are illustrated in Figs. 155 and 156.

Lifting magnets are usually suspended from cranes, and are controlled by the crane operator. The magnet is lowered onto the load to be moved; the switch is closed; the magnet with its load is moved (and lowered when the load cannot be dropped) to the desired position; and the switch then is opened and the current momentarily reversed in order to demagnetize the mag-

Fig. 155.

Fig. 156.

Fig. 155.—62-in. C-H lifting magnet holding a skull-cracker ball over material to be broken. The weight of the ball is 20,000 lb. and the height from which it dropped is 50 ft. Note the small magnetic contact between the magnet and the ball.

Fig. 156.—E. C. & M. bipolar magnet lifting 1,100 lb. of nested corrugated sheet-iron culvert piping.

net and its load to cause quick release. Resistors sometimes are inserted in the reverse side of the switch to limit the demagnetizing current strength. This cycle of operations can often be performed in one minute. These magnets handle magnetic materials which are covered with snow and frozen, as well as medium-hot iron and steel. The limit of temperature at which iron and steel can be thus lifted is about 950° F., iron becoming non-magnetic at about 1,400° F. The cost per ton of handling scrap, pig iron, and so forth is but a few cents.

Lifting magnets also are used for breaking up "skulls" which form in the bottoms of ingot molds, and scrap of various kinds. Huge balls (as in Fig. 155), called "skull-cracker balls," are dropped from the magnet upon the skulls or scrap to be broken. The same magnet is employed to place the large pieces in position and to remove the small pieces after breaking.

In an article in *Le Genie Civil*, Apr. 21, 1917, an abstract of which was published in the *Electrical World*, J. Vichniak describes three novel forms of lifting magnets, one of which is water-cooled for lifting hot billets. To increase the capacity when lifting pieces of irregular form, the Couffinal works at Saint-Étienne designed a magnet with subdivided vertically movable pole faces which, by their vertical movement, conform to the irregularities of the pieces lifted and prevent lateral motion as well as increasing the lifting capacity. Another form of magnet for lifting a bundle of iron rods or other long pieces is provided with a linkage, something after the manner of a clam-shell dredge bucket, operating arms that embrace the load held by the magnet. Once in place, the magnet is deenergized and the load is transferred to the arms.

In some foreign makes of lifting magnets aluminum coil conductors are employed, the insulation consisting only of aluminum oxide formed on the conductor. Hollow, spherical polepieces are employed for holding skull-cracker balls, while spring-mounted movable poles are used for lifting materials with very uneven surfaces.

Forms of rail and pipe magnets manufactured in the United States consist of two units widely separated so as to lift the rails or pipes at their ends.

77. Magnetic Clutches.—A magnetic clutch employs an electromagnet of the annular type. Magnetic clutches perform functions similar to those performed by mechanical clutches, but the operations do not entail end thrust and they may be made automatic, which makes them preferable to mechanical clutches in many cases. They are used extensively with motor-driven machinery.

Owing to the alternate and periodic conversions of electric energy into stored magnetic energy, and *vice versa*, in alternating-current electromagnetic systems there is an undesirable and useless or "wattless" component of the total current flowing in the electric conductors which causes the conversion of electric energy into heat therein, and makes it necessary to use larger conductors than otherwise would be required. In consequence, public utilities usually impose a power-factor penalty on power users when the power factor, or the ratio of the actual power to the total number of volt-amperes, is less than a given percentage.

One way of increasing the power factor is to use a synchronous motor, called a synchronous condenser when its field is overexcited. In cases where a great starting torque is required, the motor can be started without load and brought up to speed, after which the load can be applied gradually by closing the direct-current circuit in which the coil of the magnetic clutch is connected. In certain cases, as in a compeb mill in the cement industry,[1] the starting torque may be 150 per cent of the full motor torque on account of the high static friction.

The clutch is generally designed to transmit the maximum horsepower which the motor can develop, and may be connected directly across the line, allowing the mill to come up to speed as quickly as the clutch permits. Direct current is required for excitation at 110 or 220 volts. The exciter generator for the synchronous motor may be used as the source of energy for the magnetic clutch in many cases. The torque of the clutch increases as time elapses after the full-line voltage is applied; requiring 6 sec. in some cases, owing to the great self-inductance of the electromagnet. During this period the motor is accelerating and the clutch is gradually increasing the load on the motor. Manually operated rheostats are often used to control the excitation of the clutch during the starting period. An objection to the use of a starting rheostat is that it may be set in such a position that the clutch cannot start the mill, with the consequent unnecessary wear and heating of the clutch. The magnetic clutch thus permits the use of synchronous motor drive with a corresponding increase in the power factor from about 65 to about 90 per cent.

In one form of magnetic clutch employed to connect an engine with a generator in a combined steam and water power plant without stopping, the overall diameter and length are respectively 66 and 30 in. With a terminal voltage of 110 volts and a current strength of 8 amp. the pressure of the armature on the friction ring is 75 lb. per square inch, the resulting friction being sufficient to drive a 1,000-kw. generator at 150 r.p.m. The clutch is designed to withstand moderate periods of slipping, but has a factor of safety which permits of no slipping when the engine is running at its maximum load. Other shafting may be similarly connected and disconnected.

[1] COSTELLO, W. H., "Magnetic Clutches in the Cement Industry," *Trans.* A.I.E.E., p. 361, 1922.

In rubber mills, for instance, accidents to workmen occur at and about rubber mixing rolls, washers, crackers, calender rolls, and so forth. It is imperative that a machine shall be quickly stopped under such circumstances. Mechanical appliances designed for this purpose are more or less complicated and are not always dependable. Magnetic clutches are rapidly replacing mechanical clutches in this field and they have the further advantage that they may be controlled by any number of stop switches situated conveniently near, or remote from, the machine where the accident may occur. The safety feature also may be combined with synchronous motor drive.

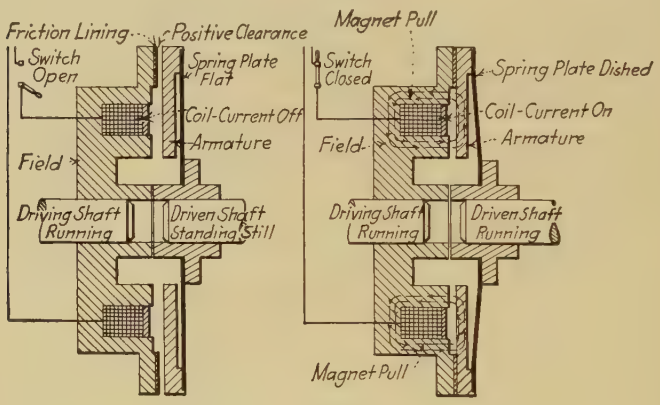


FIG. 157.—Magnetic clutch.

In Fig. 157 are shown the "on" and "off" conditions for the magnetic clutch above referred to in connection with the cement industry. Woven asbestos and brass wire commonly form the friction lining.

A magnetic coupling is essentially the same as a magnetic clutch, with the exception that it is not provided with a separate lined friction face and, therefore, is not suitable for accelerating a load, as the engagement of the two members is very sudden.

The clutch[1] at the right in Fig. 158, wherein the cover is removed to disclose the mechanism, embodies the principle of the magnetic coupling and the spring clutch. It is positive in operation and does not have to be separately mounted. The magnetic coupling at the right grips the tail end of the heavy tapered helical steel spring so that the latter clamps the driven member,

[1] Farrel Foundry & Machine Co.

Thus very little electric power is required to operate the coupling, which quickly releases when the electric circuit is opened. This device is used in connection with calenders and the like, and is built into the machine.

To eliminate danger of operators being caught in toggle presses making small molded rubber articles, in which a definite time is allowed for each cycle for removing the finished article, and insert-

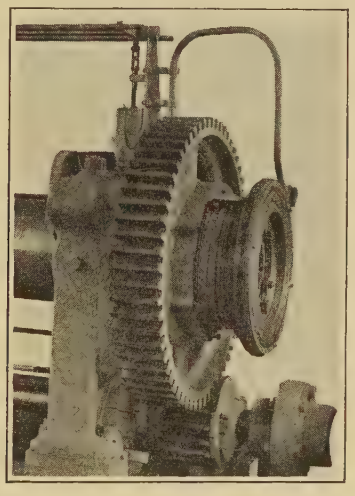


FIG. 158.—Farrel magnetic spring clutch.

ing the new material before the platen of the press starts upward, a Cleveland rubber company is using magnetic clutches operated by limit switches of the rotating cam type, according to J. K. Govan.[1] The operator must use both hands to press two push buttons to energize the clutch and start the press. The press automatically stops if the buttons are not depressed when the platen starts upward.

Small magnetic clutches are used in connection with theater dimmers and in various types of small machinery, the annular casing often consisting of pressed steel.

By using magnetic brakes in connection with magnetic clutches and couplings, so that the driven member shall be clamped by the brake when a switch is opened, it is possible to stop the driven machinery very quickly. Such combinations are respectively called clutch brakes and coupling brakes.

[1] *Elec. World*, Nov. 13, 1920.

Further reference literature is given in the footnote.[1]

78. Magnetic Separators.—The separation of magnetic materials by magnetic means is divided into three classes, as follows: concentration, reclamation, and protection.

Concentration.—Many valuable minerals contain iron, sometimes in minute quantities, but generally in sufficient amount and in a form which permits them to be influenced by an electromagnet when in either a raw or roasted condition.

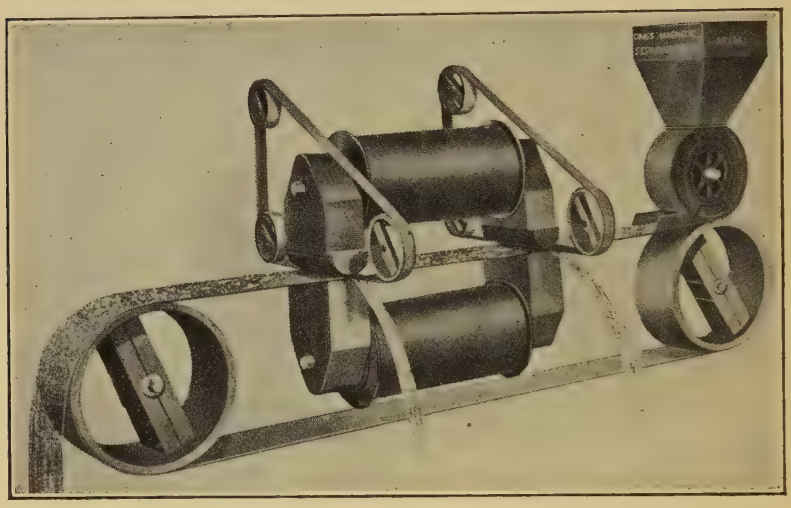


FIG. 159.—Ding's magnetic separator.

In the type of magnetic separator shown in Fig. 159 the dry magnetic material to be treated flows from the hopper onto the feed roller, which spreads it in a thin layer over the whole width of the conveyor belt as it travels towards the polepieces of the two electromagnets. The polepieces of the upper electromagnet are forged in the form of a sharp wedge, while the lower ones are flattened. This principle is characterized as "sharp against flat," and it is the high magnetic-flux density at the upper polepieces caused by the converging magnetic field which enables the magnetic separator to treat very weakly magnetic material. Since the magnetic-flux density at the upper wedge-shaped polepiece is greater than that at the lower polepiece, as the ore on the

[1] FERGUSON, C. B., "How a Magnetic Clutch Brake Solved a Synchronous-motor-drive Problem," *Elec. World*, vol. 67, p. 153, 1916; COSTELLO, W. H., "Use of Magnetic Clutches in Wisconsin Station to Operate Generators from Engines or Water Wheels," *Elec. World*, vol. 68, p. 372, 1916; "Magnetic Clutch Brake Used on Overhead Calender Drive," *Elec. Rev. and Western Electrician*, vol. 70, p. 508, 1917.

conveyor belt passes between the first polepieces, the magnetic particles jump towards the upper polepieces, but are intercepted by the cross-belt which quickly removes them from the magnetic field and allows them to drop into a receptacle provided at one side.

The use of a series of electromagnets of different strengths permits the separation from one another of materials having different permeabilities, as well as the separation of the magnetic from the non-magnetic. The separate removal of the strongest,

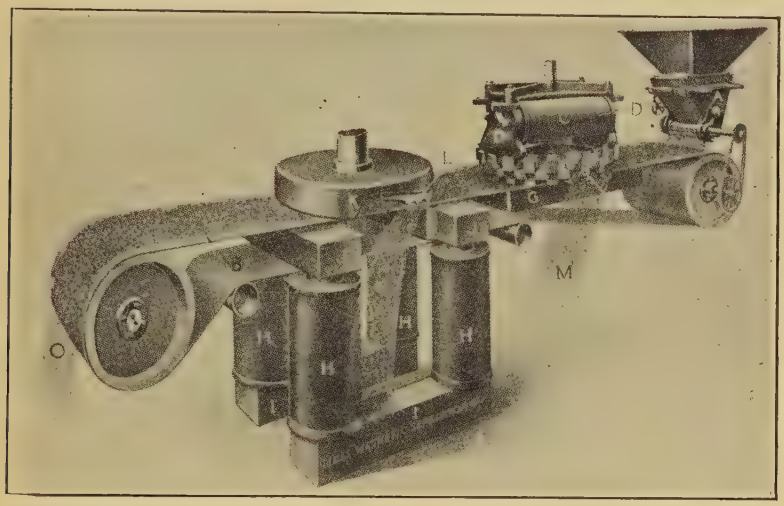


FIG. 160.—Ding's magnetic separator.

as well as the very weakest, magnetic materials is thus obtainable. As an example, in the treatment of monazite sand, the first electromagnet removes the magnetite, the second electromagnet removes the ilmenite, while the third and most powerful electromagnet lifts the monazite from the worthless sand. Although the magnetic material is lifted through a distance by the electromagnets, the holding properties of the electromagnets, particularly in other forms of magnetic separators, place magnetic separators in the portative class.

The magnetic separator in Fig. 160 comprises a feeding hopper *A*; an endless rubber-covered conveyor belt *B*; coils of electromagnet No. 1, or medium intensity, *C*; stationary polepiece *D*; revolving steel disk *E*, which is wider than the conveyor belt; auxiliary polepieces *F*, which are energized from polepieces *D*

and extract strongly magnetic materials from the conveyor belt and carry them beyond the side of the conveyor belt where they drop the magnetite; steel bar *G*, for conducting the magnetic flux from the auxiliary polepiece *F*; electromagnets *H*, including the dynamo-steel yokes *I*, for producing a very strong magnetic field; polepieces *J*; and the solid dynamo-steel armature *K*, which revolves over the conveyor belt *B*. This disk is energized from the polepieces *J* and attracts weakly magnetic materials and carries them beyond the side of the conveyor belt, where it discharges them. The grooves *L* around the periphery of the disk *K* form knife edges which cause the concentration of the magnetic flux at these points, thus producing a strong and dense magnetic field. *M* is the strongly magnetic material, *N* is the weakly magnetic material, and *O* is the non-magnetic material.

As points on the disk *K* revolve from one polepiece to the other, the polarity of each point changes midway between the two polepieces, so that when the magnetic material is midway between the two polepieces the disk at those points is magnetically neutral and, therefore, the magnetic material is no longer attracted to the disk. In fact, if there is any tendency on the part of the magnetic material to adhere to the disk at the neutral point, it will fall from the disk as that point on the disk changes polarity as it approaches the opposite polepiece. This statement applies equally to the disk *E*.

Reclamation.—Under this heading come such uses as the reclamation of shot and metal from sand in foundries, the removal of corset stays, tin cans, buttons, and other iron and steel pieces from refuse, as in city disposal plants, and other similar cases.

Protection.—In many industries it is necessary to remove iron and steel from the materials used in the manufacture of various products, and also to prevent damage to crushing machinery and pressure rolls due to tramp iron.

Magnetic separators are used in paper-pulp mills, cement mills, gypsum mills, gold mines, paper plants, lineoleum factories, rubber reclamation plants, fertilizer plants, brass and aluminum foundries, and so forth, the range running from sugar refineries, where iron oxide specks are removed from sugar, to coal mines, where coupling pins, mule shoes, and hammer and pike heads are removed from coal before it passes to the crushers.

Figure 161 illustrates the use of a magnetic separator pulley for the protection of crushing machinery, although this device

is generally used for all kinds of reclamation and protection. The general principle on which the magnetic separator pulley operates is somewhat similar to the magnetic chuck, except that the magnetic materials do not come into contact with the surface of the pulley. One form of magnetic separator pulley consists of a number of steel disks keyed to a shaft, with coils placed between alternate disks. These magnetizing coils are

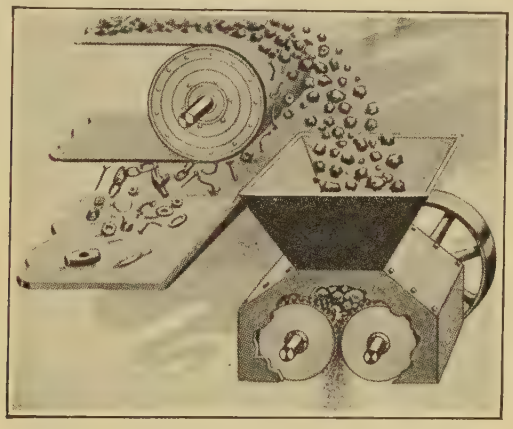


FIG. 161.—Principle of the magnetic pulley.[1]

wound on steel bobbins, which are dowled to the disks to prevent shifting. The terminal wires are brought out through a hole in the center of the shaft to a pair of slip rings located far enough beyond the edge of the pulley to permit the location of a bearing between the collector rings and the pulley. All coils are impregnated with a moisture-repelling and insulating compound, by the vacuum process, which insures the removal of all moisture and the penetration of the impregnating compound to all parts of the coil. A tubular spacer, which also performs the function of a coil shield, is placed between adjoining steel disks. When a direct current flows in the magnetizing coils, a magnetic field is established between the adjacent disks. This magnetic flux passes through the belt and causes the attraction towards the disks and against the belt of any iron or steel that may be contained in the material carried by the belt. As a result, these pieces are deflected from the course taken by other materials discharged from the belt, and are deposited in a suitable con-

[1] The Magnetic Company.

tainer, while the non-magnetic material falls into another bin. Magnetic separator pulleys are illustrated in Figs. 162 and 163.

Electromagnets also are employed to catch tramp iron which might otherwise enter cotton machinery, such as linters. Two or more electromagnets may be mounted on one assembling board

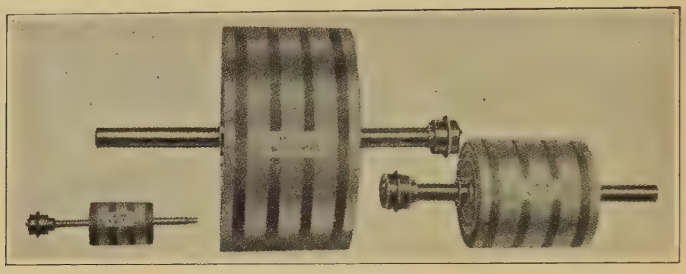


FIG. 162.—Magnetic pulleys.[1]

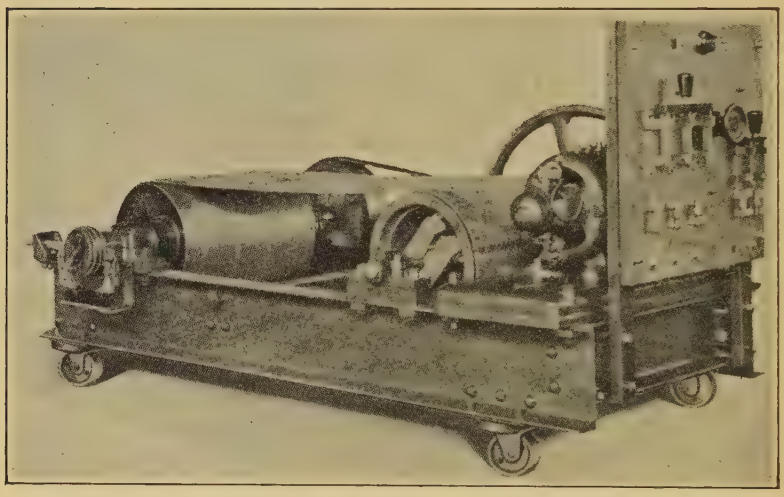


FIG. 163.—C-H portable magnetic separator. The alternating-current motor drives the pulley and a small direct-current dynamo for magnetizing the pulley.

as desired. Materials being fed to a machine are canted or slid over the faces of the polepieces of the electromagnets where any magnetic material is attracted and held until removed by hand at intervals.

The hand magnet in Fig. 153 is a convenient device for separating magnetic materials from other materials. While generally

[1] The Magnetic Company.

used in shops or shipping departments where brass and iron filings accumulate or for the removal of nails from sweepings, it has many other uses. Somewhat similar magnets also may be placed over chutes, and so forth, to catch tramp iron.

As a general rule, magnetic separators must be operated on direct current. One manufacturer of magnetic separators has announced an alternating-current magnetic separator, however.

Alternating-current electromagnets are employed in connection with inclined screen separators, wherein the vibration of the alternating-current electromagnet is applied to screening only for vibrating the screen. This is not a true magnetic separator.

79. Miscellaneous Uses.—There are many other uses for portative electromagnets. The hold-down magnet[1] for vertical

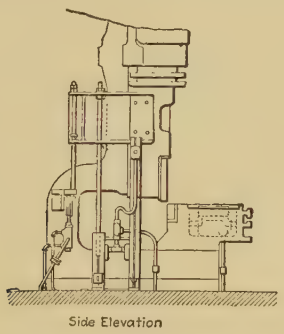


FIG. 164.—Hold-down magnet.

plate shears holds the plate while it is being cut, from one to four hold-down magnets being used in each equipment, according to requirements. Figure 164 shows the location of the magnets.

Among some of the most novel forms is the magnet hand for cripples, which, being provided with suitable straps and a ball-and-socket joint, enables one to grasp tools of magnetic material or those to which iron or steel plates may be purposely attached.

According to the *Scientific American*,[2] the railroad authorities at Berne, Switzerland, have developed a rather novel type of locomotive for switching cars which, it is claimed, has materially reduced loss of life and injuries. At the two ends of the loco-

[1] GUTHORN, S. L., "Hold-down Magnet Control for Vertical Plate Shears," *Elec. World* vol. 76, p. 1072.

[2] July 1, 1916.

motive, and directly opposite where the buffers project from the cars to be switched, are electromagnets so arranged as to engage the buffers of the cars, by what may be called a "magnetic ball and socket" under control of the engineer. Each magnet has a holding force of 3,740 lb. or 7,480 lb. for the two, with a good magnetic contact.

W. H. Addis has described in the *American Machinist* a magnetic packing plate, really a magnetic chuck, for use in carborizing operations, wherein the packer, after spacing the parts to be packed on the plate, presses a foot switch which causes the magnet to become energized. Then, placing the work in the pot, pressing it firmly into its bed of compound, and opening the foot switch, he completes the packing operation of a whole pot layer with one motion of the hands.

At least one concern manufactures an electric fishing tool for fishing bits and so forth from drilling wells. This may be described as an elongated lifting magnet of the annular type, having a sufficiently small diameter to go down a well.

A form of portative electromagnet is used to hold lamps and other objects against the iron or steel parts of machines, ship's sides, and so forth.

CHAPTER VI

SWITCHING, CONTROL, AND PROTECTIVE APPARATUS

The electrification of industries, railroads, elevators, and various equipment has developed switching, control, and protective apparatus that can only be briefly mentioned and illustrated herein.

80. Switches.—Switches are required to close electric circuits in such manners that there shall be very little electric resistance in the switch contacts, and to open the circuits in such manners that the arcing or sparking which may result shall not unduly damage or impair the contacts. In some cases the switch contacts are employed merely to close the circuit and to preserve its continuity, while in other cases they are particularly designed to break the circuit, usually under abnormal current conditions, in which case the switch is called a circuit breaker. Large switches which frequently break the circuit are provided with renewable contacts.

For certain purposes the familiar forms of manually operated knife and snap air-break switches or even telegraph keys and push buttons are suitable, but in industrial and other motor control switches magnetic blow-outs often are employed for extinguishing the arc at the switch contacts when an attempt is made to break the electric circuit. A magnetic blow-out is an electromagnet with an arc for an armature. The magnetic field of the electromagnet reacts upon the arc in such a manner as to force the arc away from the contacts and stretch it until the electric circuit will be completely broken. Magnetic blow-outs are shown on many of the switches illustrated in this chapter.

Switches employed to open and close electric power circuits often are provided with auxiliary or arcing contacts of carbon which precede the main contacts, which usually are of laminated copper. Copper coacting with graphite also is used to make a good electrical contact on closing and to prevent sticking on opening. When a switch operates to open an electric circuit, the carbon contacts maintain the continuity of the circuit after the

laminated copper contacts are separated from their coacting contacts, so that all the arcing takes place on the carbon contacts when they separate, thus protecting the main contacts from all arcing. In butt contact switches the contacts are made to roll on one another in such a manner that the arcing takes place on a different surface than that reserved to preserve the continuity of the circuit with very little electric resistance in the contacts.

The time-rate of heating of the contacts is directly proportional to (1) the ratio of the square of the active voltage to the resistance, (2) the product of the resistance and the square of the current strength, or (3) the square of the current density.

Switches also operate under oil and are called oil-immersed switches, but, while they readily rupture the arc, some of the oil is carbonized each time the circuit is broken and the oil must be renewed now and then. Furthermore, frequent ruptures of the arc under oil cause the oil and its container to become heated.

81. Definitions from A.I.E.E. Standards, 1922.—It is stated that "the A.I.E.E. Standards for Switching, Control and Protective Apparatus are the General Standards shown in Chap. II and III and the Standards in other chapters which are applicable to the devices involved, together with the modifications and extensions given in this chapter." The following[1] is taken from the text.

DEFINITIONS

Devices—General

7000. Switching and Control Apparatus. For the purpose of these Standardization Rules switching and control apparatus is defined as electric apparatus whose function is primarily to control or protect in some predetermined manner electric apparatus to which it is connected.

(7000) The "National Electrical Code" of the National Fire Protection Association deals with certain circuit breakers up to 500 volts rating and switches and fuses up to 600 volts rating for fuses.

7001. Switch. A switch is a device for making, breaking or changing the connections in an electric circuit.

7002. Master Switch. A master switch is a switch which serves to govern the operation of contactors and auxiliary devices of an electric controller.

7003. Control Switch. A control switch is a switch for controlling electrically operated switches and circuit breakers.

7004. Auxiliary Switch. An auxillary switch is a switch actuated by some main device, for signaling, interlocking, etc.

[1] The A.I.E.E. *Standards* herein quoted is given only as a guide and to encourage the use of the terms as defined. The *Standards* should be consulted for definite information.

7005. Circuit Breaker. A circuit breaker is a device (other than a fuse) constructed primarily for the interruption of a circuit under infrequent abnormal conditions.

7006. Contactor. A contactor is a device for repeatedly establishing and interrupting an electric circuit under normal conditions.

7007. Electric Controller. An electric controller is a device, or group of devices, which is designed to control in some predetermined manner the operation of the apparatus to which it is connected.

(7007) A switch (see 7001) should not be called a controller.

7008. Motorstarter. A motorstarter is an electric controller designed for accelerating a motor to normal speed in one direction of rotation.

(7008) A device for starting a motor in either direction of rotation is called a controller (see 7007).

7010. Automatic Motorstarter. An automatic motorstarter is a motorstarter designed to control automatically the acceleration of a motor.

7015. Fuse. A fuse is an element designed to melt or dissipate at a predetermined current value, and intended to protect against abnormal conditions of current.

(7015) Any terminals, tubes, etc., integral with this element, are included as part of the fuse.

Fuses may be divided into two classes:

(*a*) Those designed to protect the circuit and apparatus against definite amounts of overload (*e.g.*, fuses of the National Electric Code which open on overload in excess of 10 per cent).

(*b*) Those designed to protect the system only against short circuits (*e.g.*, expulsion fuses, which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed.

7016. Electric Protective Relay. An intermediate device, equipped with contacts to open or close an auxiliary circuit, by means of which one circuit is indirectly controlled by a change in conditions in the same or other circuits.

7017. Rheostat. A rheostat is a resistor which is provided with means for readily varying its resistance. See 3064.

7018. Protective Reactor. A protective reactor (see 3078) is a device for protecting circuits by limiting the current flow and localizing the disturbance under short-circuit conditions.

7019. Lightning Arrester. A lightning arrester is a device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration.

(7019) Lightning arresters may be divided into two classes:

(*a*) Those designed to discharge for a very short time.

(*b*) Those designed to discharge for a period of several minutes.

7020. Undervoltage or Low-voltage Release Switching and Control Apparatus. Undervoltage or low-voltage release switching and control apparatus is apparatus which, on the reduction or failure of voltage, oqerates to cause the interruption of power to the main circuit, but which does not prevent the reestablishment of the main circuit on return of voltage.

7021. Undervoltage or Low-voltage Protection Switching and Control Apparatus. Undervoltage or low-voltage protection switching

and control apparatus is apparatus which, on the reduction or failure of voltage, operates to cause and maintain the interruption of power to the main circuit.

7022. Phase-failure Protection Switching and Control Apparatus. Phase-failure protection switching and control apparatus is apparatus which, on the failure of power in one wire of a polyphase circuit, operates to cause and maintain the interruption of power on the circuit.

7023. Phase-reversal Protection Switching and Control Apparatus. Phase-reversal protection switching and control apparatus is apparatus which, on the reversal of the phase relations in a polyphase circuit, operates to cause and maintain the interruption of power on the circuit.

82. Electric Motors.—In general, the rotatable member of a direct-current motor is its armature and the stationary member is the field, while in alternating-current motors the terms rotor and stator are respectively applied. In direct-current motors the field coils are either connected in series or in shunt with the armature, or else each field coil consists of two windings respectively connected in series and in shunt with the armature. Hence, the terms series, shunt, and compound-wound motors. The only type of alternating-current motor here discussed is the induction motor, which may be either single-phase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotor. The secondary winding receives power from the primary by electromagnetic induction. Induction motors are generally of the squirrel-cage and wound-rotor types.

When the armature or the rotor of an electric motor is rotating at its rated speed with the rated voltage impressed across its terminals and the motor is delivering power at the rated output, there is a counter e.m.f. due to the generator action of the rotating member opposing the line voltage, so that the difference (algebraic for direct current, geometric for alternating current) between the line voltage and the counter e.m.f. is relatively small. However, when a motor is at rest and the full-line voltage is impressed across its terminals, there is a current inrush of abnormal strength which may not only injure the motor but also may cause current surges in the line.

83. Motor Protection at Starting.—While some direct-current motors having ratings not exceeding 10 hp., and induction motors having ratings not exceeding 5 hp., as limited by public utilities, may be directly connected to the lines by means of a switch, motors having higher ratings must be protected in starting by

means of resistors for direct-current motors, and autotransformers or resistors for alternating-current motors. As the rotating member accelerates and a sufficient counter e.m.f. is generated to limit the current strength, the auxiliary current-limiting devices are cut out or rearranged so that when the rotating member of the motor approaches full speed the full-line voltage may be applied to the motor terminals. A resistor causes a voltage drop across its terminals. An autotransformer supplies voltage of less magnitude than the line voltage to the motor. In the cases of direct-current motors, resistors connected in series are progressively cut out as the speed of the armature accelerates. In many cases, induction motors are supplied with voltages varying from 50 to 80 per cent of the line voltage until the rotor is sufficiently accelerated, when the full-line voltage is applied to the motor terminals.

84. Reversal of Motors.—The reversal of the direction of rotation of the armatures of direct-current motors is accomplished by reversing the connections between the armature and the field coils, which changes the relative polarities of the motor. In three-phase induction motors any two of the phases may be reversed or interchanged to accomplish the purpose.

85. Dynamic Braking.—When the direction of rotation of a direct-current motor is to be reversed, it may first be decelerated by opening the line switch and connecting a resistor, or resistors, across the armature. The motor then acts as an electric generator so long as the armature revolves by reason of the kinetic energy stored in it. This energy is converted into electric energy in the armature and thence into heat in the resistor.

86. Protection from Overload.—A motor "draws" current from the line in proportion to its load. When the motor becomes overloaded, the current strength becomes abnormal and is liable to damage the motor, particularly when a direct-current motor "stalls" or an induction motor "pulls out." In such cases fuses or circuit breakers interrupt the circuit.

87. Protection from Low Voltage.—When for any reason the line voltage decreases below a certain value or fails altogether, a motor should at once be disconnected from the line and again started in the usual manner when the proper line voltage is again maintained.

88. Motor Control.—The elements to be borne in mind in connection with the control of electric motors by means of apparatus

operated by electromagnets, as by automatic controllers, are:

1. The current is abnormally strong when a motor is started by switching it directly across a line.

2. The rotating member of a motor generates a counter e.m.f. which opposes the line voltage.

3. Time is required for the rotating member to accelerate from a position of rest to full speed during starting.

4. A motor draws current from the line in proportion to its load.

5. An overload may be of brief or delayed duration.

6. When the overload endures, the motor should be disconnected from the line.

7. When the line voltage becomes sufficiently reduced or fails, the motor should be disconnected from the line.

8. Mill and elevator motors are frequently started, stopped, and reversed. Hence, brakes are required for decelerating, and reversing switches are required for reversing, besides the motor-starting equipment.

9. Accidental reversal of phase may prove very disastrous.

10. Factory operators are not sufficiently skilled to start large motors by manually performing the various steps in their proper sequence or time relations.

89. Direct-current Motorstarter.—Referring to (1) and (2) in the foregoing paragraph, it is possible to start a direct-current motor safely by first connecting one or more steps of resistors in series with the armature, then closing the line switch and, as the armature accelerates and generates a counter e.m.f., successively short-circuiting the resistors until the motor is directly connected across the line and only the counter e.m.f. of the motor (besides its resistance) limits the current strength. This is the fundamental principle of all motorstarters. If, when starting a motor in the above manner, an ammeter is connected in series with the motor and a voltmeter is connected across the armature, the current strength will tend to attain a maximum when the line switch is closed and will decrease to a minimum when the normal speed of the armature is attained. But the voltage across the armature will be a minimum when the line switch is first closed, becoming a maximum when full speed is attained.

90. Alternating-current Motorstarters.—The squirrel-cage induction motor draws from five to eight times its rated current strength in starting when connected directly across the line. In

order to prevent this, an autotransformer with taps is provided so that a voltage varying from 50 to 80 per cent of the line voltage will be impressed across the motor terminals in starting. To start the motor, a lever is pushed backward to impress the lower voltage from the autotransformer across the motor terminals until the speed of the motor has approached its normal rating, when the lever is pulled forward to connect the motor directly across the line. Such a device is called a compensator.

91. Automatic Motorstarters and Controllers.—Automatic motorstarters and controllers are extensively used in connection with industrial plants, factories, electric railways, office buildings, hotels, apartments, and the like, where various machine tools, pumps, blowers, machines, trains, elevators, and so forth driven by electric motors must either be accelerated and otherwise controlled by automatic means when the master switch is operated by variations in pressure, water level, timing devices, and the like, or where the electrical equipment is too cumbersome or complex, or the ordinary employees are not sufficiently skilled to start, stop, and reverse the motors properly, as well as for safety, convenience, and economy. All such automatic devices are operated by means of electromagnets. When the master switch is situated at a distant point the term remote control is used.

By means of automatic motorstarters and controllers, a motor may be started and stopped from several different points. The fundamental devices used in such automatic control are the master switch, which is either manually or automatically operated, and the magnetic switch set in operation by the master switch. The magnetic switch is a relay with its coil connected in the pilot circuit, which is closed by the master switch. There may be a number of magnetic switches set in operation by the master switch and the phenomena due to the motor under control.

92. Direct-current Automatic Control.—The automatic control of direct-current motors is now general. The motor is started and stopped, just as lights are turned on and off, by means of a master switch which may be in the form of a push switch, which controls the first magnetic switch. A rheostat may be employed for adjusting the normal speed of the motor by varying its field strength so that any desired speed, over a considerable range, may be obtained on a machine tool, for instance. Other magnetic switches automatically cut out resistance from the armature circuit as the armature accelerates. In the cases of small motors,

only the first magnetic switch is required, since "across-the-line" starters connect the motors directly to the line.

93. Forms of Control Used in Automatic Motorstarters.—With the fundamental principles in view, the following methods are available for automatically starting motors.

1. Current-limit control, wherein the operation of the magnetic switches short-circuiting the starting resistors is controlled by the strength of the current in the motor after the master switch has been closed and as the motor accelerates.

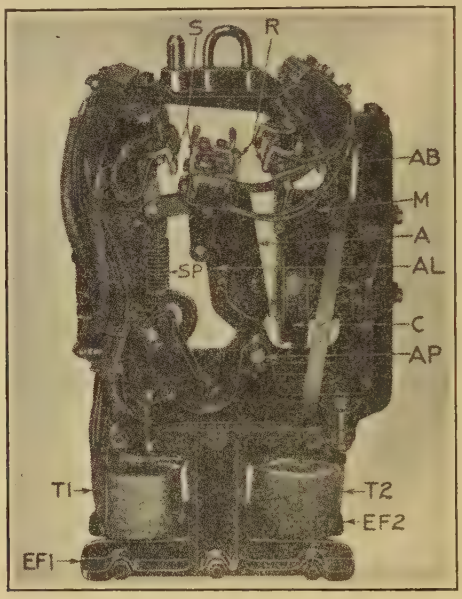


FIG. 165.—E. C. & M. automatic compensator. Note the shading coil *M* and the toggle.

2. Counter e.m.f. control, wherein the operation of the various magnetic switches is controlled by the magnitude of the counter e.m.f. of the armature.

3. A combination of current-limit and counter e.m.f. control.

4. Time-acceleration control, wherein the motor is accelerated in a given time interval regardless of the current strength.

The current-limit method of control of both direct-current and alternating-current motors is the most generally used because the motors are thereby accelerated in the minimum time interval for a given load, while the counter e.m.f. method is sometimes

used to control direct-current shunt motors. The time-acceleration method usually embodies a solenoid and some retarding means, as a dash pot or bellows, so that the pull of the solenoid will cause the contacts to close and thereby short-circuit the starting resistors in the proper order of succession and in a given time.

The automatic compensator shown in Fig. 165 is used for starting large induction motors by means of transformers having several selective voltages less than the line voltage. This is of the current-limit type of control.

In one system of direct-current control, the electromagnet of a magnetic lock-out switch is connected in series with the motor.

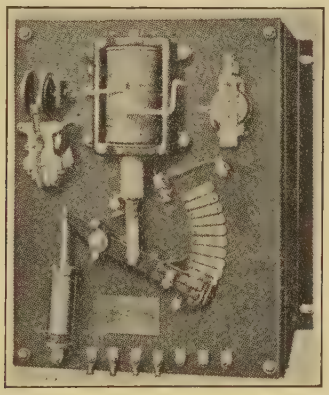


FIG. 166.—C-H special mine-duty automatic starter.

There are two or more of this type of magnetic switch, and all cut-out resistors as the armature accelerates. By means of the construction of the electromagnet, the switch is held open by the current in one portion of the electromagnet until the counter e.m.f. of the armature of the motor has so reduced the current strength that the other portion of the electromagnet can cause the contacts to close and thus cut out a resistor and transfer the control to the next magnetic lock-out switch in rotation.

A typical time-acceleration motorstarter is illustrated in Fig. 166, wherein the action of a solenoid is retarded by a dash pot.

94. Magnetic Switches.—In the air-break shunt magnetic switch in Fig. 167 the core and polepieces of the magnetic blow-out are swung back to show the butt contacts which can be easily

cleaned under these conditions. The blow-out coil consists of an edge-wound copper strip connected in series with the bottom contacts. The operating coil is wound with magnet wire of small cross-section, and is designed to be connected in shunt with the supply circuit.

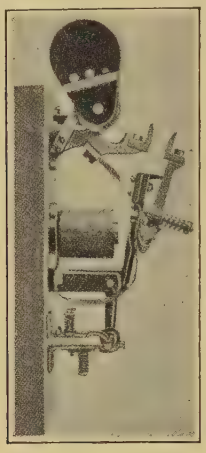


Fig. 167.—C-H shunt switch. Clapper-type electromagnet with taped coil.

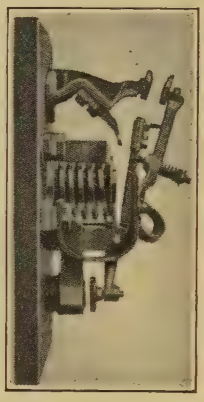


Fig. 168.—C-H magnetic series lock-out switch. The lock-out is at the bottom. Edge-wound operating coil.

In Fig. 168 is shown a magnetic lock-out switch, without the magnetic blow-out, designed for series operation in connection with current-limit control. The main operating coil consists of an edge-wound copper strip. The coil of the lock-out magnet, at the bottom of the switch, usually consists of a single turn of strap copper and is connected in series with the switch-operating coil above. The magnetic circuits and coils are so designed that when the current strength in the coils is relatively great, as when the preceding switch has just operated and the motor has not yet accelerated to a speed where the next resistor may be safely short-circuited, the lock-out magnet prevents the switch from closing, but, when the current is reduced by the acceleration of the motor armature and the consequent decrease in the current strength, the operating coil will close the switch. This type of switch usually opens when the current strength in the switch contacts is small.

In Figs. 169 and 170 are shown two types of spring-closed magnetic switches used in connection with dynamic braking.

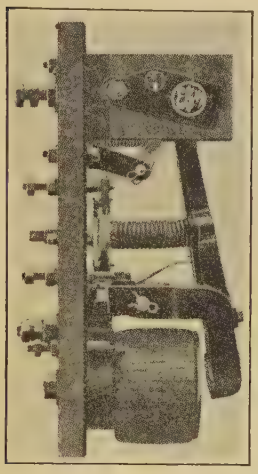


FIG. 169.—E. C. & M. single-pole spring-closed magnetic switch.

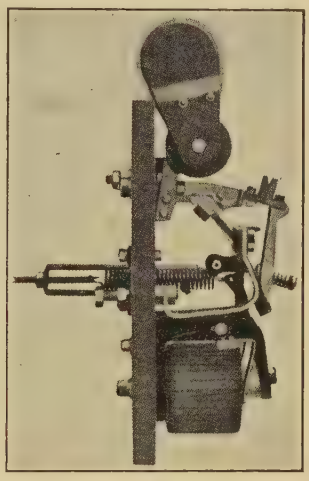


FIG. 170.—C-H spring-closed magnetically opened switch. Combination clapper and plunger type. Note method of holding the taped coil in place.

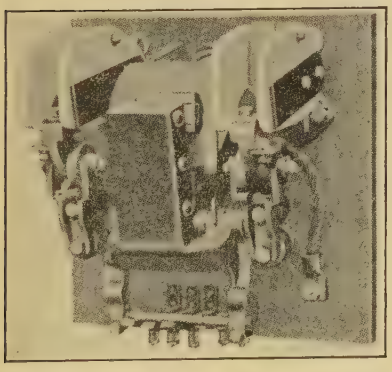


FIG. 171.—C-H standard alternating-current type magnetic switch. Clapper type. Double pole. Note shading coil in end of core.

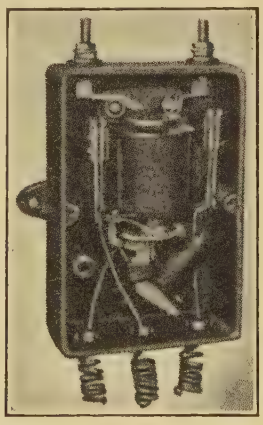


FIG. 172.—Sangamo remote-control, double-acting automatic switch.

These open only when a predetermined (or greater) voltage is impressed upon their coils.

Alternating-current magnetic switches or contactors usually are of the double-pole type, as they generally control two circuits simultaneously. Since the sealing pulls of alternating-current electromagnets of the type employed are not always sufficiently great to properly close laminated contacts, butt contacts alone are generally used. One form is shown in Fig. 171.

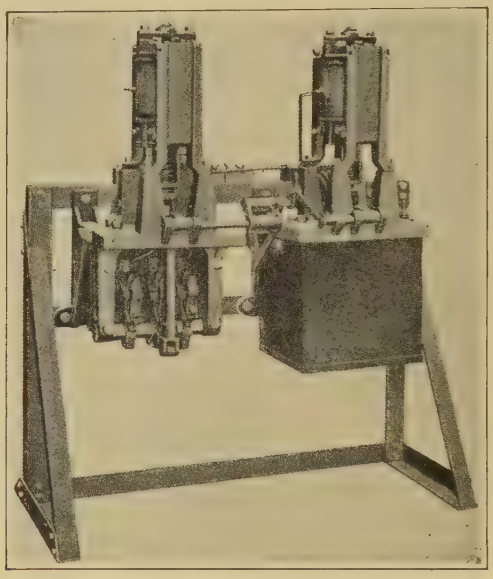


Fig. 173.—C-H high-voltage, oil-immersed reverse switch.

A double-throw, mechanically interlocked solenoid switch is so designed that should the voltage fail on one set of feeders the switch will automatically throw over to the other source of supply. This switch also is provided with magnetic blow-outs.

There are many types of magnetic switches and of various designs, but the above cases may be considered as general examples. The magnetic switch in Fig. 172 is operated by a solenoid and, besides the phosphor-bronze laminations and auxiliary arcing contact, it is provided with a ratchet device which turns the contact disk slightly each time the switch closes, so that a new surface of the disk is presented to the laminations. An example of an oil-immersed switch is shown in Fig. 173.

95. Lifting-magnet Control.—Figure 174 shows a magnetic-control equipment designed for the control of lifting magnets. In a sense, this is a form of remote control. When the lever of

the master switch, or master controller, is placed in one position, the circuit of the operating coil of the magnetic switch is closed, thus causing the circuit through the lifting magnet to be closed. By placing the control lever in another position, the circuit of the magnetic switch is opened. By placing the lever in still another position, the direction of the current in the lifting magnet, with the resistor (at the right) connected in series, may be reversed. The resistor limits the current strength so that just

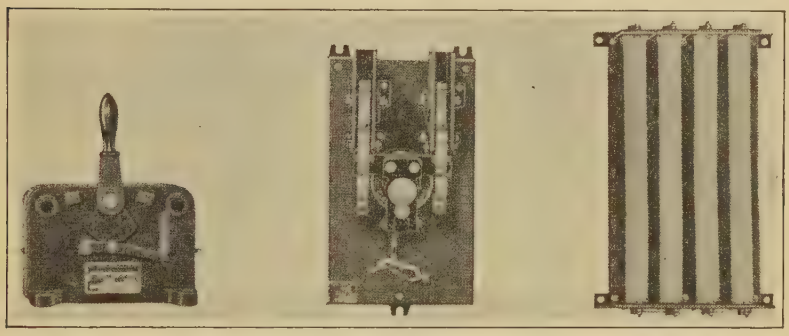


FIG. 174.—E. C. & M. lifting magnet control equipment.

enough magnetizing force will be produced to reduce the remanent magnetism to zero, or to a relatively small value, and thus cause a quick release of all of the magnetic load.

Another magnetic-control equipment for lifting magnets consists of a master controller and a magnetic-control panel with a resistor enclosed therein. The method of operating is somewhat similar to that described above, with the exception that the resistor is not connected in series with the lifting magnet when the reversed current flows therein, but is connected in shunt with the lifting magnet just before the circuit controlling the lifting magnet is broken. This causes the resistor to be in series with the coil of the lifting magnet when the latter is discharging its stored magnetic energy, so that this energy is converted into heat in the resistor rather than in an arc at the switch contacts, and undue straining of the insulation of the magnet is avoided.

96. Automatic Regulators for Railway Car Lighting.—Figure 175 shows an automatic regulator for maintaining constant voltage under the most irregular conditions. The following description was written for this treatise by Geo. E. Hulse,[1]

[1] Chief Engineer, Safety Car Heating & Lighting Co.

upon request, the subject being considered as a good example of automatic control and regulation.

In furnishing the light for a railway car, the fact that the car is continually on the move makes it necessary to obtain the energy from the stored supply on the car, or to generate it on the car itself.

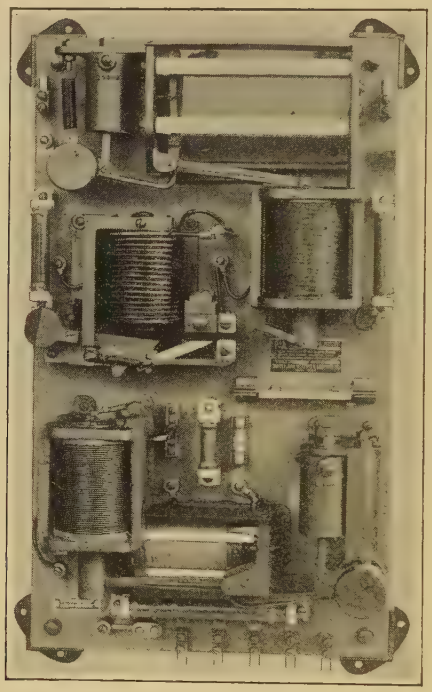


FIG. 175.—Generator and lamp regulator.

For furnishing electric light to a railway car, the system known as the *axle generator system* has become almost universally used on account of its flexibility of service and the fact that the car is not dependent on any stationary plant for its supply of energy for lighting.

With this system, the current for lighting is generated by a dynamo driven from the axle of the car. This dynamo provides current for the lights when they are in use, and also for charging the storage battery which provides current for the lights when the car is not in motion. The car provides its own power plant and has an ample supply of current available whenever the

car may be in service. Such attention as is necessary to the units of the system is easily given during its regular lay-over periods in the yard.

The essentials of such a car-lighting system are as follows:

1. The generator, mounted on either the car body or the truck, with some form of driving system between the car axle and the generator.

2. A storage battery for furnishing current when the car speed is not sufficient to drive the generator at a speed to give proper voltage for lighting.

3. A regulator to govern the output of the generator and keep it constant at varying speeds, and to charge the storage battery properly and protect it from overcharge.

4. A regulator to maintain proper voltage at the lamps.

5. Some means for keeping the polarity of the generator constant when the direction of movement of the car changes.

These conditions have been successfully met in several different ways. The majority of equipments in use have the following features:

1. The generator is mounted on the car underframe, and is driven by a belt from a pulley on the car axle.

2. The generator is controlled for output at varying speeds by a carbon pile rheostat in its field circuit, which gives a constant current output until the battery approaches full charge, when the battery is protected from overcharge by the control passing to voltage.

3. The lamp voltage is held constant by an automatic carbon pile rheostat.

4. The polarity of the generator is kept constant for properly charging the storage battery by an arrangement which shifts the brushes on the generator commutator 90 deg. each time the direction of travel of the car is reversed.

Automatic regulators for governing the generator output and the lamp voltage are, of course, a very important part of the system. An illustration of such a generator and lamp regulator is shown in Fig. 175.

As the generator is belted directly to the axle, the speed of the generator is at all times proportional to the speed of the train. As the output of a generator is directly proportional to its speed, it is necessary to have some means of controlling the output of the generator to prevent it becoming excessive at high

speeds, and causing damage to itself, or to the other parts of the equipment. It is also necessary that the generator shall deliver its full output at as low a speed as possible in order to generate enough energy to furnish light for the car at all times.

The regulation of the generator is accomplished in the most simple manner possible, that is, by placing a rheostat in series with the field of the generator. If, as the speed of the generator is increased, the current through the fields is decreased in proportional amount, the generator output will remain constant. This field rheostat consists of a short pile of thin carbon disks. The resistance of these disks is governed by the pressure which is exerted to force them together. This pressure is governed by magnets carrying generator output. In the regulator illustrated the front magnet of the solenoid type carries in its winding the total current generated. The magnet in the rear has current passing through its winding, which is proportional to the voltage generated. The lever attached to the plunger of the voltage magnet acts directly on the carbon pile. The lever attached to the plunger of the current coil acts on the carbon pile, by engaging the voltage lever. When the speed of the generator is increased above its full-load speed, the output tends to go above its full rated output, sending so much current through the windings of the current coil that the plunger is pulled into the winding, the spring against which it acts is stretched, and the pressure on the carbon pile is released, increasing its resistance, and decreasing the field current of the generator so that its output remains normal. If, for any reason, the voltage generated tends to go above its normal value, the plunger of the voltage coil is drawn into the winding and the same action takes place. The action of these two magnets, therefore, prevents the voltage or current output of the generator going above their normal values, protecting the generator from harm.

The regulator has another function—that of protecting the storage battery from overcharge, which is, of course, very harmful to a battery. This is done by limiting the voltage which can be applied to the battery.

If, with any number of cells of battery, the voltage coil of the regulator is so adjusted that a certain voltage cannot be exceeded, the battery will be automatically protected from overcharge, as the charge will automatically stop at a certain point, unless the voltage keeps on increasing.

When the generator is running below a certain speed, its voltage is lower than that of the battery. As the generator is connected directly to the battery, if this condition were allowed to exist the battery would discharge through the generator, and as the generator resistance is very low the current would be excessive, and would result in damage to the generator and quick exhaustion of the battery.

To prevent this, it is necessary to provide a switch that will disconnect the battery from the generator when the generator speed is so slow that its voltage is below that of the battery. This is accomplished by the use of a magnetic switch. This switch has two windings:

1. A winding which has passing through it a current proportional to the generator voltage.

2. A winding carrying the total current output of the generator.

The switch is so adjusted that when the generator voltage is equal to the battery voltage the voltage coil on the switch raises the armature and connects the generator to the battery. This immediately permits current to flow from the generator to the battery, and this current, acting through the current coil on the automatic switch, holds the switch tightly closed. As the air gap of this magnetic switch has been greatly decreased by the raising of the armature, the voltage at which the switch will open is much lower than that at which it will close. The current winding on this switch assists in opening the switch so that it will disconnect the generator from the battery at practically the same voltage at which it connects it. When the generator speed decreases, as its voltage falls slightly below that of the battery, the battery begins to discharge through the generator. This current, passing through the current coil of the main switch in the opposite direction to that in the voltage coil of the switch, neutralizes sufficient of the ampere-turns in the voltage coil to allow the switch to open at the proper voltage.

In order to charge the storage battery properly it is necessary that its discharge voltage be increased 25 per cent, that is, a battery which has a normal discharge voltage of 32 volts will require 40 volts before it is properly charged. The lamps which are used for lighting must have their normal voltage the same as the discharge voltage of the battery, so that they may be properly lighted when the car is standing still. The increased voltage

necessary to charge a battery would shorten the life of the lamps if it were applied to the lamps, and also would result in illumination which would be very bright when the car was running and low when it was standing still. It is necessary, therefore, to provide a regulator to keep the lamp voltage constant. This is generally of the carbon-pile type, in which the pile of carbon disks is placed in series with the lamps, between the lamps and the generator or battery.

The pressure on these disks determines their resistance. A magnet is provided which carries in its winding a current proportional to the lamp voltage. The plunger of this magnet, which may be of the solenoid type, operates a set of levers which control the pressure on the pile of carbon disks. When the voltage across the lamps tends to go above normal, the plunger of the magnet is drawn into the winding, and the pressure on the carbon pile is released, increasing the resistance in the carbon pile and holding the voltage across the lamps at normal.

On a car-lighting system with a normal voltage of 32 volts the best type of regulators will hold the voltage within 0.5 volt of normal, with a 50 per cent variation in the applied voltage and an amount of current used by the lamps varying from 3 to 50 amp.

97. Circuit Breakers.—Solenoid-operated circuit breakers are used to protect electrical apparatus from the effects due to overloads, short circuits, phase reversals, and other cases where electrical apparatus should be disconnected from the line. Consequently, there is a large variety of circuit breakers. Usually, carbon-break circuit breakers are employed, but magnetic blow-outs are used in some cases. In other cases, as in high-voltage alternating-current circuits, the circuit breaker contacts open under oil. A form much used, particularly in the smaller sizes, is known as the multiple-pole circuit breaker.

Usually, circuit breakers normally are closed and are automatically opened by a tripping coil, as the solenoid sometimes is called. In the case of the automatic reclosing circuit breaker, the functions of closing as well as tripping are automatically accomplished by means of electromagnets of the solenoid type. In this device, the core of the magnetic blow-out is movable endwise, thus also acting as an overload tripping coil. Other forms of circuit breakers are closed by means of solenoids. The tripping coil may be operable by current or voltage from the

circuit in which the circuit breaker is connected, but it usually is operated by a relay which opens or closes the tripping coil circuit.

To prevent flashing of the commutator on a rotary converter under short circuit, quick-opening circuit breakers have been developed which are capable of opening the direct current within the critical time of less than one-half cycle of the alternating current, or a period of 0.003 to 0.006 sec. according to a paper[1] by P. Forget and L. Wasser, abstracted in the *Electrical World* (vol. 82, p. 1231, 1923) and copied herein.

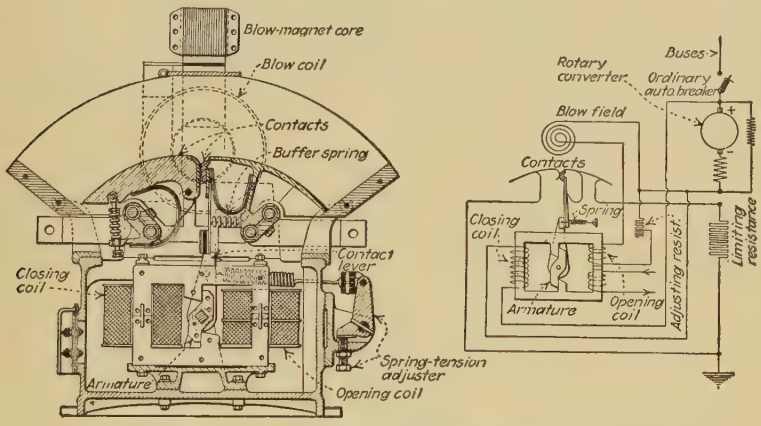


FIG. 176.—Quick-acting circuit breaker that inserts resistance into circuit within 0.003 sec.

This paper gives a very detailed description of the breaker, built and tested by an Alsatian concern. Theoretical investigations have indicated that an electromagnetic tripping of contacts under spring tension does not give a sufficient high speed. This breaker (Fig. 176) is held closed by a constant magnetic pull against heavy springs and opened by the sudden collapse of this field, achieved by an auxiliary counterfield set up by the short circuit. To obtain high speed the moving armature is made very light (less than 2 lb. for a 750-amp. breaker), the spring tension is very high (400 lb.), and the opening contact has a very short travel (about $\frac{1}{8}$ in.). These combined factors give the contact an acceleration of about 2,000 ft. per second. A powerful magnetic blow field is provided, the density of which reaches 10,000 gausses. Assuming a current strength of 7,000 amp., this blow field will exert a force of 14 lb. upon each half-inch length of arc. Tests with this breaker on an 800-kw., 500-volt rotary under a dead short

[1] *Bull.* de la Société Alsacienne de Constructions Mécaniques, Oct., 1923.

circuit disclosed, by oscillograms taken, an opening time of about 0.005 sec. This breaker is not used to open the circuit completely, but inserts into the circuit a protective resistance, normally short-circuited, which limits the current.

W. Kráska[1] has described a small automatic switch (Fig. 177) to replace the commonly used fuse, that can be screwed into the socket of an ordinary fuse block.

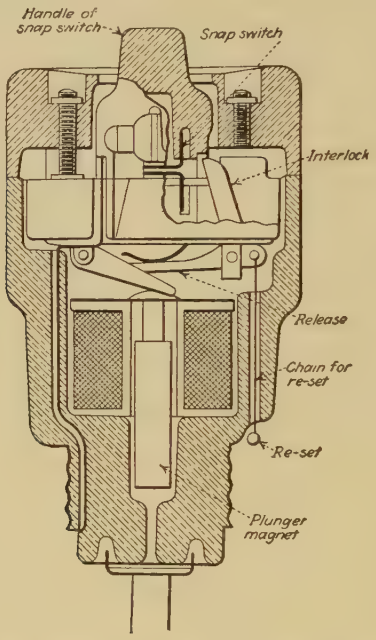


FIG. 177.—Switch that screws into fuse-plug socket gives adequate protection against overload.

It consists of a small rotary snap switch with a magnetic release by means of a plunger electromagnet placed in the bottom of the plug. A very essential detail is that the switch cannot be closed again after it has been opened by an overload. To reclose it the plug has to be first unscrewed and a small lever pulled down by hand. This prevents closing the switch upon a short circuit. The protective switch is rated at 15 amp., 250 volts.

[1] *Elektrotechnische Zeitschrist*, Oct. 11, 1923, abstract in *Elec. World*, vol. 82, p. 1181, 1923.

CHAPTER VII

RELAYS

Relays may be used for either automatically or manually controlling electrical or electromagnetic devices however remotely situated. The classifications explain the uses.

98. Electric Protective Relays—Classified According to Functions.—The following definitions[1] are taken from the A.I.E.E. *Standards*, 1922. The rule or section numbers are the ones therein given.

7026. Directional Relay. Any relay which functions in conformance with direction of power or voltage or current or phase rotation, etc.

7027. Power-directional Relay. Any relay which functions in conformance with direction of power.

(7027) This includes both unidirectional relays with single-throw contacts and duodirectional relays with double-throw contacts. The reason this name is preferred to "reverse power" is that the device is frequently used to function under normal direction of power. Furthermore, in some cases the normal condition of the system may permit power to flow in either direction. Relays for use in either alternating- or direct-current circuits are to be classed as "power-directional relays."

7028. Polarity-directional Relay. Any relay which functions by reason of a change of the direction of polarity.

7029. Phase-rotation Relay. Any relay which functions by reason of a change of the direction of phase rotation.

7030. Current Relay. Any relay which functions at a predetermined value of the current. These may be either overcurrent relays or undercurrent relays.

7031. Voltage Relay. Any relay which functions at a predetermined value of the voltage. These may be either overvoltage relays or undervoltage relays.

7032. Power Relay. Any relay which functions at a predetermined value of the power. These may be either overpower relays or underpower relays.

7033. Frequency Relay. Any relay which functions at a predetermined value of the frequency. These may be either overfrequency relays or underfrequency relays.

7034. Temperature Relay. Any relay which functions at a predetermined temperature in the apparatus protected.

[1] The A.I.E.E. *Standards* in this treatise are given only as guides and to encourage the use of the terms as defined. The *Standards* should be consulted for definite information.

7035. Open-phase Relay. Any relay which functions by reason of the opening of one phase of a polyphase circuit.

7036. Differential Relay. Any relay which functions by reason of the difference between two quantities, such as current or voltage, etc.

(7036) This term includes relays heretofore known as "ratio balance relays," "biased," and "percentage differential relays."

99. Electric Protective Relays—Classified According to Applications.

7040. Locking Relay. Any relay which renders some other relay or other device inoperative under predetermined values of current or voltage, etc.

7041. Trip-free Relay. Any relay which prevents holding in an electrically operated device, such as a circuit breaker, while an abnormal condition exists in the circuit.

7042. Auxiliary Relay. Any relay which assists another relay in the performance of its function and which operates in response to the opening or closing of its operating circuit.

7043. Signal Relay. An auxiliary relay which operates an audible or a visible signal.

100. Electric Protective Relays—General Qualifying Terms.

7050. Notching. A qualifying term applied to any relay, indicating that a number of separate impulses are required to complete operation.

7051. Inverse-time. A qualifying term applied to any relay, indicating that there is purposely introduced a delayed action, which delay decreases as the operating force increases.

7052. Definite-time. A qualifying term applied to any relay, indicating that there is purposely introduced a delayed action, which delay remains substantially constant regardless of the magnitude of the operating forces. (For forces slightly above the minimum operating value, the delay may be inverse.)

7053. Instantaneous.—A qualifying term applied to any relay, indicating that no delayed action is purposely introduced.

7054. Use of Terms. Where relays operate in response to more than one condition, all functions should be mentioned.

101. Telephone Relays.

12550. Relay. A relay is a device by means of which contacts in one circuit are operated by a change in conditions in the same circuit or in one or more associated circuits.

12251. Polar Relay. A polar relay is a relay which operates in response to a change in the direction of the current in the controlling circuit.

12252. Quick-operating Relay. A quick-operating relay is one which operates its contacts within a specified brief time limit.

12253. Quick-release Relay. A quick-release relay is one which releases its contacts within a specified brief time limit.

12254. Quick-acting Relay. A quick-acting relay is one which has the properties of both a quick-operating and a quick-release relay.

12255. Slow-operating Relay. A slow-operating relay is one which will not operate until after a specified delay.

12256. Slow-release Relay. A slow-release relay is one which when operated will not release until after a specified delay.

12257. Slow-acting Relay. A slow-acting relay is one which has the properties of both a slow-operating and a slow-release relay.

12258. Line Relay. A line relay is one whose coil is normally in the line circuit.

12259. Cut-off Relay. A cut-off relay is one which when operated disconnects from a line apparatus normally connected to it.

12260. Relay Coil Section. A relay coil section is one of two or more windings of a coil on one and the same core. The several sections may be concentric or placed side by side on the core.

102. Telegraph Relays.

12500. Relay. A relay is a device by means of which contacts in one circuit are operated by a change in conditions in the same circuit or in one or more associate circuits.

12501. Polar Relay. A polar relay is a relay which operates in response to a change in the direction of the current in the controlling circuit.

12502. Non-polar Relay, or Neutral Relay. A non-polar relay is a relay which operates in response to a change in the strength of the current in the controlling circuit, irrespective of the direction of the current.

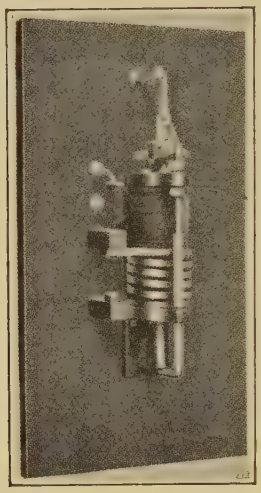
103. General Forms of Relays.—Relays operate on the principles of the galvanometer, watt-hour meter, and the induction motor, as well as on straight, balanced, differential, polarized, or other electromagnetic principles.

Electric protective relays operate circuit breakers to disconnect sections of line or apparatus which may be defective or threatening, and thus maintain the service in the majority of sections. Such relays may be either current relays, voltage relays, power relays, or phase-rotation relays, and may be of the inverse-time, definite-time, or instantaneous type. Many of these relays are operated by electromagnets of the solenoid type, current and voltage adjustments being made by adjusting the positions of the plungers.

An overcurrent relay is shown in Fig. 178. The shunt coil at the top is for resetting from the master controller. Figure 179 shows an inverse-time, overvoltage relay. A current-directional, low-voltage relay is illustrated in Fig. 180. A high-resistance resistor is connected in series with the shunt-polarizing coil at the bottom. The relay is closed by temporarily short-circuiting the resistor. The relay opens on either a reversal of current or failure of voltage. One form of phase-rotation relay operates on the principle of the squirrel-cage induction motor with operating coils corresponding to the stator and a non-rotatable hollow

aluminum cylinder functioning as a rotor, but in a straight line, moving up or down according to the direction of phase rotation.

The object of the time element in connection with relays is to hold the circuit closed as long as practicable after a disturbance



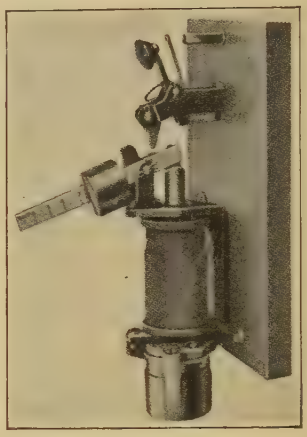


Fig. 178. Fig. 179.

Fig. 178.—C-H double-coil overcurrent relay. The series coil trips the relay and the shunt coil at the top is used for resetting from the master controller.

Fig. 179.—C-H inverse-time overcurrent relay with hand reset. Current adjustment 125 per cent load current. Time adjustment—minimum—15 sec. at 125 per cent load; 2 sec. at 500 per cent load. Time adjustment—maximum —120 sec. at 125 per cent load; 15 sec. at 500 per cent load.

occurs before opening the circuit in order to allow normal conditions to prevail. Definite time, as secured with clockwork or bellows, may hold the circuit closed for several seconds, say from 2 to 8 sec.; depending upon the circuit and the apparatus. In the solenoid types, bellows or oil pots cause the operating time to vary inversely as the pull of the solenoid. If such an inverse-time relay operates in 10 sec. when the current strength is 100 amp., it will operate in approximately 5 sec. when the current strength is 1,000 amp. Directional relays operating on the reversal or current or voltage are generally of the polarized type.

A simple form of differential relay has two coils on each core. Assuming that the number of turns in the coils are equal, and that they are equally positioned with respect to the cores, when two currents of equal strengths flow in the respective coils, but in

opposite directions, the m.m.fs. will be equally opposed and no magnetic field will result to operate the relay. Conversely, if currents flow in similar directions in both coils, an additive condition will prevail.

The contacts of relays operating circuit breakers are usually reset by hand, although they are also reset electromagnetically, as in Figs. 178 and 180. One form of hesitating control relay used

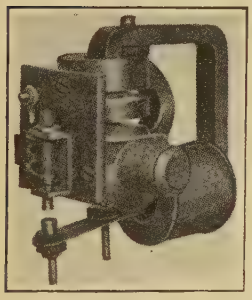


FIG. 180.—A. R. C. B. polarity-directional relay.

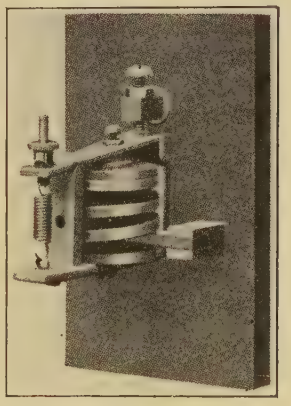


FIG. 181.—C-H direct-current series relay.

to close circuit breakers electrically has its coil circuit opened by an auxiliary switch on the circuit breaker as soon as the latter is closed. The relay holds its contacts closed about 1 sec. after its coil circuit is opened. This is accomplished by surrounding the relay plunger, inside the coil, with a heavy copper tube, which acts as a closed secondary coil. When the coil circuit is broken, the induced current in the copper tube tends to maintain the magnetic flux until the energy is converted into heat. When the greater part of the previously stored magnetic energy is exhausted, the plunger falls and opens the relay contacts.

Acceleration relays operate in conjunction with shunt-wound magnetic switches for accelerating electric motors. One type is shown in Fig. 181.

The function of an automatic generator voltage regulator is to maintain the voltage between close limits by varying the current strength in the field coils. With high field strength, the voltage of the generator will be too great and the voltage regulator functions to reduce the field strength, as by introducing

resistance. With low field strength, the voltage becomes too low and the voltage regulator functions to increase the field strength, as by cutting out resistance. Automatic generator voltage regulators are usually operated by electric motors controlled by relays of the contact-voltmeter type. In Fig. 182 the plunger of the voltmeter solenoid rises with increasing voltage and lowers with decreasing voltage. The two electromagnets

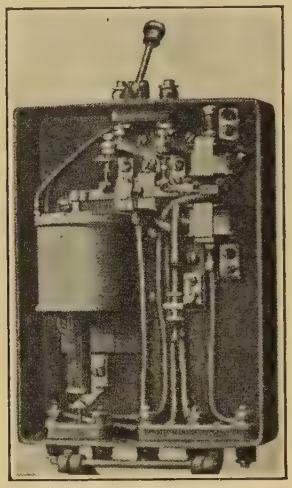


Fig. 182.—W. E. & M. contact-type voltage relay

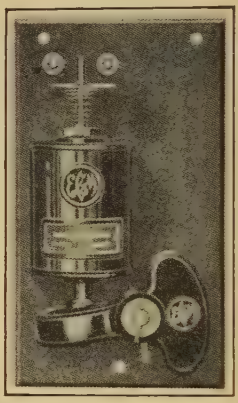


Fig. 183.—E. C. & M. shunt-field protective relay.

at the right assist the voltmeter solenoid in closing the motor circuit. The plunger of the alternating-current solenoid in Fig. 182 consists of soft-iron or steel wires. A cut-out relay is used in connection with high-voltage regulators to protect the system against voltage rise in case of failure of the voltage regulator.

A shunt field protective relay is shown in Fig. 183. The mass connected to the plunger at the bottom of the relay is to prevent the relay from responding to sudden fluctuations in the current strength and, being balanced, no additional force is required to move it when sufficient time is allowed.

Signal relays vary in size from small relays with contacts tipped with platinum, tungsten, or silver to the large magnetic switches previously described, according to the load on the cir-

cuit to be opened and closed. In code-calling systems in industrial plants and the like, a relatively large number of bells and other sound-producing devices may be connected in series on a lighting circuit. In such a case, medium-sized magnetic switches of the general class described are used. Signal relays of this class are controlled by small automatic master switches. Relays used in connection with fire and burglar alarms are of the closed-circuit type and function when their coil circuits are open.

Numerous forms of telephone relays are used for various purposes. In general, these relays must operate on relatively weak currents and must release when the current strength is a certain percentage of the specified operating current.

Track relays in railway signaling are short-circuited when a train enters the block across the rails of which they are connected. These relays may be of the watt-hour-meter type for alternating currents, or of the horseshoe-electromagnet type for direct currents. Because of the poor contacts which may sometimes be made across the rails, the low-voltage track relays must function between relatively small limits of current strength. The usual 4-ohm, direct-current track relay will pick up its armature on about 70 milliamp. and will release it when the current strength is halved.

Alternating-current telegraph relays are now being used on short lines. However, the current strength in such relays is proportional to the length of the air gaps in the relays in the line. Special forms of relays are designed for quiet operation on alternating current.

In the Sundh high- and low-level alarm device for totally enclosed tanks, two coils are placed over a gage glass at approximately the high and low positions, respectively. Inside the glass and riding on the liquid is a copper-and-nickel-plated laminated-iron cylindrical float. When the float moves within either of the coils surrounding the gage glass, it acts as a core and attracts an armature to close or open the alarm circuit, depending upon whether the alarm circuit is of the normally open or normally closed type. When the float is between the two coils the respective armatures are in their "safe" positions and no alarm is given.

104. Power-directional Relay Transformer.—H. C. A. Kortlandt[1] has described a system for protecting parallel feeders

[1] *Sterkstroom*, Aug. 15, 1923, Abstract in *Elec. World*, vol. 82, p. 1027, 1923

using straight overload protection at the generating end and power-directional relays at the receiving end of the line. Enough voltage for operation of the power-directional element even under

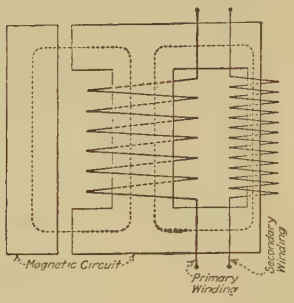


Fig. 184.—Power-directional relay transformer.

short-circuit conditions is obtained through the small transformer illustrated in Fig 184. The construction insures a high ratio at low voltages and a low ratio at higher voltage.

CHAPTER VIII

MISCELLANEOUS ELECTROMAGNETIC DEVICES

A large number of general and specific uses and devices are given, covering all uses and devices mentioned in this treatise, as well as a great many which are not herein discussed. The list is by no means complete. Then follow descriptions of specific applications of electromagnets in common use, as well as relatively new uses, as in the lumber industry.

105. Some Uses and Names of Devices.

Advertising devices.
Air brakes—to operate all brakes simultaneously.
Alarms—burglar, lightning-arrester, fire.
Annunciators.
Arc lamps.
Battery chargers.
Bells.
Blasting machines.
Blow-outs—magnetic.
Brakes—magnetic, mechanical.
Breaking filaments of used lamps.
Buzzers.
Cable locks.
Cars—elevator, passenger, street.
Change makers.
Chimes, tower.
Chronographs—for measuring the velocities of projectiles.
Chucks, magnetic.
Circuit breakers.
Clamps, magnetic.
Clocks—mantel, master, program, secondary, time.
Coils—induction.
Coin-operated devices.
Control—automatic, manual, remote, speed.
Crossings—electric, at grade.
Cutting-off machines.
Demagnetizers—tool, watch, work.
Door openers—for cars and apartments.
Electrotherapeutic apparatus.
Fields—magnetic, for dynamos, generators, and motors.
Fishing tool, magnetic—for drilling wells.
Flashers, sign.

Gear shifts—for automobiles.
Hair clippers.
Hammers, electric.
Hand—magnetic.
Handy magnets—for automobilists and machinists.
Heaters.
Holding devices.
Horns—electric.
Ignition systems.
Indicators—electric.
Instruments—indicating, measuring, recording, scientific.
Lamp holders.
Lifting magnets.
Locks—time.
Loud speakers.
Magnetizers—for magnetizing permanent magnets.
Magnetos.
Measuring apparatus.
Meters—electric, demand.
Motion-picture apparatus.
Musical instruments—banjos, organs, pianos.
Pendulums—electric.
Permeameters.
Phonic wheels.
Phonograph motors.
Pick-up magnets—for automatic train stops.
Printers—electric, telegraph.
Protective devices.
Punches—magnetic.
Push-button switches—for hospitals.
Radio apparatus.
Registers—cash, fare, telegraph.
Regulators—feeder, furnace, temperature, voltage.
Relays.
Reversing mechanisms.
Safety devices.
Saws—for raising and lowering.
Selectors—telegraph, telephone.
Separators—magnetic.
Signals—audible, telegraph, telephone, visual.
Spark plug—magnetic.
Starters—motor.
Stamps, time.
Stock listers.
Stops—automatic, engine, loom, train.
Switches—automatic, magnetic, time, track.
Switching cars.
Surgical magnets—eye, flesh.
Telegraph—printing, radio.

Telephone—automatic, exchange, radio, subscriber.
Testing—magnetic.
Tool holders—magnetic
Toys.
Trains—lighting.
Tuning forks—electric.
Valves—magnetic—for air, fluid, steam, vapor.
Vibrators—induction coils, massage, screens.
Whistles—fire-alarm, fog, steamship, and wherein the electromagnet compresses the air.

A portion of the above list was published in the *Electrical Record.*

106. Magnet Brakes.—A magnet brake is a friction brake controlled by electromagnetic means. Magnet brakes may be generally classified as shoe, band, and multiple-disk magnet brakes.

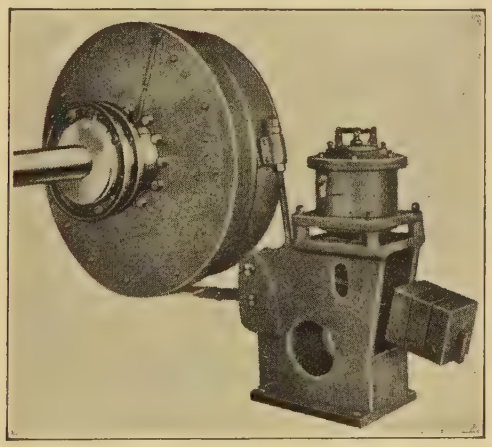


FIG. 185.—C-H band-type magnetic-clutch brake.

The most general application of magnet brakes is in connection with electric cranes, elevators, and hoists of all kinds, in mines, steel mills, and other industrial plants, but they are also used on lift and draw bridges, coal and ore bridges, turntables, printing presses, line shafting, and so forth. The electromagnets are connected in series with the motor on direct-current installations, and in shunt with the motor-control circuit on alternating-current installations.

Figure 185 shows a typical clutch brake which is used in connection with magnetic clutches. However, most magnet brakes are of the shoe-type. The weights in the magnet brake in Fig.

185 are lifted while the motor is operating. When the current is switched off, the total weight of the lifted parts is released to apply the brake.

Springs, instead of weights, are employed in the magnet brakes in Figs. 186 and 187.

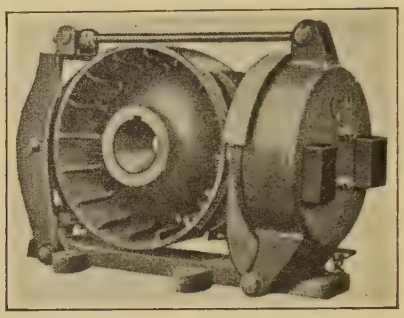


FIG. 186.—C-H magnet brake.

In the G. E. automatic solenoid brake the length of the air gap is automatically held constant by a ratchet wheel and pawl as the shoes wear. Constant shoe clearance is automatically maintained by means of a split-toggle clamp, spring, and set

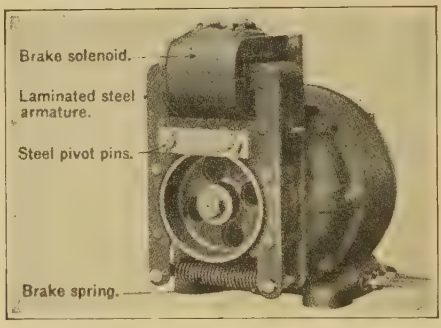


FIG. 187.—W. E. & M. quick-acting magnet brake.

screws. On release, the plunger strikes a floating stop, thus eliminating the hammer blow. On application, the moving parts descend smoothly through the action of a spring or dash pot, the plunger dropping on springs interposed between the core and lever in the smaller sizes, and a simple form of oil dash pot being used in the larger sizes. The dash pot is single-acting in the downward direction. The devices described overcome jar

and shock in the application of the brake. The dash pot is adjustable and can be used to obtain time application if required. No additional weights are mounted on the brake lever in the alternating-current solenoid brake, as the movable yokes of the solenoid are very massive.

In all types of magnet brakes it is desirable to eliminate the hammer blow and prevent jar and shock in the application of

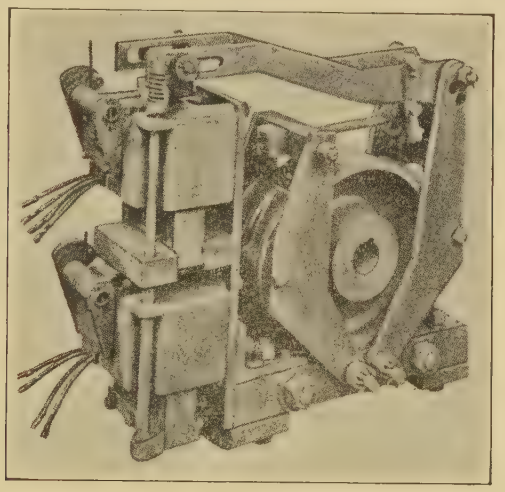


FIG. 188.—G. E. double-solenoid brake.

the brake. Coils should be rain- and moistureproof, and still be ventilated. Hence, they are impregnated and often are enclosed within a steel frame. The frame is a good thermal conductor and has a greater cooling surface than the coil.

The double solenoid brake in Fig. 188 is designed for graduated braking of alternating-current motors. The solenoids have two or more windings, which usually are connected across one phase of the line. Numerous combinations of braking may be obtained with this brake, which is applicable to cranes, such as revolving hammer heads, traveling gantry cranes, and fast trolleys with swinging loads, to prevent backlashing of the gears, in the first case, and to prevent gantrys from being blown along the track. A similar brake is used for the control of crane-hoist induction motors. This general class is called the multiple-magnet brake.

In the magnet brake in Fig. 186 the armature is an integral part of one of the arms carrying the brake shoes. A short air

gap greatly reduces or eliminates shock on setting and hammer blow on release.

A form of multiple disk magnet brake is shown in Fig. 189. Such a magnet brake consists of a friction case and an electromagnet of the iron-clad type. The former contains stationary friction plates and rotating friction plates secured to and keyed to a hub. A compression spring normally forces the plates

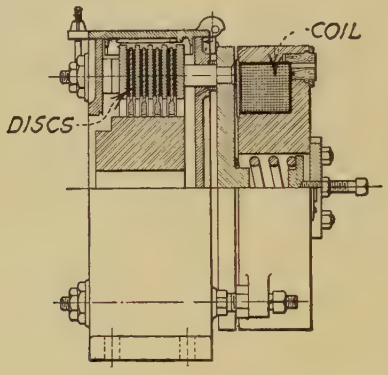


Fig. 189.—Disk brake.

or disks together, thus applying the brake. The brake is released by energizing the electromagnet. Among the advantages claimed for the multiple-disk magnet brake is independence from non-uniform wear of brake members which might cause a side strain on the shaft.

Magnet brakes are rated in pounds retarding torque at 1-ft. radius on the brake shaft. The term "retarding torque" is applied only to a revolving shaft condition, since the coefficient of friction is somewhat greater when the brake is at rest than when it is revolving.

107. Magnetic Valves.—These are usually solenoid-operated and either operate directly or control the operation of valves for steam, air, gas, liquids, and so forth. Solenoid-operated valves are used for controlling the temperature in passenger cars and buildings to some extent and are controlled by thermostats.

The solenoid-operated valve in Fig. 190 is used in connection with steam heating. The valve is normally held open by a spring and is closed by the solenoid.

In Fig. 191 is shown the method of mounting the plunger of a solenoid-operated valve having a capacity up to 200 lb. per square inch, and designed for use where it is desired to operate the valve from remote points or automatically in connection with motor-driven pumps, compressors, and so forth. The valve is kept tightly closed by fluid pressure which is admitted on top

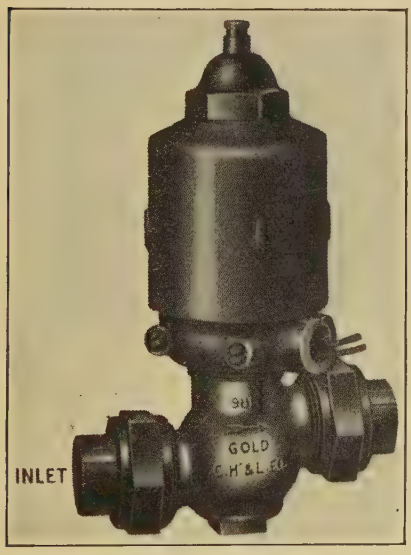


FIG. 190.—Gold magnetic valve.

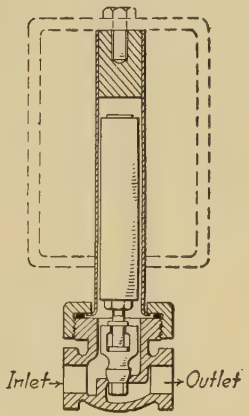


FIG. 191.—C-H magnetic valve.

of the valve. When opening, the plunger lifts with a short, free travel, delivering a hammer blow to unseat the valve. The pressure becomes nearly balanced after the valve is started. The weight of the plunger closes the valve, after which fluid pressure tightly seals it.

In one form of automatic regulator for pulp-wood grinders the control is obtained through the use of two alternating-current solenoids. In another type of valve the solenoid controls the valve to which it is connected by operating a smaller valve which allows steam to operate a piston within the larger valve, thereby causing it to open or to close.

Steamship and other whistles are controlled by solenoid-operated valves, either by a manually operated switch or automatically by code-calling or other signaling mechanisms. In the smaller types, the solenoid may be fastened to the valve stem of the whistle.

The magnetic valves of large pipe organs comprise disks attracted by "hair-pin" horseshoe electromagnets, although solenoid-operated valves are now being used for this purpose. The valves must instantly respond to the pressure on any key and are called upon to function in rapid succession. The small magnetic valves control air-operated valves which, in turn, permit the air to flow to the pipes.

In a semi-automatic hydro station described by E. R. Stauffacher and G. Clingwald,[1] a solenoid operates to open a control valve which causes the governor to act to shut down the water wheel.

108. Solenoids in a Sawmill.—A. H. Onstad, of the Weyerhauser Timber Company, gave the following information in a letter in October, 1919, with permission to publish it. The mill in question was remotely situated in the state of Washington and difficulty was experienced in obtaining equipment in wartime. Therefore, Mr. Onstad designed the solenoids which were constructed from his sketches. This is but one example of the many possible uses of electromagnets, but it is a commendable example of what may be done in this branch of the woodworking industry, particularly under the circumstances.

We are using solenoids for two different purposes, one is for operating the valves on steam cylinders. Through the mill we use a large number of steam cylinders, in sizes varying from 4-in. diameter and 8-in. stroke to 18-in. diameter and 42-in. stroke, the functions of these cylinders being to operate log rollers, loaders, and also for moving timbers sidewise off the rolls, or in other cases from transfer chains onto the rolls.

In large mills it is frequently desirable to operate or to be able to control the operations of one cylinder from two or more widely separated points. We therefore reached the conclusion that the most satisfactory way to do this would be solenoids, and when we found we could not buy them we built a solenoid having an effective pull of 2 in., capable of exerting 25 lb. at the start of the plunger travel, and a pull of 75 lb. through the last half-inch of the stroke.

These solenoids worked so successfully that on a mill which we built later on we decided to apply solenoids instead of air cylinders for raising and lowering saws used in the trimmer table. This trimmer table consists of a horizontal table about 9 ft. wide and 24 ft. long, under which are located 13 saws spaced 2 ft. center to center. Across this trimmer table and at an angle of 90 deg. from the line of the saw arbors are transfer chains on which the lumber is placed, an operator then raising one or

[1] "Semi-automatic Operation Economical in Small Hydro Station," *Elec. World*, vol. 78, p. 213, 1921.

FIG. 192.—Trimmer solenoids

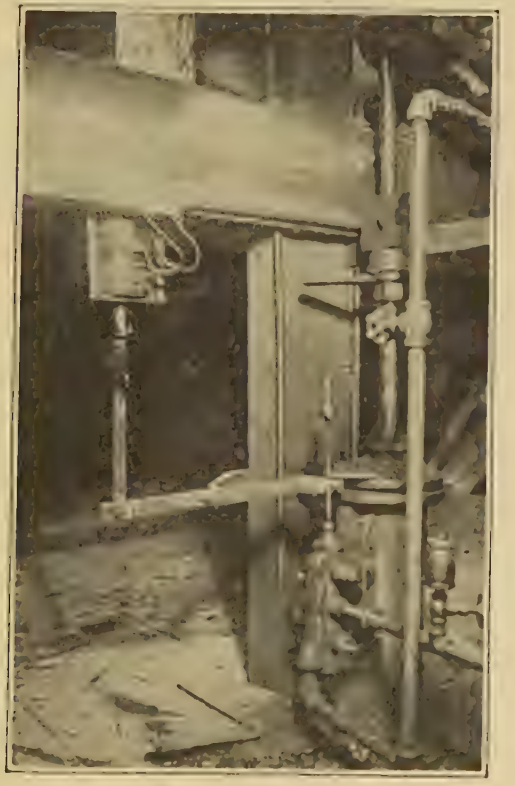


FIG. 193.—Valve-operating magnet.

more saws as he may desire to cut the lumber into desired lengths, and also cut out defects.

Formerly, these saws were either raised by hand, through a system of bell cranks and levers, or raised with air cylinders located under the trimmer table and cords controlling the valves brought to a central point convenient for the operator. During the cold weather these air cylinders gave more or less trouble, due to the valves freezing, so I decided to apply a solenoid to these saws to replace the air cylinders.

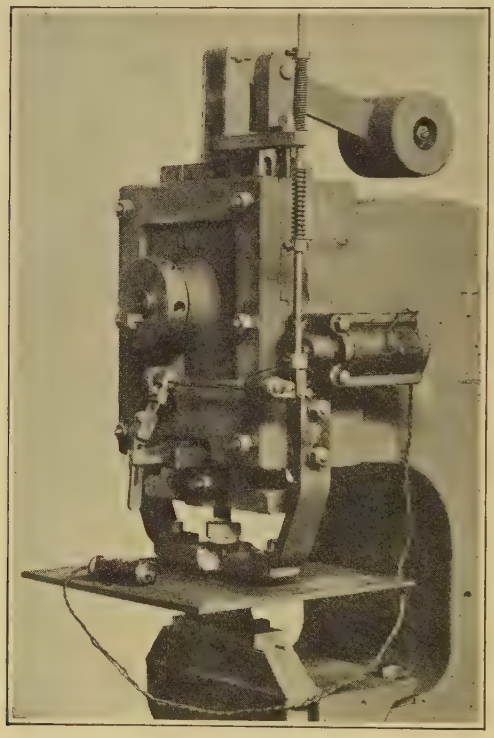


FIG. 194.—Safety solenoid.

The conditions for which I found it necessary to provide were a 50-lb pull effective through a stroke of 4½ in., 220-volt direct current was available, so I wound some coils and applied them as solenoids to this trimmer. Barring a little trouble we had in connection with our switch which we have overcome, these solenoids have now operated successfully for just about a year and a half. The operation of these solenoids is very frequent, as for long periods boards are put into this trimmer at the rate of 45 per minute, and, as nearly every board must be cut in a different manner, two of the solenoids are usually brought into action for

every board that goes over the trimmer. On the next mill which we build, we will use more solenoids for various controls than we have up to the present.

Figure 192 shows the group of solenoids beneath the trimmer, the rods being connected to the saw frames and the conical-shaped pans serving to keep sawdust from the solenoids. Each solenoid is protected by the insertion of resistance at the completion of the stroke. The helical springs on the plungers were used to offset the effects of sticking. The method of tripping the valves on steam cylinders is illustrated in Fig. 193.

109. Solenoid Control for Gags of Punching Machines.—By the simple application of the solenoid to the machine in Fig. 194, no punching operation can be performed until the switch is purposely closed. The illustration was furnished by the late W. T. Sears, who stated that he was a joint inventor.

110. Surgical Magnets.—Steel and iron workers frequently get bits of these materials in their eyes or flesh. The removal of such pieces by ordinary surgical methods requires preliminary probing, which often is very painful. Should the piece strike the eye with sufficient force to become embedded, its removal by the use of the knife is likely to be difficult as well as painful. The protecting coating of the eye must be cut, and there is danger that, instead of removing the particle, it may be pushed further into the eye. A large surgical magnet[1] in use in the relief department of one large electrical manufacturer is supplied with electric energy at the rate of 4,000 watts (4 kw.), and is designed for operation at 70 volts. Such electromagnets usually are equipped with special attachments, like conical and bent removable polepieces, which may be used for the particular operation for which the attachment is designed. The principle of the magnetic separator, characterized as "sharp against flat," may be advantageously applied to operations affecting the hand.

111. Lamp-filament Breaker.[2]—Among the thousands of incandescent lamps returned as having been burned out by lighting customers are many which, while not entirely useless, are unfit for further service. To prevent purchasers from sorting the units and reselling those with the filaments intact, the Boston Edison Company constructed an electromagnet with a laminated U-shaped core, weighing about 80 lb. The core was wound with 55

[1] *Iron Age*, p. 452, Feb. 25, 1915.
[2] *Elec. World*, vol. 68, p. 233, 1916.

lb. of No. 16 cotton-covered magnet wire. The number of ampere-turns was about 7,600. By means of a quick-acting socket the lamps could be placed between the poles of the electro-

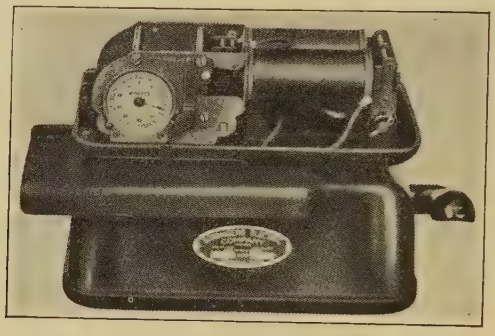


FIG. 195.—Stromberg time-stamp mechanism

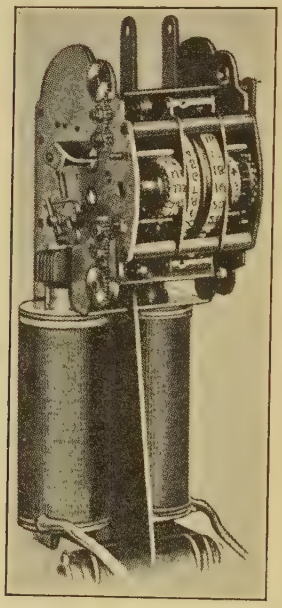


FIG. 196.—Stromberg time-clock mechanism

magnet where the filament was instantly broken by the mechanical force developed between the current in the filament and the magnetic field of the electromagnet.

112. Time-recording Devices.—Figures 195 and 196 respectively illustrate the mechanism of a time stamp and a time clock.

113. The Weiss Electromagnet.—In the type of electromagnet, designed by Prof. P. Weiss for scientific purposes, the coils are wound with hollow copper through which circulates cooling water. In consequence, the temperature of the coils may be maintained constant during experiments. This electromagnet, which is scientifically designed and constructed, produces a field strength of 52,500 gausses between its opposed and tapered poles when the length of the air gap is 1.5 mm. and its diameter is 3 mm.

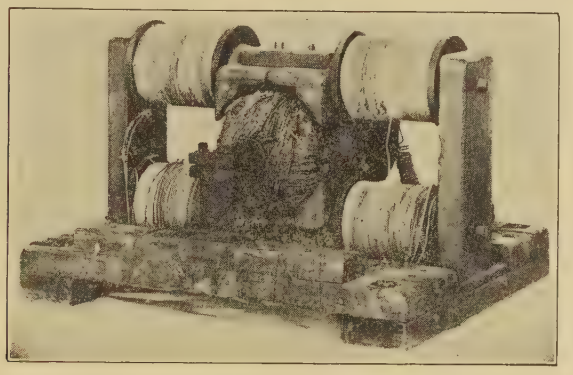


FIG. 197.—Elihu Thomson's first arc-light generator.

114. Original Thomson Arc-light Generator.—The arc-light generator in Fig. 197 is of historical interest, being one of the first important inventions of Prof. Elihu Thomson, and built entirely by him in 1878 to 1879. This machine was described by C. D. Wagoner in the *Electrical Review* (June 12, 1920), to whom the author is indebted for the cut. The machine weighed about 350 lb. with its base of planks bolted together, and is now preserved in Lynn, Mass. Because of the difficulty in securing sheet-iron plates, the armature core was assembled from a number of thin, soft castings. This machine is said to have supplied transformers from its alternating-current side early in 1879.

One object of placing the description of this most important invention in the present chapter is to emphasize the fact that all static and rotating or other movable electromagnetic apparatus and machinery are, after all, combinations of various forms of electromagnets.

CHAPTER IX

SOLENOIDS

Magnetic leakage is referred to and the magnetic-field and mechanical-force characteristics of direct-current solenoids without plungers are then discussed.

115. Magnetic Linkage and Magnetic Leakage.—The equations generally given for magnetizing force, self-inductance, and permeance are based on the conditions for a toroidal electromagnet, that is, where all of the magnetic flux in the core hole produced by an electric current in the coil is assumed to be completely linked with all of the turns of conductor in which said electric current flows. The magnetic flux in the core hole of a toroidal solenoid links with all of the turns whether only space, air, and other materials of unit permeability are present or whether the core hole is filled with a uniform magnetic material. In the latter case, the structural flux joins the electromagnetic flux to increase the number of complete linkages, that is, the total number of ampere-weber-turns. The total magnetic energy in joules stored in the core hole of a toroidal electromagnet is equal to one-half the number of ampere-weber-turns.

In the case of a straight solenoid, a bar electromagnet, or in about all of the various forms of commercial electromagnets, some (often very much) of the total magnetic flux takes the most natural path back to some place of opposite polarity and, therefore, does not link with all of the turns. Some of the flux produced in the coil wall does not enter the core hole at all. Magnetic flux also is said to leak when it does not pass to, or link with, some magnetic material forming a part of the electromagnetic system, although all of this portion of the magnetic flux may be linked with the turns. As the result of magnetic leakage in a straight sloenoid, the magnetizing force is greatest at the middle or neutral-polarity plane of the solenoid, since it is there that the linkage of magnetic flux with turns of conductor is most complete. However, the m.m.f. in gilberts is always equal to 1.257 times the product of the number of turns and the current strength in

amperes in said turns, that is, to 1.257 times the number of ampere-turns.

In the following pages, factors may be introduced to allow for the magnetic leakage so that the lengths may be expressed directly in terms of solenoid lengths and solenoid radii, and cross-sections may be expressed in terms of the cross-section of the core hole.

116. Magnetizing Force along the Axis.—The magnetizing force (electromagnetic flux density) at the center of a single turn of conductor of small cross-section (as a small round wire) is

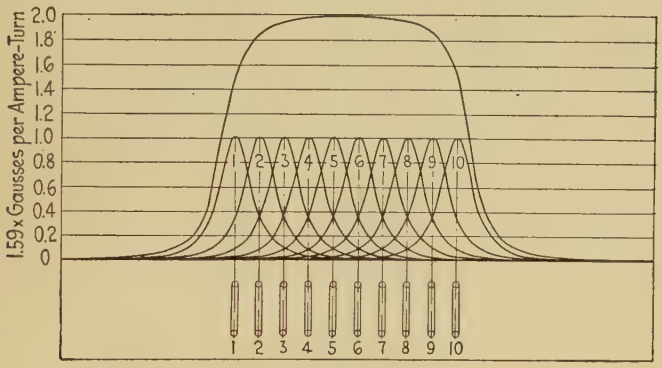


FIG. 198.—Gausses per ampere-turn for a solenoid.

equal to $0.2 \times 3.1416 = 0.628\ 32$ times the ratio of the number of ampere-turns to the average radius r_a, in centimeters, of the turn. Hence, the ratio of the magnetizing force to the number of ampere-turns, or the number of gausses per ampere-turn, is equal to $\frac{0.628\ 32}{r_a}$. At points along the axis of the turn the number of gausses per ampere-turn decreases as the distance from the center of the turn increases. This is illustrated in Fig. 198, wherein the average radius r_a is taken as 1 cm. In this illustration ten similar turns are placed side by side and 1 cm. apart, center to center, on a common axis, and the number of gausses per ampere-turn for all of the turns are added together at each point along the axis to give the total number of gausses per ampere-turn for the ten turns at each point. Since the turns are 1 cm. apart, ½ cm. must be added to each end for average conditions, when the ten turns constitute a single-layer solenoid 10 cm. long.[1]

[1] See so THOMPSON, S. P., "The Electromagnet," London, 1892.

Instead of employing a single layer of spaced turns, groups[1] of turns may be arranged concentrically and side by side to form a commercial coil or solenoid, and the number of gausses per ampere-turn may be determined in a manner similar to that for finding the number of gausses per ampere-turn for the single layer of spaced turns.

In a finite solenoid, the number of gausses per ampere-turn varies inversely as the length of the solenoid, being equal to the product of what is herein termed the magnetizing-force factor h and the ratio $\frac{0.628\ 32}{L}$, or equal to $0.628\ 32\ \frac{h}{L}$, where L is the length of the solenoid in centimeters. In order to deal simultaneously with average radii r_a and lengths of solenoid L, it is convenient to consider solenoid lengths L in terms of average radii r_a by introducing solenoid sections,[2] each section being a solenoid or a portion thereof having a length equal to its average radius. Then the length L of the solenoid is equal to the product of the average radius r_a and the number of sections M, or $L = r_a M$. In consequence, the number of gausses per ampere-turn is equal to $0.628\ 32\ \frac{h}{r_a M} = 0.628\ 32\ \frac{h}{L}$.

The magnetizing-force factor h is equal to 1.59 times the product of the length L of the solenoid in centimeters and the number of gausses per ampere-turn, and is equal to 1 at the center of a single turn, and 2 at the middle of an indefinitely long solenoid, but it is equal to 2 throughout the entire core hole of a toroidal coil. Thus $100\ \frac{h}{2}$, or $50\ h$, expresses the percentage of complete linkages of electromagnetic flux in the core hole with the turns at any point along the axis of any uniform coil.

The above method may best be utilized by finding the magnetizing force as described by Dr. C. W. Burrows,[3] wherein a finite solenoid is first assumed to be a part of an indefinitely long solenoid and then is removed therefrom. The method is herein somewhat elaborated on. At the end of an indefinitely long solenoid the value of h is 1, while at its middle the value of h is 2.

[1] Underhill, C. R., "Solenoids," New York, 1910.
[2] Underhill, C. R., "Solenoids," *Trans.* A.I.E.E., 1914.
[3] *Trans.* A.I.E.E., 1914.

Figure 199 shows the values of h at the abutting ends of the two halves of an indefinitely long solenoid and at points along the axis therefrom. The sum of the increasing values of h within

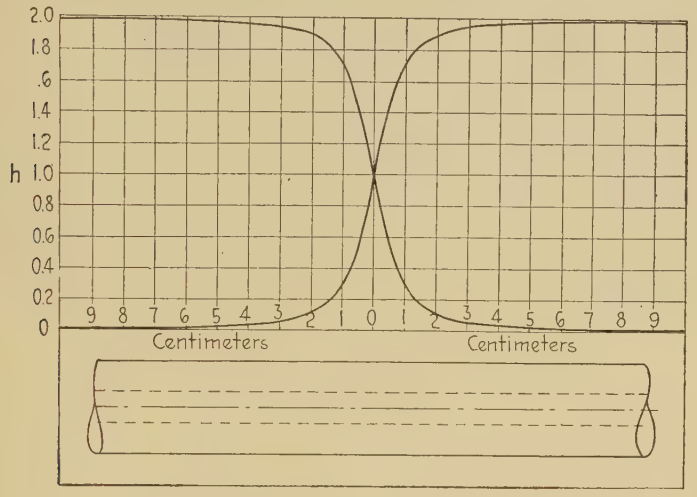


FIG. 199.—Values of h at the abutting ends of two long solenoids, herein shown as one continuous solenoid.

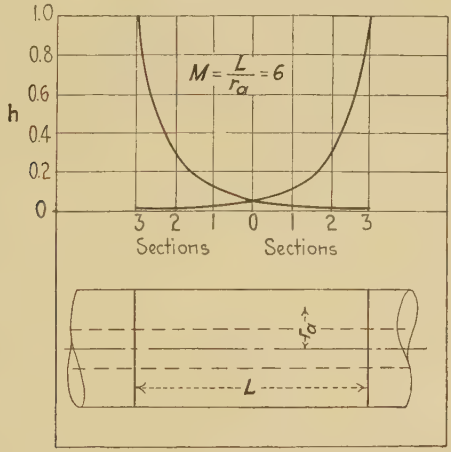


FIG. 200.—Detaching a finite solenoid from the middle of an indefinitely long solenoid.

either half solenoid and the decreasing values of h outside of the adjacent half solenoid, at any point along the axis, is always equal to 2. Each half solenoid is assumed to be indefinitely

long, so each may be called a separate solenoid. In Fig. 200 the two solenoids have been separated to represent the detachment of a finite solenoid therefrom—a six-section solenoid in this case. The method of finding the number of gausses per ampere-turn for this six-section solenoid is given as an example for finding similar data for other solenoids.

The data in Table I refer to the values of h along the axis outside of an indefinitely long solenoid, as measured from either end.

TABLE I.—VALUES OF THE MAGNETIZING-FORCE FACTOR h OUTSIDE OF AN INDEFINITELY LONG SOLENOID

Distance in radii, r_a	h	Distance in radii, r_a	h
0.0	1.0000	2.5	0.0715
0.1	0.9005	3.0	0.0511
0.2	0.8039	3.5	0.0385
0.3	0.7126	4.0	0.0300
0.4	0.6286	4.5	0.0238
0.5	0.5528	5.0	0.0195
0.6	0.4855	6.0	0.0136
0.7	0.4266	7.0	0.0102
0.8	0.3755	8.0	0.0080
0.9	0.3310	9.0	0.0064
1.0	0.2928	10.0	0.0050
1.5	0.1681		
2.0	0.1056		

The method consists in adding together the values of h for the two indefinitely long solenoids *beyond their ends* and then subtracting these sums from 2. The values only need to be calculated up to the middle section of the finite solenoid, since the values of h on each side of the middle or the neutral-polarity plane where the magnetizing force is the greatest are similar at equal distances from the middle. Referring to Table I, and to Fig. 200,

h at end of either indefinitely long solenoid. $= 1.0000 + 0.0136 = 1.0136$
h at 1 radius distant from end of either indefinitely long solenoid..... $= 0.2928 + 0.0195 = 0.3123$
h at 2 radii distant from end of either indefinitely long solenoid..... $= 0.1056 + 0.0300 = 0.1356$
h at 3 radii distant from end of either indefinitely long solenoid..... $= 0.0511 + 0.0511 = 0.1022$

Subtracting the above values of h for both indefinitely long solenoids from 2 gives the values of h along the axis of the finite solenoid, as follows:

h at either end	0.9864
h at 1 radius from end	1.6877
h at 2 radii from end	1.8644
h at middle	1.8978

h is equal to 1.59 times the product of the length L of the solenoid in centimeters and the number of gausses per ampere-turn.

If the average radius r_a of this six-section solenoid is 3 cm., the length of the solenoid is equal to $3 \times 6 = 18$ cm. (7.1 in.). The number of gausses per ampere-turn at the various points along the axis may be calculated as previously described, that is, by multiplying the above values of h by $\frac{0.628\,32}{18} = 0.0349$. This gives the following results:

Six-section Solenoid	Gausses per Ampere-turn
At either end	0.0347
At 1 radius from end	0.0588
At 2 radii from end	0.0650
At middle	0.0661

Following the foregoing general procedure, the values of h at the ends and centers of finite solenoids consisting of from one to ten sections have been calculated, as in Table II.

Table II.—Values of the Magnetizing-force Factor h at the Ends and Middles of Finite Solenoids

Number of sections M	h at ends	h at middles
1	0.7072	0.8944
2	0.8944	1.4144
3	0.9489	1.6638
4	0.9700	1.7888
5	0.9805	1.8570
6	0.9864	1.8978
7	0.9898	1.9230
8	0.9920	1.9400
9	0.9936	1.9524
10	0.9950	1.9610

From the data in Table II it is observed that the value of h at the middle or neutral-polarity plane of any solenoid is double that at either end of any other solenoid containing one-half as many sections. For instance, the value of h at the middle of a 20-section solenoid is equal to $2 \times 0.9950 = 1.9900$. The data in Table II are plotted in Fig. 201.

117. M.m.f. along the Axis.—In all cases, the m.m.f. in gilberts is equal to 1.257 times the number of ampere-turns.

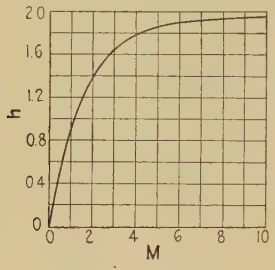


Fig. 201.—Values of h at the middles of solenoids.

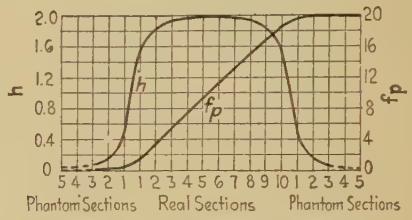


Fig. 202.—10-section solenoid.

In a toroidal solenoid the magnetizing force in gausses is equal to the ratio of the magnetizing force in gilberts to the average length L of the solenoid. Consequently, the total m.m.f in gilberts is equal to the product of the magnetizing force in gausses and the average length of the toroidal solenoid.

Figure 202[1] shows the values of h along the axis of a 10-section straight solenoid, both within and outside of the core hole. M is the number of "real sections" of which the solenoid is composed. The "phantom sections" are distances in average radii beyond the middles of the end sections, and need not be considered in the approximate method soon to be described. In all cases, the perpendicular lines represent the middles of the respective sections. A phantom section possesses a smaller magnitude of magnetizing force at any point within it than does a real section.

Because some of the magnetizing force exists outside of the core hole of a straight solenoid as well as within it, the magnetizing-force factor h is introduced to facilitate the calculation of the magnetizing force at any point along the axis. In a similar manner, the magnetomotive-force factor (or m.m.f. factor) f is here intro-

[1] Underhill, C. R., "Solenoids," *Trans.* A.I.E.E., 1914.

duced to facilitate the calculation of the m.m.f as it progressively increases from a point remote from the conventional left-hand end of the solenoid to some other point along the axis, as a point within the core hole of the solenoid. In all cases, the maximum value of h approaches 2, while the maximum value of f *is* 2.

The m.m.f. factor f is equal to the ratio of f_p (Fig. 202) to the number of real sections M, that is, $f = \frac{f_p}{M}$. The maximum value of f_p in Fig. 202 is seen to be equal to twice the number of sections M. The value of f_p at any point along the axis expresses the area under the curve h (in terms of $h \times M$) up to that point on the axis, as measured from a point remote from the left-hand end of the solenoid, and represents the cumulative products of all values of h and the corresponding infinitesimal lengths expressed in terms of radii wherever the magnetizing force exists up to the point of reference, that is, f_p is the integral or the whole of the product of the magnetizing-force factor h and the length expressed in radii over which it exists. But f_p is equal to the product of the number of real sections M and the m.m.f. factor f. Consequently, $f = \frac{f_p}{M}$, as previously stated.

Multiplying 0.628 32 times the ratio of the total number of ampere-turns to the total length of the solenoid in centimeters by the value of h at any point along the axis, within or outside of the core hole, gives the value of the magnetizing force (or the electromagnetic flux density) in gausses at that point. Multiplying 0.628 32 times the total number of ampere-turns by the value of $f\left(=\frac{f_p}{M}\right)$ at any point on the axis, within or outside of the core hole, gives the value of the m.m.f. in gilberts at that point, as measured from a point remote from the left-hand end of the solenoid.

The number of gausses per gilbert is the permeance per square centimeter of core hole. The ratio $\frac{h}{f}$ at any point along the axis is equal to the product of the average radius r_a in centimeters and the number of gausses per gilbert at that point. Referring to Fig. 202, the middle of the 10-section solenoid is between the middles of sections 5 and 6. At this point $f_p = 10$ and, therefore, $f = 1$. At the same point, $h = 1.96$. Hence, at the middle of a 10-section solenoid the number of gausses per gilbert is equal to $\frac{1.96}{r_a}$.

The fact that the curve f_p is so nearly a straight line throughout the length of the actual solenoid makes it possible to obtain quite accurate averages of $f = \frac{f_p}{M}$ for solenoids containing various numbers of sections, that is, having various lengths as measured in terms of average radii. Figure 203 shows the average values of $\frac{f}{M}$ for solenoids containing any number of sections up to 10. To apply the data in Fig. 203, divide the length of the solenoid

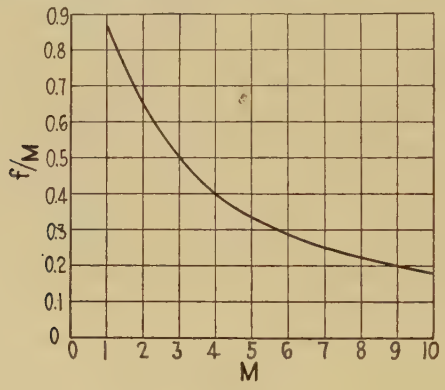


FIG. 203.—Average values of $\frac{f}{M}$.

in centimeters by the average radius in centimeters (or the ratio of the length in inches to the average radii in inches can be taken) to obtain the number of sections M. By means of Fig. 203, find the value of $\frac{f}{M}$. Multiplying the value of $\frac{f}{M}$ thus found by the number of the section, as conventionally reckoned from the left-hand end of the solenoid, gives the approximate value of f at the middle of that section. The m.m.f. in gilberts at any point within the solenoid is then approximately equal to 0.62832 times the product of the total number of ampere-turns and the value of f at that point.

As an example, let a solenoid contain six sections. Referring to Fig. 203, when $M = 6$, $\frac{f}{M} = 0.29$, and this is the approximate value of f at the middle of the left-hand end section. At the middle of the second section, $f = 2 \times 0.29 = 0.58$, and so forth. The approximate m.m.f. in gilberts at the middle of the fifth section is equal to $5 \times 0.29 \times 0.628\ 32 \times$ ampere-turns $= 0.91 \times$

ampere-turns, that is, the number of ampere-turns per gilbert is equal to 0.91. The total number of ampere-turns per gilbert is 1.257. Hence, $\frac{0.91}{1.257}$ = about 73 per cent of the total m.m.f. is required to produce the magnetizing force along the axis up to the middle of the fifth section in this case. When a steel plunger is placed in the core hole of the solenoid so that its entering end has moved from the left-hand end to the middle of the fifth section, the above 73 per cent of the total m.m.f. is available for magnetizing the plunger, the entire or 100 per cent m.m.f. being utilized in producing the total magnetizing force in all cases.

The value of f at the middle of any solenoid is 1. Its maximum value 2 is at a point remote from the conventional right-hand end of the solenoid, but the value 2 is approximated at a point but a few radii distant from that end.

118. Stored Magnetic Energy.—The magnetic energy in joules stored in a toroidal solenoid is equal to one-half the product of the number of ampere-turns and the uniform magnitude of the magnetic flux in maxwells, divided by 100,000,000. There are 10,000,000 ergs or dyne-centimeters in one joule. Hence, the energy in ergs or dyne-centimeters is equal to 0.05 times the number of ampere-maxwell-turns. The above holds for all points along the axis of a toroidal solenoid and also for all finite distances along the axis on either side of the middle of an indefinitely long straight solenoid.

119. Density of Stored Magnetic Energy.—The density of stored magnetic energy in ergs per cubic centimeter at all points along the core hole of a toroidal solenoid is equal to 0.05 times the product of the uniform ampere-turn gradient in ampere-turns per centimeter and the uniform magnetic flux density in gausses. In this case, the uniform ampere-turn gradient is equal to 0.7958 times the uniform m.m.f. gradient, and the magnetic flux density is the magnetizing force. Hence, the uniform density of stored magnetic energy in ergs per cubic centimeter of core hole of a toroidal solenoid is equal to $0.05 \times 0.7958 \left(= 0.0398 = \frac{1}{8\pi}\right)$ times the product of the m.m.f. gradient and the magnetizing force. But the m.m.f. gradient is equal to the magnetizing force. Hence, the uniform density of stored magnetic energy in ergs per cubic centimeter along the core hole of a toroidal solenoid is equal to 0.0398 times the square of the magnetizing force in gausses.

In straight solenoids, the density of stored magnetic energy is greatest at the middle and least at the ends.

120. Mechanical Pressures and Forces between Portions of Solenoids.—In a free medium, as in the core hole of a solenoid, the mechanical pressure, or mechanical force per unit of area or cross-section, is equal to the density of stored magnetic energy. The erg is equal to 1 dyne times 1 cm. Unit volume is equal to 1 sq. cm. times 1 cm. Hence, unit mechanical energy per unit volume is 1 dyne per square centimeter. Since the area or cross-section here referred to is that of the core hole, the mechanical pressure is expressed in terms of dynes per square centimeter of cross-section of core hole.

The density of stored magnetic energy at the middle of an indefinitely extended (infinite) solenoid is the same as that at any point along the core hole of a toroidal solenoid when the m.m.f. gradients and the average radii are equal. As noted above, in the reduction of density of energy to pressure the length cancels out. Hence, this length may have any value whatever. It may be a finite quantity or it may be smaller than any assignable quantity. In the case of a toroidal solenoid, this length may be either the average length of the solenoid or an element of its length so small as to be undistinguishable from zero. Either value may be used in the case of a toroidal solenoid, since the density of energy is uniform in all portions of the core hole. However, in the cases of finite straight solenoids, the length of the solenoid cannot be used (except as hereinafter described), because the density of stored magnetic energy is not uniform throughout the length of the core hole since some of the magnetic flux exists at points beyond the ends of the solenoid and all of the magnetic flux in the core hole does not link with all of the turns. Therefore, the infinitesimal elements of length must be used in such cases, or else averages for the density of magnetic energy may be taken between two points close together. The net result is that the expressions for pressures now refer to points along the axis instead of for the entire solenoid.

Since the magnetizing force per ampere-turn, in gausses per ampere-turn, at any point along the axis of any uniform straight solenoid is equal to $0.628\,32\,\frac{h}{L}$ (page 162), wherein h is the magnetizing-force factor and L is the length of the solenoid in centimeters, the density of energy in ergs per cubic centimeter or the intensity

of pressure in dynes per square centimeter of core hole at any point along the axis of any uniform straight solenoid is equal to 0.015 71 times the square of the product of the number of ampere-turns and the ratio $\frac{h}{L}$ at that point on the axis, and this rule holds for the pressures produced between the abutting components of a single solenoid (as when the components may

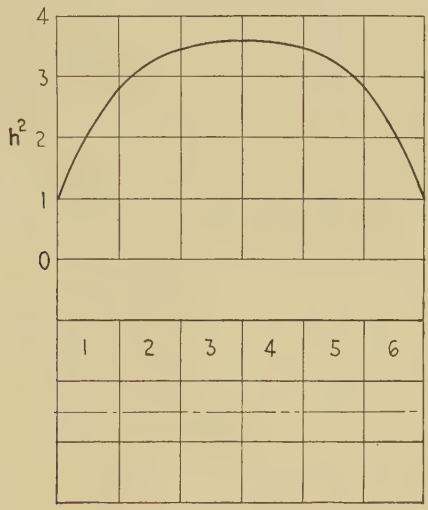


FIG. 204.—Values of h^2.

be separated) at any point along the axis within the limits of the length of the solenoid. Figure 204 shows the values of h^2 (representing the relative intensity of pressure or the density of energy) along the axis of a six-section solenoid, as plotted from the data on page 165. The maximum possible value of h^2 (under all circumstances) is 4. In a toroidal solenoid $h^2 = 4$ throughout the entire core hole. However, the equivalent process is to find the value of h as previously described, then calculate the magnetizing force and multiply its square by 0.0398 to obtain the density of energy in ergs per cubic centimeter or the intensity of pressure in dynes per square centimeter of core hole.

One kilogram = 980,596.6 dynes. Consequently, 1 dyne is equal to 0.000 001 019 8 kg. To find the number of kilograms per square centimeter, multiply the number of dynes per square centimeter, by 0.000 001 019 8, or multiply the square of the

magnetizing force in gausses by 0.000 000 040 58. One kilogram per square centimeter is equal to 14.223 lb. per square inch. Therefore, to find the number of pounds per square inch, multiply the square of the magnetizing force in gausses by 0.000 000 557 1. The above figures are unnecessarily accurate for ordinary work. One kilogram is commonly said to be equal to 981,000 dynes.

The mutual attraction between any two solenoids having common radii and on a common axis with their ends abutting may be found as in the following example. Let two solenoids, each containing three sections and having an average radius of 3 cm. and a core hole 6.45 sq. cm. (1 sq. in.) in cross-section, be placed end to end and electrically energized with 1,000 amp.-turns per centimeter length of solenoid. The total length of the two solenoids, which are the equivalent of a single solenoid separated at its middle, is (page 162) equal to six sections $\times$ 3 cm. average radius = 18 cm. (7.08 in.). Referring to Table II, page 165, the value of h at the neutral-polarity plane where the ends abutt is equal to 1.8978. The magnetizing force at the middle of the solenoid at the bounding plane or neutral-polarity plane between the two halves is equal to 0.628 32 $\times$ 1.8978 $\times$ 1,000 = 1,192 gausses, and its square is 1,420,900. Therefore, the pressure exerted in pulling the two halves of the solenoid together is equal to 1,420,900 $\times$ 0.000 000 040 58 = 0.0576 kg. per square centimeter, and the force is equal to 0.0576 $\times$ 6.45 = 0.372 kg., 372 grams, or 0.82 lb. since 1 kg. is equal to 2.205 lb. To calculate any other pull due to such a solenoid with any other number of ampere-turns, simply divide by the square of the number of ampere-turns per centimeter length of solenoid and multiply by the square of any other number of ampere-turns applied per centimeter length of solenoid. The pulls between various portions of the same solenoid are found in a similar manner.

The number of ampere-turns per unit length of solenoid and, therefore, the magnetizing force, is closely related to the current density in the solenoid conductor. With a current density as great as that due to 1,000 amp.-turns per centimeter, the coil would become very hot in a short time.

It is possible to lift a solenoid by means of another solenoid held in the hand when the current is strong and the lifting is done before the coil becomes too hot to be held.

When the polarities of the abutting ends are similar, mutual repulsion results.

121. The Self-inductance of Air-cored Solenoids.—There are a number of equations available for calculating the self-inductance of straight solenoids, but the method by Prof. L. A. Doggett[1] as herein reproduced in the main is very simple and quite accurate.

To calculate the self-inductance of a solenoid by this method proceed as follows:

1. Find the ratio of the average radius to the length of the solenoid. This is $\frac{a}{l}$ in Fig. 205.

2. Take the ratio of the coil-wall thickness to the length of the solenoid. This is $\frac{c}{l}$ in Fig. 205.

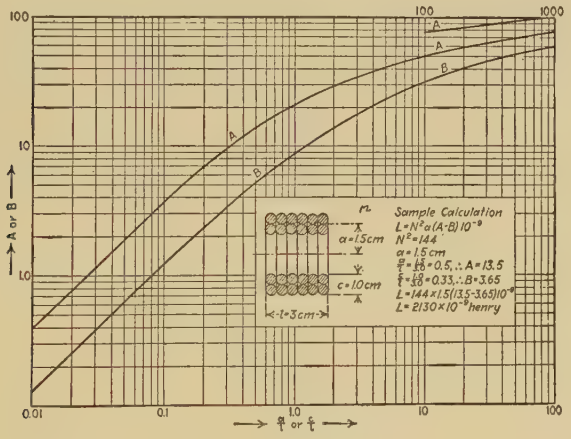


Fig. 205.—Data for calculating the self-inductance of a solenoid.

3. Find the value of A from the ratio $\frac{a}{l}$.

4. Find the value of B from the ratio $\frac{c}{l}$.

5. Subtract B from A.

6. The self-inductance in henries is then equal to the continued product of $(A - B)$, the average radius of the solenoid, and the square of the number of turns, divided by 1,000,000,000.

With the exception of slight changes to make the article conform to the arrangement of the text, the former is practically copied herein.

[1] "The Inductance of Air-cored Solenoids," *Elec. World*, vol. 63, p. 259, 1914.

In order to test the accuracy of the formula, comparisons were made between the calculated and measured values of self-inductance for 50 coils. The data on 14 of these are shown in Tables III and IV, the aim having been to select values of $\frac{a}{l}$ and $\frac{c}{l}$, not those that happened to yield the most accurate results but those that covered as wide a range of these ratios as possible. The average inaccuracy for the 50 coils was about plus or minus 2 per cent.

The foregoing formula will serve for most ordinary cases where the space factor, defined as the ratio of the copper area to the product of the length times the winding depth, is above 35 per cent. For lower values a correction factor needs to be applied which is 1.01 for a space factor of $\frac{1}{3}$, 1.04 for $\frac{1}{4}$, 1.08 for $\frac{1}{5}$, 1.12 for $\frac{1}{6}$, 1.21 for $\frac{1}{8}$, 1.28 for $\frac{1}{10}$, 1.47 for $\frac{1}{15}$, and 1.62 for $\frac{1}{20}$.

TABLE III.—CALCULATED AND MEASURED VALUES OF SELF-INDUCTANCE L

Reference No.[1]	Number of turns	Mean rad. a, in centimeters	$\frac{a}{l}$	$\frac{c}{l}$	Calculated L, in henries	Measured L
1	144	45.1	0.987	0.583	1.39×10^{-2}	1.481×10^{-2}
1	126	43.18	0.945	0.556	1.012×10^{-2}	1.074×10^{-2}
3	2	4.97	22.25	0.425	0.115×10^{-5}	0.1092×10^{-5}
2	784	6.19	1.932	1.00	7.57×10^{-2}	7.574×10^{-2}
3	8	1.505	1.545	0.098	0.243×10^{-5}	0.2479×10^{-5}
2	594	3.25	1.0833	0.53	1.865×10^{-2}	1.8476×10^{-2}
4	30	6.5	0.507	0.0162	0.795×10^{-4}	0.82×10^{-4}
3	20	0.98	0.227	0.0221	0.287×10^{-5}	0.2867×10^{-5}
3	24	0.98	0.1885	0.0181	0.348×10^{-5}	0.3458×10^{-5}

To illustrate the application of this point, the self-inductance of a current-limiting reactor, described in the *Electrical World* for Dec. 21, 1912, page 1,333, was calculated. The constants of the coil are: number of turns $N = 34$, $a = 42.6$, $\frac{a}{l} = 0.58$, $\frac{c}{l} = 0.36$, space factor = 21 per cent. The calculated value of the self-inductance was 0.000 554, compared with 0.000 583, the measured value, in henry.

In explanation of the curves A and B, it is to be noted that the A curve corresponds to the assumption of a current sheet of radius a and length l. In vol. 8, No. 1, of the *Bulletin* of the

TABLE IV.—CALCULATED AND MEASURED VALUES OF SELF-INDUCTANCE L

Reference No.[1]	Number of turns	Mean rad. a, in centimeters	$\frac{a}{l}$	$\frac{c}{l}$	Calculated L, in henries	L, calculated by Brooks' formula
2	80	3.18	2.5	3.2	2.79×10^{-4}	2.89×10^{-4}
2	50	5.08	4.0	2.0	3.01×10^{-4}	3.01×10^{-4}
2	8	31.8	62.5	2.0	1.18×10^{-4}	1.19×10^{-4}
2	4	63.5	250.0	4.0	0.666×10^{-4}	0.69×10^{-4}
2	2	127.0	500.0	2.0	0.404×10^{-4}	0.42×10^{-4}

[1] References: (1) "Current-limiting Reactances," *Trans.* N.E.L.A., vol. 2, p. 144, 1909, (2) BROOKS AND TURNER, *Bull.* 53, Univ. Illinois; (3) STRASSER, B., *Wied. Annalen*, p. 763; 1905; (4) PIERCE, G. W., "Wireless Coil," *Phys. Rev.*, vol. 24, p. 153.

Bureau of Standards, this curve has been calculated for a range of $\frac{a}{l}$ from 0.1 to 2, while in the accompanying curve sheet (Fig. 205) the range has been extended from 0.01 to 1,000. Similarly, the B curve, which was obtainable from this same *Bulletin* in the range of $\frac{c}{l}$ from 0.0333 to 1, has been extended in range from 0.01 to 100. The extensions of the above curves have made the formula universal and rendered it possible to include the solution of the self-inductance of any air-cored solenoid on one page, as shown on the curve sheet.

CHAPTER X

CONSTANT-CURRENT SOLENOIDS WITH PLUNGERS

Maxwell's hypothesis, based on the density of energy, is described and its modified application to a single solenoid with a single solid plunger is discussed. Then follows a discussion of the respective magnetic fields, and the effects due to changes in the relative lengths of the solenoid and the plunger, as well as those due to saturation, on the position of maximum net pull.

122. The Mutual Attractions between Solenoids and Plungers. The mutual attraction or pressure between any two parts of a solenoid, as described in Art. 120, is the basis of all mechanical pressures due to magnetic flux linked with ampere-turns, with magnetic materials, or with both.

A solenoid having within its core hole a single steel core or plunger whose length is equal to that of the solenoid, so that the respective ends of the plunger are even with the corresponding ends of the solenoids, constitutes one form of bar electromagnet. As a magnet, a bar electromagnet is very similar to a solenoid, the fundamental difference being that the magnetizing force of the solenoid causes the structural flux of the steel to join the electromagnetic flux and link with at least a portion of the ampere-turns. The sum of the two magnetic flux densities is the induction.

As in the cases of solenoids having common radii, the structural flux density at the neutral-polarity plane of any bar electromagnet as above is double that at either end of a bar electromagnet of one-half the length. Hence, when two similar indefinitely long bar electromagnets are placed end to end on a common axis the structural flux density at the neutral-polarity or bounding plane between the abutting ends of the two electromagnets will be double that at the respective ends. The two abutting bar electromagnets may be considered as a single bar electromagnet with a zero air gap at its middle or neutral-polarity plane, but with all of the magnetic flux passing through said plane.

All of the electromagnetic flux in the core hole (equal to the product of the magnetizing force and the cross-section of the core

hole) and all of the structural flux (equal to 12.57 times the product of the intensity of magnetization and the cross-section of the steel core or plunger) passes through the neutral-polarity or bounding plane a between the half solenoids s_1 and s_2 and the steel half cores c_1 and c_2, as in Fig. 206.

In all cases, wherever magnetic flux passes straight across a bounding plane in a common direction, the pressure is always proportional to the product of the number of gausses entering the

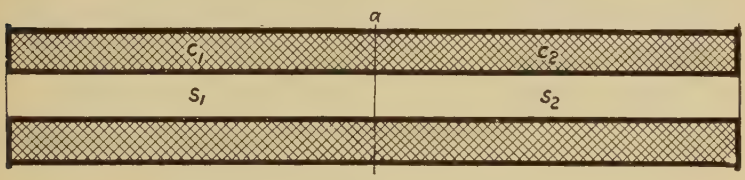


FIG. 206.—Divided bar electromagnet.

plane on one side and the number of gausses leaving the plane on the other side, provided that all of the flux mutually links the two objects being pulled together. This subject was briefly discussed on page 99.

The electromagnetic flux may be thought of as being attached to the turns (in which an electric current is flowing) of the solenoid wherein it is produced, while the structural flux may be thought of as being attached to the steel (or to the molecular currents therein) from which it emerges, for the purpose of forming conceptions regarding the mutual attractions between two solenoids, between two cores or plungers, between a solenoid and a plunger, or between electric conductors, magnetic materials, or between electric conductors and magnetic materials, in general. However, all such attractions are due to the mutual linkages of magnetic flux with the various forms of electric currents, and the above attachments occur in this manner. In each case, mutual attractions occur when the magnetic flux linked with one magnetic-flux-producing element links with another magnetic-flux-producing element.

The following hypothesis, due to Maxwell, is based on the ideal condition that the magnetizing force in gausses is equal to 1.256 64 (1.257 is used herein) times the number of ampere-turns per centimeter length of solenoid. This condition holds in the case of a toroidal electromagnet, or when the solenoid or bar electromagnet is made indefinitely long. In such cases,

practically all of the magnetic flux links with all of the ampere-turns and then, and only then, the magnetizing force is equal to 1.257 times the number of ampere-turns per centimeter length of solenoid. Practically, the condition for maximum magnetic flux density in air at the neutral-polarity plane, for a given ampere-turn gradient, is approximated when the length of the solenoid is about 20 times its average radius, that is, when $\frac{L}{r_a} = M = 20$ (pages 162 and 164). However, by the use of the magnetizing-force factor h, Maxwell's hypothesis may be made applicable to uniform solenoids and electromagnets of any length.

Designating the magnetizing force or electromagnetic flux density, in gausses, by H and the structural or ferromagnetic flux density, in gausses, by b, their sum $(H + b)$ is the induction B. Then the force in dynes between the abutting ends of the two halves of bar electromagnets like those in Fig. 206, but having indefinite lengths, is equal to 0.0398 times the product of the common cross-section of the core hole and steel core and the square of the induction in gausses. Likewise, the pressure in dynes per square centimeter tending to bring the abutting ends closer together is equal to 0.0398 times the square of the induction B in gausses.

Since the induction B is equal to the sum of the magnetizing force H and the structural flux density b, the square of the induction B is equal to $H^2 + 2Hb + b^2$. Then the total pressure in dynes per square centimeter, equal to $0.0398\ B^2$, consists of the following components:

$0.0398\ H^2$ is the pressure between the two half solenoids, as in the example on page 172.

$0.0398\ Hb$ is the pressure between solenoid s_1 and core c_2, and also between solenoid s_2 and core c_1. Hence, there are two of these pressures and their sum is equal to $0.0398 \times 2\ Hb$.

$0.0398\ b^2$ is the pressure between the two half-cores, or between core c_1 and core c_2.

The magnetic field of a solenoid is not altered by the presence of magnetic material, that is, the electromagnetic field is not altered. What is changed is the resultant magnetic field. The magnetizing force, as measured along the axis of the solenoid, is always the same at any given point for a constant and continuous m.m.f., whether or not magnetic materials are present.

This magnetizing force causes the structural flux to link at least partially with the ampere-turns and thus cause a reaction between the solenoid and the steel. The magnetizing-force factor h at any point along the common axis expresses the relative amount of magnetic flux linked with all of the ampere-turns at that point. When the middle of the core or plunger coincides with the middle of the solenoid in the case illustrated in Fig. 206, the product of h and a constant quantity expresses the relative linkage of the structural flux with all of the ampere-turns at that point.

Referring to Fig. 206 and the description thereof, if the solenoid is considered as being fixed in position, the two half-cores tend to move in opposite directions with respect to the entire solenoid as a unit under the influences of equally opposed mechanical forces. Hence, the net or useful attraction, pressure, or force is equal to zero and, even when the whole core or plunger is freely mounted within the core hole of the solenoid, no relative motion between the solenoid and the plunger can result when the middle of the solenoid and the middle of the plunger coincide. As a matter of fact, there is never a net or useful attraction between a solenoid and a plunger at the neutral-polarity plane of the plunger, although mutual attractions exist between the solenoid and the plunger at whatever point along the common axis the neutral-polarity plane of the plunger may be, and whether or not the plunger is actually separated at its neutral-polarity plane, so long as the length of the gap is actually (or hypothetically) equal to zero. In all such cases, the product of the magnetizing force and the structural flux density at each of the abutting ends of the plunger portions (with zero gap) is the same for both of the abutting ends, and each force exactly balances the other.

In the case of a single solenoid with a single plunger, the mutual attractions between portions of the solenoid are neither available nor relatively important (page 172) and there are no available mutual attractions between steel and steel. If the plunger is freely mounted in the core hole of a solenoid of equal length, the solenoid and the plunger will tend to remain in such relative positions that their respective ends will be flush with one another. Barring friction and unevenly distributed remanent magnetism, if either the solenoid or the plunger is forcibly moved in either direction along the common axis with respect

to the other, they will, when released, again assume their original and normally natural relative positions with the neutral-polarity plane of the plunger coinciding with the neutral-polarity plane of the solenoid. The linkage of magnetic flux with ampere-turns is then a maximum, and the maximum amount of magnetic energy is stored in the system.

The tendency of a plunger to move with respect to a solenoid so that their neutral-polarity planes shall coincide is due to unbalanced pressures or forces at the respective ends of the plunger. Each of these pressures is proportional to the product of the magnetizing force and the structural flux density at the end of the flat-faced plunger at that point on the axis where the end of the plunger then is. The net or useful pressure, as that observed during a test with the solenoid and plunger in a horizontal position and with friction allowed for, is the difference between the opposing pressures at the respective ends of the plunger. Hence, the solenoid is a differential device.

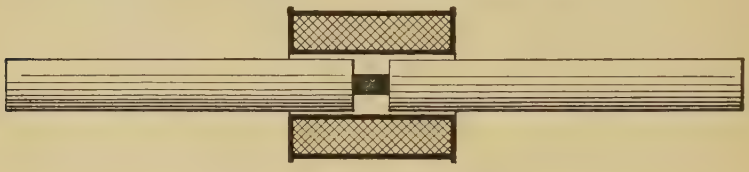


Fig. 207.—Differential solenoid.

To illustrate further the difference between the conditions for Maxwell's hypothesis and those for commercial solenoids with plungers, one form of differential solenoid is here discussed. While this form generally has two coils (Figs. 28 to 30) which may have different magnetizing forces in similar or opposed directions, a single solenoid is here described as illustrated in Fig. 207. The solenoid is not divided, but the plunger is separated for a finite distance at its middle and a piece of non-magnetic material is inserted in the gap in such a manner that the entire plunger is still a mechanical unit. The length of each half plunger is assumed to be at least double that of the solenoid in order that the pressures at their extreme ends may be neglected. Some of the ferromagnetic flux passes through the neutral-polarity plane of the solenoid, since it passes through the gap between the ends of the two half plungers, but much of it leaks back to the portions of opposite polarity at or near the respective ends of the solenoid. The longer the gap the greater

is the leakage and the smaller becomes the linkage of structural flux with the ampere-turns in the opposite half of the solenoid.

Let the middle of the gap between the half-plungers be slightly displaced from the middle of the solenoid. If the length of the magnetic gap was equal to zero, that is, if it did not exist, the net pressure due to the difference in the opposed pressures at the gap, capable of producing relative motion between the solenoid and the plunger to return them to their original positions, would be equal to zero. However, with a finite length of gap, the leakage of the structural flux increases as the length of the gap increases, less structural flux passes through the gap, thus causing a decrease in the useless mutual attraction between

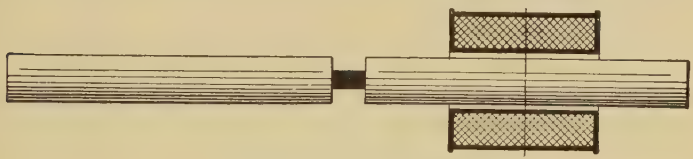


FIG. 208.—Stable condition.

the two half plungers, and the electromagnetic system tends to be divided into two distinct systems. Consequently, when the middle of the gap is not coincident with the middle of the solenoid, the opposing pressures are unequal, because the products of the magnetizing force and the structural flux density at the ends of the respective half plungers are unequal, and a state of unequal equilibrium exists, with the result that the favored half plunger forces the other completely out of the core hole, the net pressure at any point during the travel being proportional to the difference in the products of the magnetizing force and the structural flux density at the respective faces of the half plungers at the ends of the gap. This effect is illustrated in Fig. 208.

Much has been written about the demagnetizing effect of a solenoid on the entering end of a plunger after it has passed the middle of neutral-polarity plane of the solenoid. This so-called demagnetizing effect is nothing but the effect due to the leakage of magnetic flux and the opposed pulls on the plunger. The idea of the so-called demagnetizing effect appears to have arisen from the fact that "like poles mutually repel, while unlike poles mutually attract one another." Now mutual repulsion only results when the relative movement between two parts of an

electromagnetic system would permit an increase in the stored magnetic energy of the system *if the two parts moved away from each other.* Mutual attraction always results when the stored magnetic energy of the system would be increased by the relative movement of the two parts towards each other. Polarity is a matter of relative direction. If a solenoid is divided into any number of portions of small length, as in Fig. 91 each portion will have an S and N end. Likewise, if the core of a bar electromagnet or the plunger of a solenoid-and-plunger is divided into any number of small portions, along the axis, each portion will have an S and N end, and when they are all fitted together again the plunger will have one S end and one N end. The solenoid surrounding the core of a bar electromagnet does not demagnetize any portion of that core. On the contrary, it magnetizes the core. In a similar manner, the farther the entering end of a plunger passes into the core hole of a solenoid the more does the plunger become magnetized. A solenoid never demagnetizes its plunger, but always magnetizes it under the conditions herein set forth The reason that the pull decreases as the entering end of a plunger (having a length greater than that of the solenoid) approaches the farther end of the solenoid is because the magnetizing force decreases as the distance along the axis from the middle of the solenoid increases, and this is due to magnetic leakage

Referring again to Maxwell's hypothesis, when one of the cores or plungers is removed from the entire solenoid of indefinite length in Fig. 206, so as to form a very long solenoid with but one half plunger contained within it, as in Fig. 209, the conditions are entirely different, for the following reasons:

1. The structural flux links with only one-half of the total number of ampere-turns.

2. The density of magnetic energy for the entire system is but one-half as great as in the case illustrated in Fig. 206.

4. The pressures are no longer balanced, and the pressure at the end of the solenoid where the end of the plunger is flush with it must be subtracted from the pressure at the middle of the solenoid in order to obtain the net or useful pressure.

In a solenoid with only one-half the amount of steel assumed in Maxwell's hypothesis twice the amount of stored magnetic energy per unit volume is required to produce the same pull. Hence, the pressure in dynes per square centimeter at either end

of the half-plunger in Fig. 209 is equal to 0.0796 $\left(\frac{1}{4\pi}\right)$ times the product of the magnetizing force and the structural flux density, both in gausses, and the net or useful pressure is the algebraic sum of the two, calling that in the right-hand direction *plus*, and the left-hand pull *minus*.

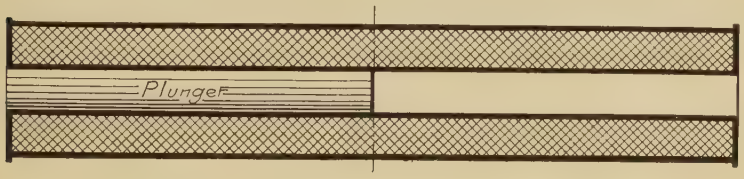


FIG. 209.—Single solenoid with 50 per cent plunger.

For any other relative length of solenoid to the length of plunger within its core hole, when the percentage of length of solenoid which the end of the plunger has entered expresses the percentage of linkage of ferromagnetic flux with the total number of ampere-turns, the pressure in dynes per square centimeter of cross-section of plunger at any point along the axis is equal to 0.0398 times the continued product of (1) the magnetizing force in gausses, (2) the ferromagnetic flux density in gausses, and (3) the ratio of the length of the solenoid to the length of the plunger within its core hole, as measured at the end of the flat-faced plunger along the axis. The ratio of the length of the solenoid to the length of the plunger within the solenoid may be taken as the ferromagnetic linkage factor.

123. Effects Due to Varying the Relative Lengths of Coacting Solenoids-and-plungers.—When a permanent magnet having a length equal to one-half that of the solenoid is employed as a plunger, the maximum net pull occurs at the middle of the solenoid when the value of h at the middle of the solenoid is double that at the end, provided the permanent magnet remains uniformly magnetized, as when the magnetizing force is small. This is a mean condition.

As one extreme condition, consider an exceedingly short plunger, as a thin disk of steel, and let it be assumed that the intensity of magnetization is constant. If this disk is so fastened on a slidable non-magnetic rod that its entering side shall represent the entering end of a very short flat-faced plunger, and friction can be neglected, Fig. 210 will quite accurately represent the force-distance characteristics, and the maximum pull will

occur at or near the left-hand end of the solenoid where h is increasing at the maximum rate with respect to distances measured in the right-hand direction on the axis, that is, at the point on the axis where the greatest difference exists between opposite sides of the disk. Hence, the greatest difference in the pulls on the opposite sides of the disk will occur at this place. As usual, the net pull will become equal to zero when the neutral-

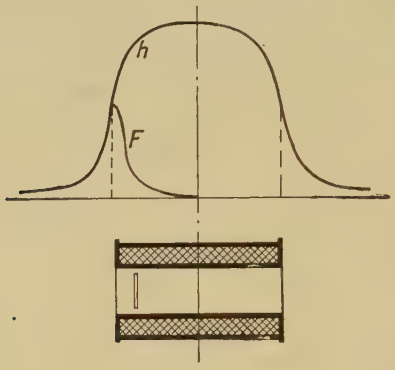


Fig. 210.—Effect of an extremely short plunger.

polarity plane of the disk coincides with that of the solenoid. If the thickness of the disk (the length of the plunger) was made equal to zero, there would be no attraction at all, because there would then be no magnetic material present. However, if the disk approached zero thickness, the attraction in each direction would be equal to that in the opposite direction.

Going to the other extreme, consider a permanent magnet as a plunger many times longer than the solenoid with which it is to coact. Then the end of the plunger which remains outside of the solenoid will always be so far away from that end of the solenoid that the value of h at that point on the axis will always be so small that the pull on that end of the plunger may be entirely neglected. The shape of the pull curve will be approximately the same as that of the h curve.

Now substitute a long soft steel plunger for the permanent magnet. Take the 10-section solenoid in Fig. 202 as a basis, and make the assumption that the ferromagnetic flux density at the entering end of the plunger increases in direct proportion to the m.m.f., that is, in direct proportion to f_p (page 167). Since the m.m.f. varies directly with f_p, and the magnetizing force

varies directly with h, the product hf_p will represent the net pull in terms of h at all points along the axis. It is to be understood that, while only relative values are used, the values may be made real by multiplying them by the proper conversion factors. The values of h and f_p in Fig. 202 are reproduced in Fig. 211. The product hf_p, expressed in terms of h, shows the relation of the pull to the position of the entering end of the plunger along the axis. This pull is a maximum when the entering end of the

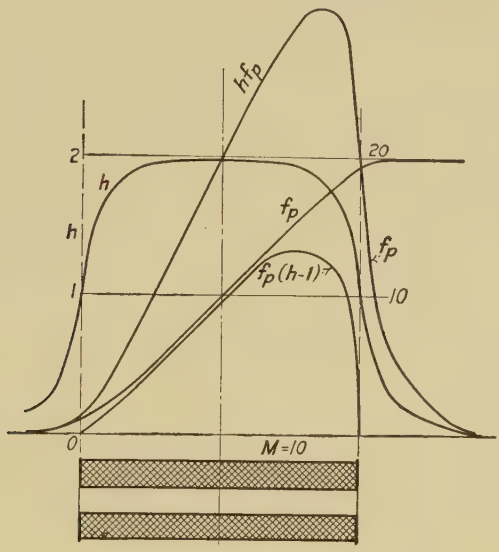


FIG. 211.—h, f_p, and their product.

plunger has traversed about 85 per cent of the length of the solenoid, and represents the extreme position for a flat-faced plunger under the premises. The curve $f_p(h - 1)$ is similar to that for a solenoid-and-plunger having equal lengths.

As will be presently shown, while the maximum net attraction between a solenoid and a plunger does not vary greatly when the plunger is longer than the solenoid and the plunger is saturated, the relative range or possible distance of travel of either the plunger or the solenoid when one is fixed in position, as measured from the end of the solenoid at which the plunger enters, is greater when the plunger is longer than the solenoid than when their lengths are equal.

The maximum relative range of travel as measured from the end of the solenoid at which the plunger enters is equal to (1)

one-half the sum of the lengths of the solenoid and the plunger, (2) the length of the solenoid plus one-half the difference between the length of the plunger and the length of the solenoid, or (3) the length of the plunger plus one-half the difference between the length of the solenoid and the length of the plunger. Generally, the solenoid is assumed to be fixed in position and only the plunger moves.

Let the solenoid be 10 in. long. If the plunger is 20 in. long, the maximum range will be equal to $\left(\frac{10 + 20}{2}\right) = 15$ in., or 1.5 times the length of the solenoid. If the plunger is 5 in. long, the

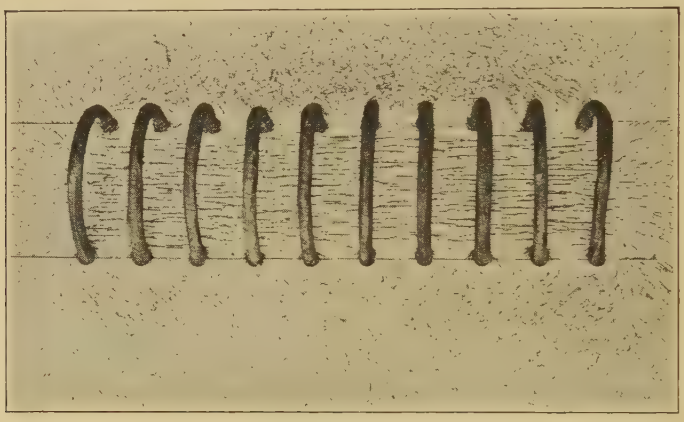


Fig. 212.—Magnetic field of a solenoid.

maximum range will be 7.5 in., or three-quarters of the length of the solenoid. The range here given is the maximum possible range, under the influence of the mechanical force of mutual attraction, as measured from the end of the solenoid at which the plunger enters, and is not the useful range, which is much shorter, being about 60 per cent of the length of the solenoid as a maximum.

The position of maximum net pull with respect to the axis of the solenoid is influenced by (1) the ratio of length of plunger to length of solenoid, (2) the amount of magnetic material inside the core hole, and (3) the magnetizing force. Condition (1) has been partially considered. The neutral-polarity plane of the plunger tends to be near the middle of that portion of the plunger which is within the core hole of the solenoid. This is

evident in Fig. 213, wherein a piece of steel has been inserted inside of the solenoid in Fig. 212, while the neutral-polarity plane of a uniform solenoid always is at the middle of the solenoid. When the neutral-polarity plane of the plunger coincides

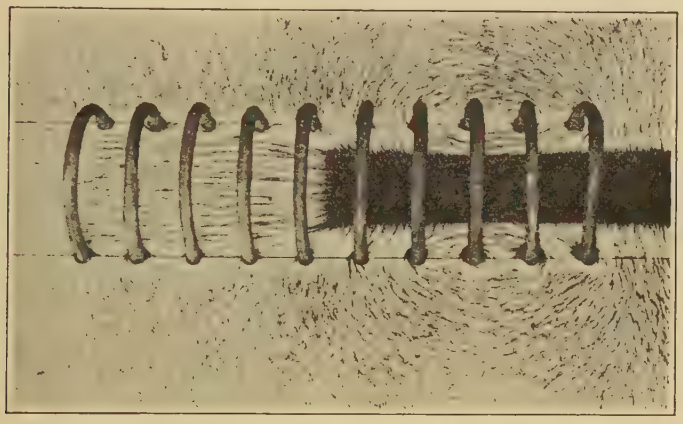


FIG. 213.—Position of neutral-polarity plane of plunger.

FIG. 214.—Plunger inside of solenoid.

with that of the solenoid, as in Fig. 214, the net mechanical pressure is equal to zero.

Let it be assumed that the solenoid is fixed in the horizontal position and that the plunger is free to move endwise along the axis common to both. Let the end of the solenoid adjacent to the end of the plunger be of *S* polarity. Then the end of the

plunger nearest that end of the solenoid will be of *N* polarity through magnetic induction. Mutual attraction results and, barring friction, the plunger will move towards the solenoid.

There are two ways of viewing the cause of the phenomenon of mutual attraction between the solenoid and the plunger under these circumstances. One way is to state that the plunger is mutually attracted by the solenoid because any relative movement would cause a greater linkage of magnetic flux with the ampere-turns in the solenoid, that is, the self-inductance and the permeance of the system thereby would be increased, the net mutual attraction becoming a maximum when the rate of change in the linkage, with respect to corresponding changes in relative position, is a maximum, and that the net pull is a minimum when the rate of change in the linkage, with respect to corresponding changes in position, is a minimum. Another way of explaining the phenomenon is to state that mutual attraction results in proportion to the product of the magnetizing force and the ferromagnetic flux density at the face of the plunger, regardless of the polarity of the solenoid, as previously mentioned. Both statements are correct and amount to the same thing.

Before the end of the plunger enters the core hole of the solenoid, that end may be so slightly magnetized that the mutual attraction is very small, but as the plunger enters the solenoid for greater distances it becomes increasingly magnetized, due to the greater m.m.f. available to overcome the external reluctance of the ferromagnetic circuit and the reduction of said reluctance, due to the greater area of plunger on the entering side of the neutral-polarity plane.

If the plunger is at least as long as the solenoid, after the entering *N* end of the plunger passes the neutral-polarity plane of the solenoid it enters the *S* magnetic side and becomes increasingly demagnetized as the structural or ferromagnetic flux increasingly leaks from it, as it moves on towards the extreme end of the solenoid.

124. Magnetic Poles and the Neutral-polarity Plane.—Right here is a good place to discuss magnetic poles. There are no magnetic poles (in the ordinary sense) in a uniform toroidal solenoid or electromagnet. Magnetic poles are in evidence where the leakage flux density is the greatest—not where the total flux density is the greatest. The place of greatest magnetic flux density in a solenoid, a bar electromagnet, or a bar per-

manent magnet (or in the horseshoe types) is at the neutral-polarity plane in each case.

The weaker the magnetizing force along the axis within the solenoid the more pronounced is the polarity. The polarity is a maximum when the length-rate of change in the magnetizing force is a maximum, and the polarity disappears when the length-rate of change in the magnetizing force is equal to zero. The term *polarity* as here used refers to poles and not specifically to direction.

125. Effect of Saturation on the Position of Maximum Pull.— The fact that a soft-iron or steel plunger is magnetized to different extents as it moves into the core hole of a solenoid causes the position of maximum pull to vary along the axis of the solenoid. While the entering end of the plunger may be saturated when it is at or near the middle of the solenoid, it never is saturated when at the extreme end of the solenoid. Consequently, when the plunger is longer than the solenoid, the position of maximum pull may occur at various points between the middle and the extreme end of the solenoid. Likewise, when the solenoid is longer than the plunger, the position of maximum pull may occur at various points between the middle and the entering end of the solenoid.

The calculation of the pressures due to the reactions between a solenoid and a plunger is somewhat complicated due to this changing in the intensity of magnetization in the plunger with respect to changes in the relative positions of the plunger and the solenoid. However, as shown in Chap. XI, the force-distance characteristics of solenoid-and-plungers may be quite closely approximated.

CHAPTER XI

TEST CHARACTERISTICS OF SOLENOIDS WITH PLUNGERS

Characteristics of solenoids with plungers as obtained by actual test are herein shown and discussed, the solenoids and the plungers having various dimensions.

126. Varying the Length of the Solenoid.—Years ago the author published results of tests[1] of various solenoids with

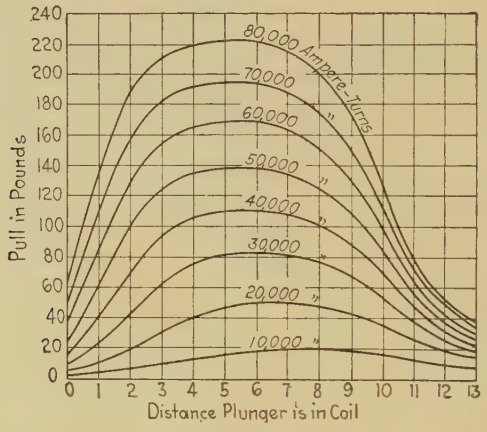


FIG. 215.—Pull characteristics of 10-in. solenoid.

plungers of different lengths and cross-sections, and with various magnetizing forces. These were later published in book form[2] and have otherwise been widely published. Figure 215[3] is a good illustration of the characteristic force-distance relations of solenoids in general with flat-faced plungers of uniform cross-section and longer than the solenoid, the magnetizing force and the intensity of magnetization being generally high. In the case of Fig. 215, the coil contained 122 lb. of No. 13 d.c.c. magnet

[1] UNDERHILL, C. R., "Practical Design of the Solenoid," *Elec. World and Engineer*, Apr. 29, 1905.

[2] UNDERHILL, C. R., "Solenoids," New York, 1910.

[3] UNDERHILL, C. R., "Calculation of Pull Due to Solenoids," *Elec. World and Engineer*, May 13, 1905.

wire, and was 10 in. long and 8.75 in. outside diameter. The plunger was of cold-rolled steel, 24 in. long and 1⅞ in. in diameter, making the cross-section 2.76 sq. in. The average radius of the solenoid was 2.7 in.

Referring to Fig. 215, the maximum net pulls do not exceed the net pulls at the middle of the solenoid when the number of ampere-turns per inch of length of solenoid is 2,000 or greater. The term *net pull* as herein used refers to the pulls as given, and is the observable pull after corrections have been made for friction and the weight of the plunger (if tested in a vertical position). This is the usual definition.

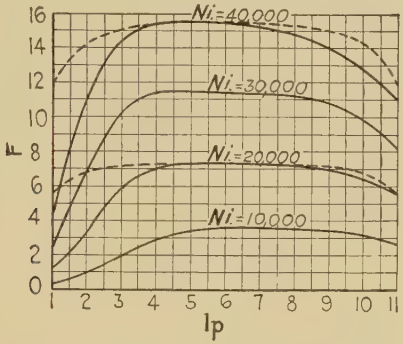


FIG. 216.—Comparison of F and h curves.

A group of other solenoids tested were respectively 1, 3, 6, 9, 12, and 18 in. long. All but the 1-in. solenoid (outside diameter—3.5 in., average radius—1.2 in.) were 3⅛ in. in outside diameter and had a common average radius of 1.1 in. A Swedish iron plunger about 3 ft. or 1 meter long was employed in the tests about to be described. This plunger was 1⅛ in. in diameter. Hence, its cross-section was 1 sq. in.

Figure 216[1] shows the test characteristics of the 12-in. solenoid above referred to. This solenoid contained 11 sections, as determined from the average radius. l_p is the distance, in section lengths, which the entering end of the plunger has passed into the core hole. Each numbered value of l_p corresponds to the middle of that section. $l_p = 6$ is the middle of the solenoid and the position of the neutral-polarity plane of the solenoid. F is the net force or pull in kilograms (1 kg. = 2.205 lb.) The two dotted curves represent relative values of magnetizing force

[1] UNDERHILL, C. R., "Solenoids," *Trans.* A.I.E.E., 1914.

(or of h) plotted to such scales as to be readily compared with the corresponding net pulls. The effects due to saturation are more noticeable as the number of ampere-turns (Ni) increases. The fact that the plunger is approximately saturated at its neutral-polarity plane when the number of ampere-turns is great causes the effects of magnetic leakage from the plunger to be more apparent as it approaches the farther end of the solenoid. With relatively low magnetizing forces, the structural flux density tends to continue increasing after the entering end of the plunger has passed the middle of the solenoid, due to the greater m.m.f. acting to increase it.

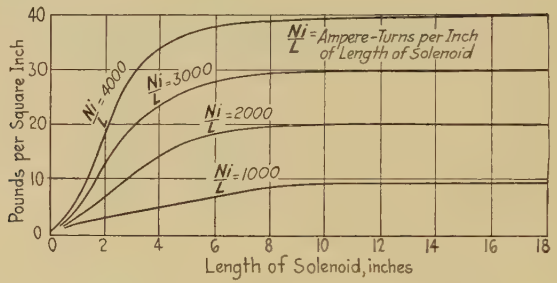


Fig. 217.—Ratio of pressure to maximum ampere-turn gradient.

In all cases the general test characteristics were about as in Fig. 215, the maximum net pulls tending to occur at the middle of the solenoid as the ampere-turn gradient was increased.

Figure 217 shows the net pounds pull per square inch of plunger corresponding to various ampere-turn gradients or numbers of ampere-turns per inch of length of solenoid. When the proper conversion factors are used, this amounts to the mechanical pressure per unit magnetizing force. Ratios of maximum net pounds pull per square inch to the number of ampere-turns per inch of length of solenoid, for various solenoid lengths, are plotted against ampere-turns in Fig. 218, the data being taken from Fig. 217. When the maximum net pounds pull per square inch of plunger per ampere-turn per inch is equal to 0.01, and h is equal to 2 at the middle of the solenoid, the structural flux density at the entering end of the plunger (much longer than the solenoid) is equal to $\frac{3{,}502{,}000}{2} \times 0.01 = 17{,}510$ gausses, as explained below. The highest structural

flux density found in pure iron,[1] even with extremely high magnetizing forces, is 21,100 gausses.

The data usually available or desired in connection with the testing or design of solenoids with plungers are the net pull per unit cross-section of plunger per ampere-turn per unit length of solenoid. The maximum net pull in dynes per square centi-

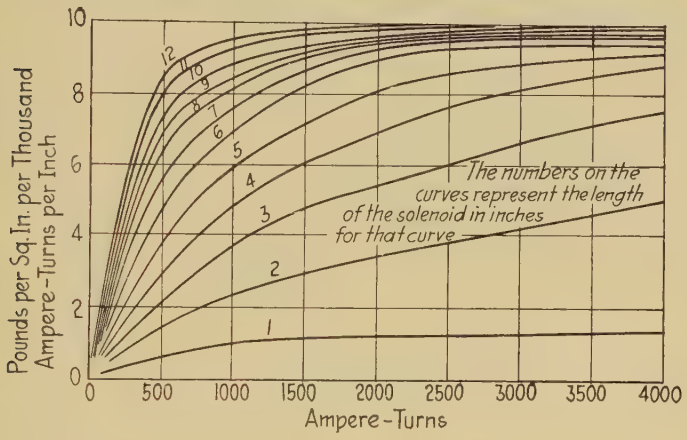


Fig. 218.—Ratio of pressure to maximum ampere-turn gradient.

meter of cross-section of plunger per ampere-turn per centimeter length of solenoid is equal to 0.05 times the product of h and the ferromagnetic flux density in gausses; in the case of a relatively long solenoid with its plunger approximately saturated, on the basis that the net pull or attraction is a maximum when the entering end of the plunger is at the middle of the solenoid.

The maximum net pull in kilograms per square centimeter of plunger per ampere-turn per centimeter of length of solenoid is equal to 0.000 000 51 times the product of h and the ferromagnetic flux density in gausses, while the maximum net pull in pounds per square inch of plunger per ampere-turn per inch of length of solenoid is equal to 0.000 000 285 times the product of h and the ferromagnetic flux density in gausses.

The ferromagnetic flux density in gausses is always taken at the middle of the solenoid with the entering end of the flat-faced plunger at that place. This may be calculated by multiplying the net pull in kilograms per square centimeter of plunger per ampere-turn per centimeter length of solenoid by 1,961,000 and

[1] Hadfield and Hopkinson, *J. Elec. Eng.*, p. 237, 1911.

dividing by h, or by multiplying the net pounds pull per square inch of plunger per ampere-turn per inch of length of solenoid by 3,502,000 and dividing by h. Figures 217 and 218 may be studied in this connection.

127. Solenoids with Constant Dimensions.—Contingencies like litigation often are productive of test data which otherwise might not be available. The author is much indebted to George E. Hulse[1] for data showing the results of tests of solenoids with

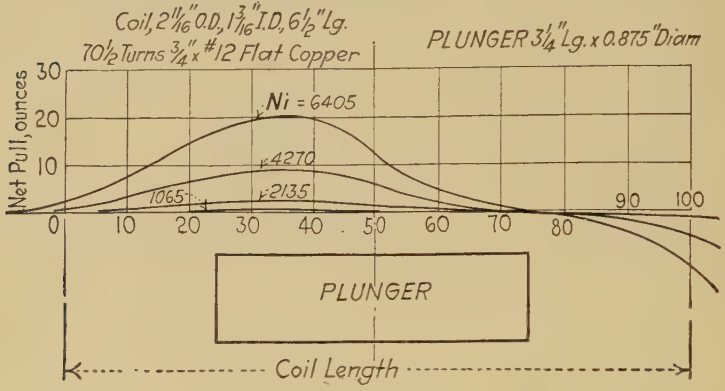


Fig. 219.—Test characteristics with 50 per cent plunger length.

plungers of various lengths, as well as those due to two similar solenoids with an intervening space between them, but both acting on a common plunger. While the data as presented are in a form slightly different than herein given, the measurements being taken from a point outside of the core hole and the distances in inches of travel of the plunger taken from that point, the distance along the axis is herein measured from the end of the solenoid at which the plunger enters and the distances are expressed in terms of length of solenoid. However, the curves are all plotted through the original points.

In all cases, the solenoid was $6\frac{1}{2}$ in. long, $2\frac{11}{16}$ in. outside diameter, $1\frac{3}{16}$ in. inside diameter, and contained 70.5 turns of $\frac{3}{4}$- by 0.081-in. copper, edge-wound. A simple calculation shows that the space factor was about 87 per cent. In each case, the diameter of the plunger was $\frac{7}{8}$ in. and its cross-section was 0.6. sq. in. The lengths in inches of the plungers were 3.25, 5.31, 6.5, 9.75, and 13, respectively, or, expressed in terms of

[1] Chief Engineer, Safety Car Heating & Lighting Co.

solenoid length, their respective lengths were 0.5, 0.817, 1, 1.5, and 2. Figures 219 to 223 show the original test data with the slight changes in arrangement referred to in the preceding para-

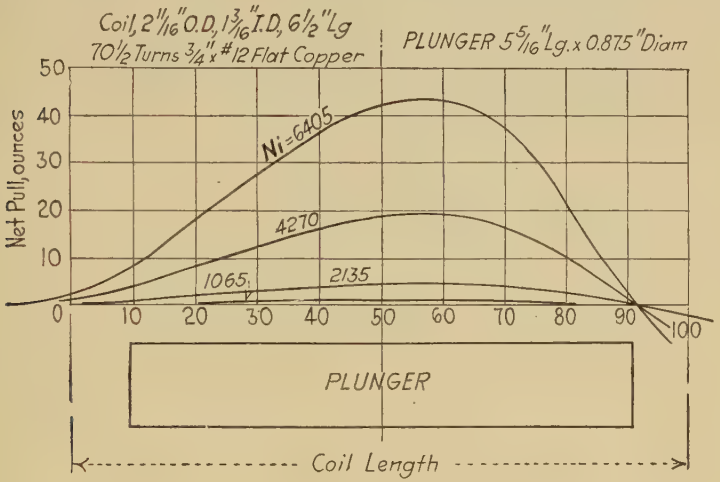


FIG. 220.—Test characteristics with 82 per cent plunger length.

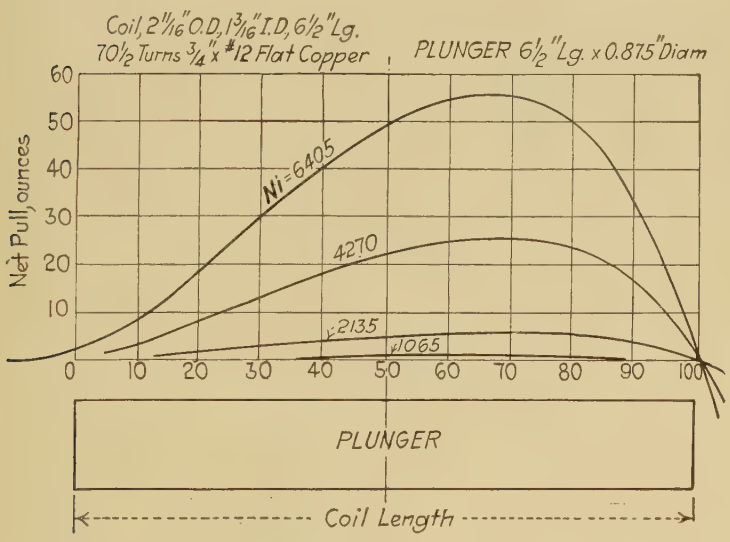


FIG. 221.—Test characteristics with 100 per cent plunger length.

graph. Furthermore, in these illustrations the middle of the plunger in each case is shown coincident with the middle of the solenoid. The right-hand end of each plunger is shown as having

entered the left-hand end of the solenoid and then having traveled to the position shown, leaving its pulling record behind it. The curves to the right of the right-hand end of the plunger show

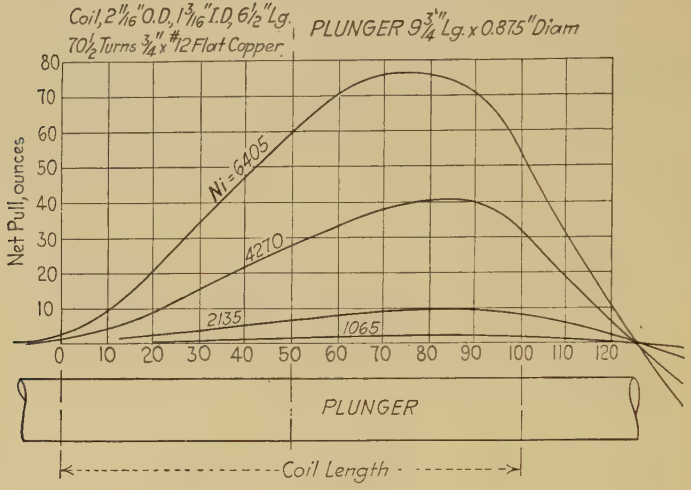


FIG. 222.—Test characteristics with 150 per cent plunger length.

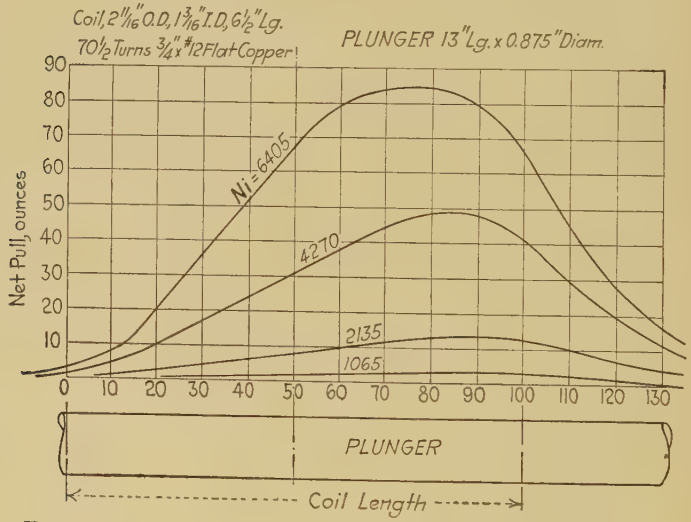


FIG. 223.—Test characteristics with 200 per cent plunger length.

that the pulls are in an opposite direction, and represent the tendency of the plunger to move back to the position of balanced pull between the solenoid and the plunger. These right-hand

negative pulls are exactly the same as though the plunger was moved back again over its original line of motion, except that the pulls would then be in a positive direction as shown. The constant direct-current strengths in amperes were 15, 30, 60, and 90, respectively, making the respective numbers of ampere-turns 1,068, 2,135, 4,270, and 6,405, and the ampere-turn gradients were 164, 328, 657. and 976 ampere-turns per inch, respectively.

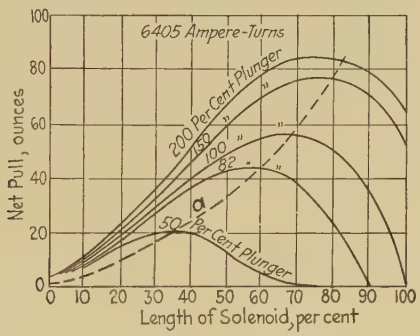

6405 Ampere-Turns
200 Per Cent Plunger
100 "
82 "
50 Per Cent Plunger
a
Net Pull, ounces
Length of Solenoid, per cent
0 10 20 30 40 50 60 70 80 90 100
0 20 40 60 80 100

FIG. 224.—Comparison of test characteristics.

Whereas the author's published data referred chiefly to groups of solenoids with plungers longer than the solenoids, and with magnetizing forces due to 1,000 ampere-turns per inch and higher,

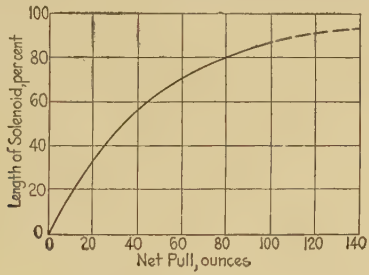


FIG. 225.—Curve *a* from Fig. 224.

the above data refer to a solenoid of constant dimensions with plungers of various lengths, and with magnetizing forces due to about 1,000 amp.-turns per inch and lower. Consequently, the two groups go well together.

Figure 224 was constructed from data taken from Figs. 219 to 223, and shows that the *increase* in maximum net pull, for the same solenoid but with different lengths of plunger, becomes less and

less as the plunger becomes longer and longer. The dotted curve in Fig. 224 is shown rearranged in Fig. 225 and indicates that the tendency of the left-hand end of the solenoid to attract the plunger in that direction decreases as the length of the plunger extending from that end of the solenoid increases.

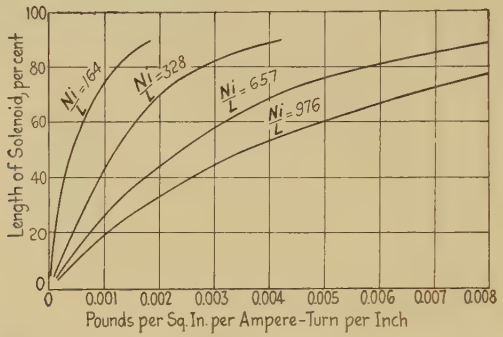


Fig. 226.—Pressure per maximum ampere-turn gradient for the entire group.

In the above analysis of the test data, the trial curve so closely resembled an exponential curve that the curve purposely was made to follow that law. The fact that the maximum pulls for plungers longer than the solenoid occur when the entering ends of

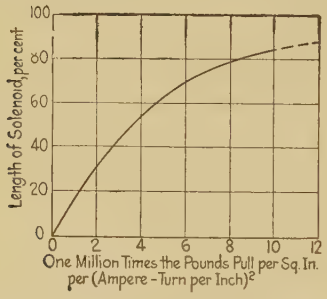


Fig. 227.—Relation common to all of the groups.

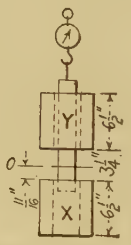


Fig. 228.—Differential solenoid.

the plungers are more than 70 per cent of the solenoid length inside of the solenoid shows that the plungers are not saturated.

Figure 226 shows the maximum net pull in pounds per square inch of plunger per ampere-turn per inch of length of solenoid for the entire group with the various magnetizing forces, and indicates that the plungers are magnetized in proportion to the square

of the number of ampere-turns per inch. Dividing again by the number of ampere-turns per inch, and averaging up the slight differences in the positions of maximum pull for each of Figs. 219 to 223, the ratios of the net pounds pull per square inch of plunger

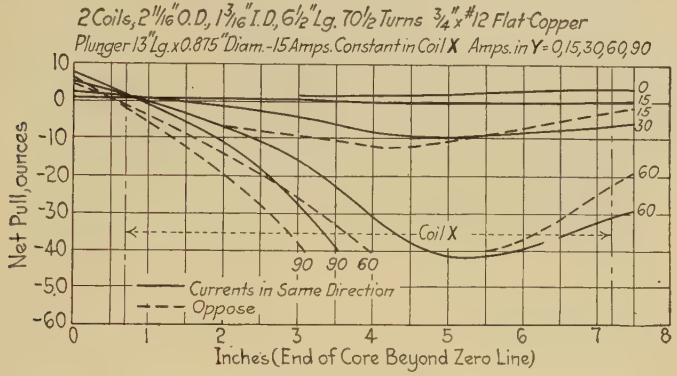

2 Coils, 2 1/16" O.D, 1 3/16" I.D, 6 1/2" Lg. 70 1/2 Turns 3/4" x #12 Flat Copper
Plunger 13" Lg. x 0.875" Diam. -15 Amps. Constant in Coil X Amps. in Y = 0, 15, 30, 60, 90
Net Pull, ounces
Inches (End of Core Beyond Zero Line)
Coil X
90 90 60
Currents in Same Direction
Oppose
0
15
15
30
60
60

FIG. 229.—Test characteristics of differential solenoid.

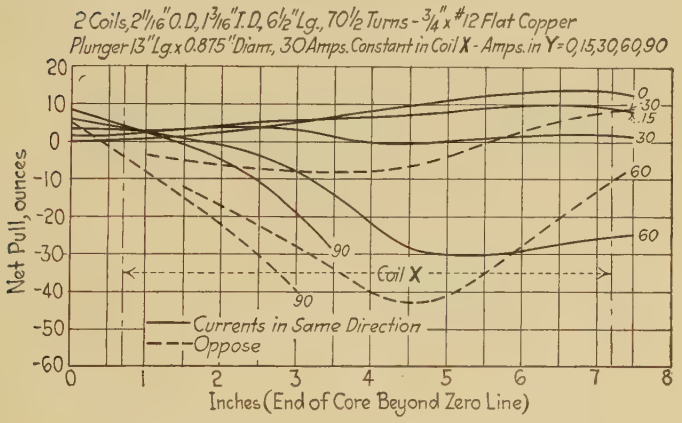

2 Coils, 2 1/16" O.D, 1 3/16" I.D, 6 1/2" Lg., 70 1/2 Turns - 3/4" x #12 Flat Copper
Plunger 13" Lg. x 0.875" Diam., 30 Amps. Constant in Coil X - Amps. in Y = 0, 15, 30, 60, 90
Net Pull, ounces
Inches (End of Core Beyond Zero Line)
Coil X
90
90
Currents in Same Direction
Oppose
0
30
15
30
60
60

FIG. 230.—Test characteristics of differential solenoid.

to the square of the number of ampere-turns per inch of solenoid in each case are so nearly alike as to show that the intensity of magnetization varies directly with the magnetizing force in all cases and that saturation is not approached. This statement is qualified to the extent that some variation in the intensity of magnetization may be included in the exponential relation between the apparent intensity of magnetization and the distance

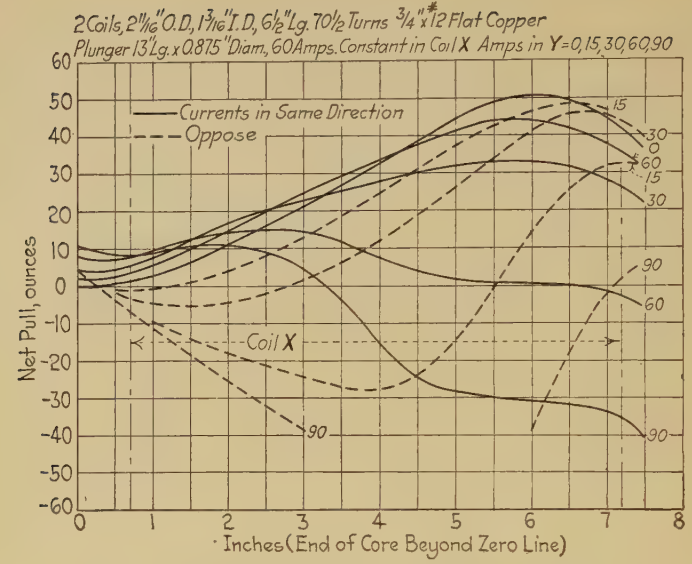

2 Coils, 2 1/16" O.D., 1 3/16" I.D, 6 1/2" Lg. 70 1/2 Turns 3/4" x #12 Flat Copper
Plunger 13" Lg. x 0.875" Diam, 60 Amps. Constant in Coil X Amps in Y=0, 15, 30, 60, 90
Currents in Same Direction
Oppose
Coil X
Net Pull, ounces
Inches (End of Core Beyond Zero Line)

FIG. 231.—Test characteristics of differential solenoid.

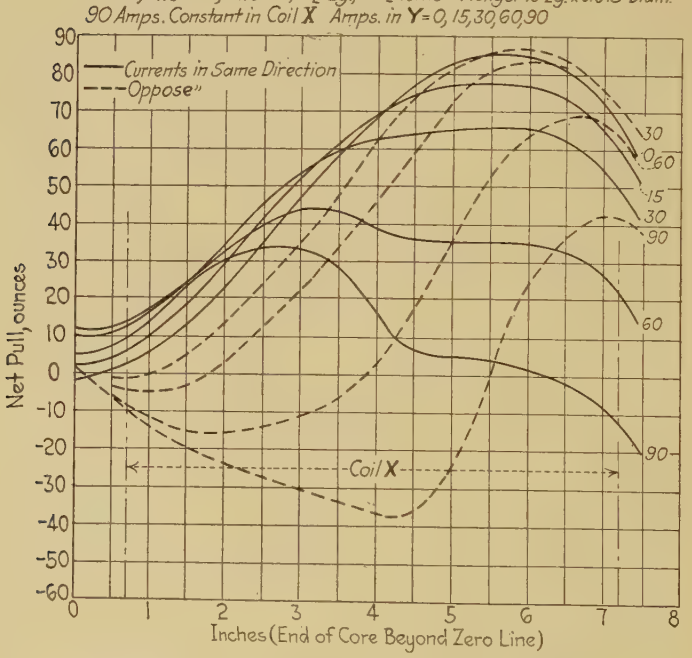

2 Coils, 2 1/16" O.D., 1 3/16" I.D, 6 1/2" Lg., 70 1/2 Turns Plunger 13" Lg. x 0.875" Diam.
90 Amps. Constant in Coil X Amps. in Y=0, 15, 30, 60, 90
Currents in Same Direction
Oppose"
Coil X
Net Pull, ounces
Inches (End of Core Beyond Zero Line)

FIG. 232.—Test characteristics of differential solenoid.

inside of the solenoid in terms of per cent length of solenoid, at which the maximum pulls occur. After performing the above calculation, the common relation is as shown in Fig. 227.

The test characteristics in Figs. 229 to 232 were obtained with two similar solenoids acting on the 13-in. plunger, as referred to on page 194. Figure 228 shows the arrangement. Both solenoids are the same as in Figs. 219 to 223 and the diameter of the plunger was 0.875 in., as in the previous tests. The pulls refer to positions of the lower end of the plunger below the point *O* in Fig. 232. Negative pulls are pushes, or the extent of the tendency of the plunger to move in a direction opposite to that usually assumed. As usual, the weight of the plunger is deducted from the total downward pull to obtain the net pull. Whatever friction may appear also is allowed for.

When the currents in the two solenoids are equal and in the same direction, the difference between the pulls on the opposite ends of the plunger is the net pull. The fact that the ends of the two solenoids do not abut is immaterial as to principle, but the plunger is not so thoroughly magnetized as though the intervening space was active.

CHAPTER XII

GENERAL CHARACTERISTICS OF SOLENOIDS WITH PLUNGERS

The present chapter discusses the energy, force, self-inductance and other characteristics of solenoids with plungers, the general methods herein described being applicable to all forms of electromagnets.

128. Variations in the Magnetization of the Plunger.—As the entering end of the plunger moves towards the middle of the solenoid, that end of the plunger becomes increasingly magnetized. But after the end of the plunger passes beyond the middle of the solenoid it simultaneously becomes further magnetized and loses some of its magnetization, due to the leakage of the structural flux, as previously described. It is this effect which causes the maximum pull to occur beyond the middle of the solenoid and yet within the limit of the core hole.

This effect is clearly indicated by placing the plunger in various positions with respect to the solenoid and then varying the magnetizing force without permitting the plunger to move. These data may be obtained from Fig. 215, wherein the solenoid is 10 in. long and the plunger is 2.76 sq. in. in cross-section. When the entering end of the plunger is 1 in. inside of the solenoid, it is in 10 per cent of the length of the solenoid. When it is in 10 in. it is in 100 per cent of the length of the solenoid. From these data, the net pressures in pounds per square inch have been plotted against the number of ampere-turns per inch of length of solenoid in Figs. 233 and 234, the former showing the characteristics for various relative positions up to the middle of the solenoid, and the latter from the middle to the extreme end of the solenoid. A comparison of Figs. 233 and 234 shows that the rate of magnetization of the plunger with respect to corresponding increases in the magnetizing force is greater before the entering end of the plunger arrives at the middle of the solenoid than after it has passed that point. This relative difference is particularly noticeable at 0 and 100 per cent.

129. Work Done by a Solenoid-and-plunger.—In what follows, the solenoid is assumed to be fixed in position while the plunger is movable. The work done by an electromagnet is equal to the product of (1) the average force acting along the path of motion and (2) the length or distance traversed by the movable part along said path under the influence of that force. The work done, or the mechanical energy expended, in moving the plunger of a solenoid along the axis from a point remote from the conventional left-hand end of the solenoid to any point of reference

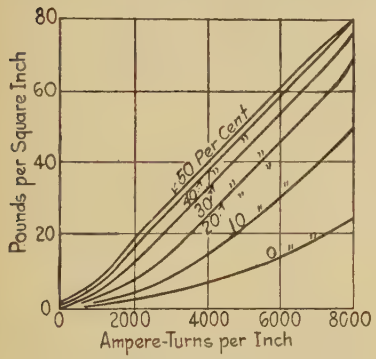


FIG. 233.—Effects due to magnetization of plunger.

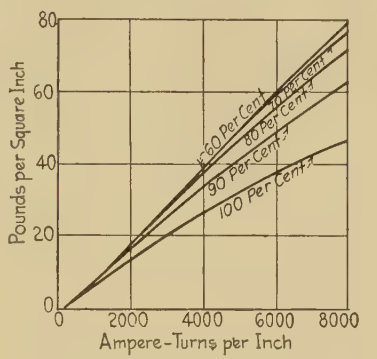


FIG. 234.—Effects due to leakage from plunger-and-coil.

on the axis is equal to the product of the average force and the total distance, as measured from the remote point to the point of reference, and this is equal to the magnetic energy stored during the process. Therefore, by taking the net static pulls over ranges beginning where the pull is equal to zero, the magnitude of the stored magnetic energy may be calculated after measuring the area under the curve with a planimeter.

In all cases, the amount of mechanical energy expended and the equal amount of magnetic stored are equal to the area under the force-distance curve between the initial and final positions of the plunger. If the initial position of the plunger is at the entering end of the solenoid and its final position is at the extreme end of the solenoid, the mechanical energy expended and the magnetic energy stored during the movement of the plunger throughout the length of the solenoid in overcoming mechanical resistance, under the influence of the mechanical force developed by the solenoid-and-plunger, are equal to the product of the average mechanical force and the length of the solenoid. Under

these conditions, the work per unit volume of plunger within the core hole of the solenoid is equal to the product of the average force and the length of the solenoid, divided by the product of the cross-section of the plunger and the length of the solenoid, or to the average net pressure throughout the length of the solenoid.

The force-distance curves in Fig. 235 are taken from Fig. 215. The areas under these curves to the left of the end of the solenoid have been taken, so that the number of inch-pounds at

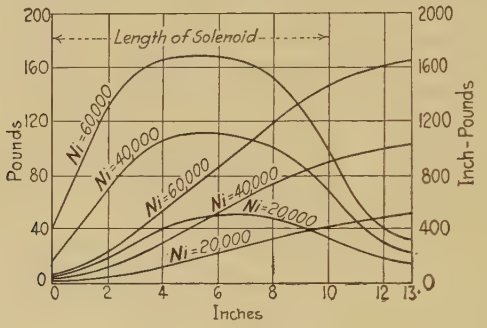


Fig. 235.—Stored magnetic energy due to solenoid-and-plunger.

the initial position of the plunger at the left-hand end of the solenoid shows that some work already has been done in moving the plunger to what now is its initial position.

The inch-pound or work curves in Fig. 235 may be viewed in two distinct manners, that is, they may be viewed as expressing the energy expended up to any point along the axis in overcoming mechanical resistance along the path of motion of the plunger with the number of ampere-turns constant in each case, or they may be regarded as the magnetic energy stored in the system with the plunger at rest at the various positions, the number of ampere-turns increasing from 0 to the values given while storing the magnetic energy. The latter view is the basis of the equation for stored magnetic energy, that is, the current strength increases from 0 to a definite value, so that the average current strength is really meant when the self-inductance is constant, and this leads to the equation $W = \frac{1}{2} Li^2$.

The force-distance curves, showing the pull in pounds at any given position of the plunger with various numbers of ampere-turns, express the rate of change in the number of inch-pounds of work or of stored magnetic energy with respect to correspond-

ing small changes in the position of the plunger. The work or stored-energy curves express the integral of the force-distance characteristics.

For each unit of mechanical energy expended in an electromagnetic system, two units of electric energy are expended, one replacing the magnetic energy converted into mechanical energy and expended, and the other increasing the stored magnetic energy by one unit. When there are eddy-current and hysteresis losses, more electric energy is abstracted from the electric circuit to overcome or to supply these losses.

All of the above will be understood by referring to Fig. 235. With 40,000 amp.-turns (4,000 amp.-turns per inch), the pull may be taken as 112 lb. on the average, between the limits 5 and 6 in. within the solenoid. Consequently, the mechanical energy expended in moving the plunger from the initial position where its entering end is 5 in. within the core hole of the solenoid to its final position, 6 in. within the core hole, is equal to (6 in. − 5 in.) × 112 lb. = 112 in.-lb. For the same number of ampere-turns, the difference in the energy expended or the work done between the limits 5 and 6 in. is equal to 520 in.-lb. (at 6 in.) − 408 in.-lb. (at 5 in.) = 112 in.-lb. In a like manner, the number of inch-pounds is found for the first inch. Then the number of inch-pounds is found for the second inch, and this number is added to the first to give the total for the second inch, and so on. However, the exact method is to (1) make the differences in length so small that they cannot be distinguished from zero, (2) multiply the force at each conceivable point by that difference in length, and (3) add together all of the products. The result is the integral, and the process is called integration. This operation may be mechanically performed by means of a planimeter.

As shown above, the ratio

$$\frac{\text{difference in work}}{\text{difference in length}}$$

gives the average force for the corresponding difference in length. Likewise, the product of the average force and the difference in length gives the difference in work, or the energy expended or stored for that difference in length.

In all cases, the mechanical energy expended represents an increase in the stored magnetic energy equal to that expended.

130. The Amount of Magnetic Energy Stored in a Solenoid-and-plunger.—One inch–pound = 0.112 98 joule. One joule = 8.851 34 in.-lb. Consequently, for every inch-pound of mechanical energy expended by a solenoid-and-plunger, or by any other form of electromagnet, 0.112 98 joule is abstracted from the electric circuit, converted into magnetic energy, and thence converted into mechanical energy, another equal amount of electric energy being converted into magnetic energy and stored in the magnetic circuit. Hence, for every inch-pound of mechanical energy expended there are 2 in.-lb., or $2 \times 0.112\ 98 = 0.225\ 96$ joule of electric energy abstracted from the electric circuit and converted into magnetic energy, one-half being expended as mechanical energy and the other stored as magnetic energy besides any additional electric energy required to supply losses.

Referring to Fig. 235, when the entering end of the plunger is placed 5 in. within the core hole of the solenoid and the number of ampere-turns is then increased from 0 to 40,000, the amount of electric energy converted into magnetic energy and stored in the magnetic circuit is equal to 408 in.-lb., or to $408 \times 0.112\ 98$ joule = 46.1 joules. When the entering end of the plunger is moved by the mechanical force produced between the solenoid and the plunger against mechanical resistance from a point remote from the left-hand end of the solenoid, 408 in.-lb., or 46.1 joules, of mechanical energy have been expended in doing work and $46.1 \times 2 = 92.2$ joules or volt-coulombs of energy have been abstracted from the electric circuit, one-half of which is expended in doing work, while the other half is stored as magnetic energy in the magnetic circuit. Likewise, if the plunger is moved 1 in. farther into the core hole by the force produced between the solenoid and the plunger, and this mechanical energy is due to its movement, 112 in.-lb. (page 205) of mechanical energy will be expended during the movement, $112 \times 0.111\ 98 = 12.65$ joules of electric energy are abstracted from the electric circuit to supply the mechanical energy, and 12.65 more joules of electric energy are abstracted from the electric circuit to be stored in the magnetic circuit. Thus $2 \times 12.65 = 25.3$ joules of electric energy must be expended in any form of direct-current electromagnet in doing 12.65 joules, or 112 in.-lb., of work.

From the foregoing, the electric energy expended when mechanical work is done is double that converted into magnetic energy and stored in the magnetic circuit. The electric energy in joules

converted into magnetic energy and stored in the magnetic circuit is equal to one-half the number of ampere-weber-turns, and also is equal to the product of (1) the constant voltage, (2) the average amperage, and (3) the elapsed time in seconds, that is, to the number of volt-coulombs. Therefore, the constant or average voltage, amperage, and the elapsed time and quantity of electricity are related to the pounds or kilograms of mechanical force and the distance or length in inches or centimeters through the common point of contact *energy*.

The work data in Fig. 235 are plotted in Fig. 236 in terms of joules instead of in mechanical units.

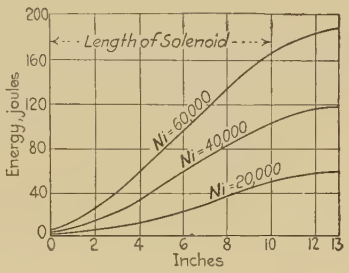


FIG. 236.—Energy in joules.

131. The Self-inductance of a Solenoid-and-plunger.—The self-inductance of a solenoid-and-plunger may be approximately determined from the work characteristics on the basis that the additional stored magnetic energy is equal to the work done due to the movement of the plunger in overcoming mechanical resistance, and the fact that the stored magnetic energy in joules is equal to one-half the product of the self-inductance in henries and the square of the current strength in amperes. Consequently, the self-inductance in henries for any relative position of the solenoid and the plunger is approximately equal to twice the ratio of the work in joules to the square of the current strength in amperes, and the ratio of the self-inductance in henries to the square of the number of turns is approximately equal to twice the ratio of the work in joules to the square of the number of ampere-turns.

As an example, the approximate self-inductance of the above solenoid (Art. 130) will be calculated on the basis of 5,000 turns. Then for 40,000 amp.-turns, the current strength is 8 amp. and its square is 64. When the plunger is 5 in. inside of the core hole, the work is equal to 408 in.-lb., or 46.1 joules. Hence, the approximate self-inductance is equal to $\frac{(2 \times 46.1)}{64} = 1.44$ henries. If there were no eddy-current or hysteresis losses, and there was no current-wave or voltage-wave distortion (as there really are),

the r.m.s. current strength in amperes with the solenoid connected across a 110-volt, 60-cycle circuit would be equal to $\frac{110}{377 \times 1.44}$ $= \frac{110}{543} = 0.202$ amp., and the number of r.m.s. ampere-turns would be equal to $0.202 \times 5{,}000 = 1{,}010$, instead of 40,000 with a continuous voltage of about 125 volts. In order that the number of r.m.s. ampere-turns should be equal to 40,000 at 60 cycles, the r.m.s. voltage would have to be equal to $377 \times 1.44 \times 8 =$ 4,350 volts.

Figure 237 shows the approximate values of self-inductance in henries for the 10-in. solenoid under discussion, for various numbers of ampere-turns and at various relative positions of the solenoid and the plunger. As the plunger becomes more and more magnetized, the number of weber-turns per ampere (self-inductance in henries) becomes smaller and smaller. Therefore, the self-inductance diminishes as the number of ampere-turns increases when saturation is approached, or at any point above the knee of the normal saturation curve. Below the knee, the self-inductance is greater.

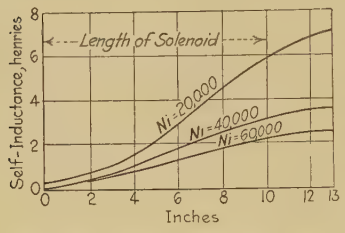


FIG. 237.—Self-inductance of a solenoid-and-plunger.

132. Mechanical Force as a Function of the Length-rate of Change in the Self-inductance.—Since the difference in energy due to a corresponding change in position, or difference in length, along the axis of a solenoid-and-plunger is equal to one-half the product of the square of the constant-current strength and the differences in self-inductance corresponding to the same difference in length, and since

$$\frac{\text{difference in energy}}{\text{difference in length}}$$

is the average mechanical force over the difference in length, it follows that the average mechanical force is equal to one-half the product of the square of the constant-current strength and the ratio

$$\frac{\text{difference in self-inductance}}{\text{difference in length}}.$$

One joule is equal to 8.851 34 in.-lb. Consequently, the average mechanical force in pounds is equal to 4.426 times the product of the square of the constant-current strength in amperes and the

$$\frac{\text{difference in self-inductance in henries}}{\text{difference in length in inches}}.$$

Small differences in length should be taken where the curvature of the force-distance curve is great, but larger differences in length may be taken where the curve is nearly straight. Referring to Fig. 237, when the end of the plunger is 5 in. inside of the solenoid and the current strength is 8 amp. on a 5,000-turn basis, making the number of ampere-turns 40,000, the self-inductance is 1.44 henries. When the plunger is 6 in. inside the solenoid, the self-inductance is 1.83 henries. The difference in the self-inductance is 0.39 henry, the difference in length is 1 in., and the

$$\frac{\text{difference in self-inductance}}{\text{difference in length}}$$

is 0.39 henry per inch. The square of the current strength in amperes is 64. Therefore, the average force during the 1-in. movement of the plunger is equal to $4.426 \times 64 \times 0.39 = 111$ lb. Referring to page 205, the average force during the same movement was taken as 112 lb., as may be observed by reference to Fig. 235.

When the difference in self-inductance and the corresponding difference in length are made so small that they cannot be distinguished from zero, the ratio is the differential coefficient $\frac{dL}{dl}$, or the length-rate of change in the self-inductance, and the average force is the actual force at the point of reference. The differential coefficient $\frac{dL}{dl}$ is equal to the tangent of the angle of slope of the self-inductance-distance curve at the point of reference, when the scale is corrected so that units of self-inductance shall have the same length on the paper as units of length or distance. When the angle of slope is a minimum the mechanical force is a minimum. When the angle of slope is a maximum the mechanical force is a maximum.

133. E.m.f. as a Function of the Time-rate of Change in the Self-inductance.—The self-induced voltage is equal to the product of the constant current strength in amperes and the time-rate of change in the self-inductance in henries per second.

In the case of the example in Art. 132, the length rate of change in the self-inductance is 0.39 henry per inch. If the average velocity of the plunger throughout the inch of travel is 100 in. per second the time-rate of change in the self-inductance will be equal to 39 henries per second, since the time of travel is 0.01 sec. Then the average self-induced voltage for a constant current of 8 amp. will be equal to $8 \times 39 = 312$ volts.

The equation is as follows: The self-induced voltage is equal to the product of (1) the length-rate of change in the self-inductance and (2) the product of the current strength in amperes and the velocity of the plunger or armature. Any unit of length may be used. Since the unit here used is the inch, the length-rate of change of self-inductance is in henries per inch and the velocity is in inches per second.

The average time of travel is found from an equation which is obtained by rearranging the above equation, as follows: The velocity is equal to the ratio of (1) the self-induced voltage to (2) the product of the amperage and the length-rate of change in the self-inductance. Hence, if the impressed voltage is 110 volts and the self-induced voltage may be assumed to exactly balance the impressed voltage, if the plunger should move from the 5-in. position to the 6-in. position at a uniform velocity while overcoming friction, that velocity could not exceed $\frac{110}{8 \times 0.39} = \frac{110}{3.12}$ $= 35.2$ in. per second, and its time of travel between the two positions would be equal to $\frac{1}{35.2} = 0.0284$ sec.

134. Self-inductance as a Function of the Length and Maximum Net Pull of a Solenoid-and-plunger.—For solenoids having lengths of 8 in. or more, and with plungers considerably longer than the solenoids, the maximum net pounds pull per square inch of plunger is about 0.01 times the number of ampere-turns per inch of length of solenoid (or the ampere-turn gradient) when the latter is 1,000 or more. Stated in another manner, the maximum net pounds pull per square inch of plunger is about 10 when the ampere-turn gradient is 1,000 amp.-turns per inch or greater.

Figure 218 should be consulted for varying the above statement and for finding the maximum net pounds pull per square inch of plunger for other solenoid lengths and ampere-turn gradients. Excellent data also will be found in Figs. 219 to 227.

In any case, it is well to allow a factor of safety, since the heating of the solenoid increases the electric resistance and decreases the calculated direct-current strength for a given voltage, and the line voltage may vary. Furthermore, the steel used may not have the same permeability as the specimens used in the tests.

Figure 238[1] shows the average of the test characteristics of the solenoids having lengths of 6, 9, 12, and 18 in. respectively,

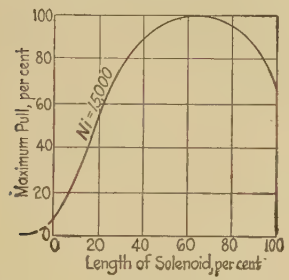


FIG. 238.—Average test characteristics of solenoid-and-plunger.

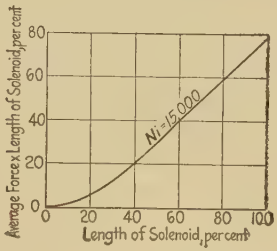


FIG. 239.—Average work curve of solenoid-and-plunger.

referred to in Art. 126, and also shows that the per cent maximum net pull is 8 when the entering end of the plunger is at the conventional left-hand end of the solenoid, 90 at 40 per cent length of solenoid, 100 at 65 per cent, 86 at 90 per cent, and 67 at 100 per cent length of solenoid. By this method, the maximum net pull may be considered as the average pull between the limits 60 and 70 per cent length of solenoid.

The general method about to be described consists in first taking the product of the net maximum pull and the length of the solenoid, and then multiplying this product by the percentage of the above product as obtained by taking the actual area under the per cent force—per cent distance curve. Figure 239 expresses the area under the curve in Fig. 238, including that portion beyond the left-hand end (not shown in Fig. 238). The curve in Fig. 239 shows the approximate per cent work for different per cent lengths of solenoid on the basis that the plunger has been moved from a point remote from the left-hand end of the solenoid up to the various per cent lengths of solenoid. It is to be noted that Fig. 239 is based on 15,000 amp.-turns, regardless of the length of the solenoid.

[1] UNDERHILL, C. R., "Characteristics of the Solenoid," *Elec. World*, vol. 50, p. 1907, 1907.

The effect of increasing the number of ampere-turns is to increase the useful range and to shift the position of maximum pull. This also increases the per cent work values. To use Fig. 239 in this connection, deduct 6 per cent from the work values in Fig. 239 for 10,000 amp.-turns, and add 6 per cent for 20,000 amp.-turns, 12 per cent for 30,000 amp.-turns, 18 per cent for 40,000 amp.-turns, and 24 per cent for 50,000 amp.-turns. None of the values thus obtained are to be considered more than approximate, but they are very useful in finding the approximate self-inductances of solenoids with plungers, bar electromagnets, and some other forms.

As an example of the use of Fig. 239, let the dimensions be as follows: length of solenoid, 10 in., cross-section of plunger, 2 sq. in., and length of plunger, say, 20 in. If the ampere-turn gradient is 1,500 amp.-turns per inch, the total number of ampere-turns will be 15,000 and the maximum net pull in pounds for each square inch will be 15, the force in pounds being equal to $15 \times 2 = 30$ when the entering end of the plunger is 65 per cent of 10 in., or 6.5 in., inside of the solenoid. The product of the maximum net pull in pounds and the length of the solenoid is equal to $30 \times 10 = 300$ in.-lb., and the work done in moving the plunger up to the 6.5-in. point when the number of ampere-turns is 15,000 is equal to 45 per cent of this value (as shown in Fig. 239), or to 135 in.-lb., or to $135 \times 0.112\ 98 = 16.6$ joules, and 16.6 joules is equal to one-half the product of the self-inductance in henries and the square of the constant direct-current strength in amperes, or to one-half the product of the square of the number of ampere-turns and the ratio of the self-inductance in henries to the square of the number of turns. Consequently, the ratio of the self-inductance to the square of the number of turns is equal to $\frac{2 \times 16.6}{15{,}000^2} = \frac{33.2}{225{,}000{,}000} = 0.000\ 000\ 148$. If the number of turns is 4,000, the self-inductance in henries is equal to $0.000\ 000\ 148 \times 16{,}000{,}000 = 2.37$. If the number of turns is 2,000, the self-inductance is equal to $0.000\ 000\ 148 \times 4{,}000{,}000 = 0.592$ henry.

The data obtained from the $3\frac{1}{8}$-in. diameter solenoids with 1-sq. in. plungers will now be applied to the 10-in. solenoid described in the preceding articles, using the same number of turns (5,000) and ampere-turns. Since the total number of ampere-turns is 40,000 and the cross-section of the plunger is 2.76

sq. in., the maximum net pull is equal to $4 \times 10 \times 2.76 = 110$ lb. (Fig. 215), and this occurs when 5 in. of the plunger are inside of the solenoid. One hundred and ten pounds $\times$ 10 in. = 1,110 in.-lb., or $1{,}110 \times 0.112\ 98 = 125.3$ joules. This is the work which would be done if the maximum net pull extended throughout the entire length of the solenoid. Referring to Fig. 239, when 5 in. of the plunger are inside of the solenoid, the work done with 15,000 amp.-turns is equal to 30 per cent of 125.3 joules, or to 37.6 joules. For 40,000 amp.-turns, add 18 per cent (page 212). Then the work in joules is equal to $37.6 \times 1.18 = 44.4$ instead of 46.1, as calculated on page 206, the difference being 4 per cent. However, the force on page 206 was taken as 112 lb. instead of 110 lb. a difference of 2 per cent. Hence, the calculation of the self-inductance by either method gives approximately the same results.

The method is of great value in connection with alternating-current solenoids when allowances are made for the usual losses. It also is useful in cases where the approximate time-constant must be known in order to determine the time which must elapse before the solenoid can pick up a load.

When the kilogram-centimeter is the unit of work, as when the pull is expressed in kilograms and the length of the solenoid is expressed in centimeters, the rule for the maximum net pull is as follows: The maximum net pull in kilograms per square centimeter of plunger is 0.72 when the ampere-turn gradient is 400 amp.-turns per centimeter, or greater. For instance, the length of the 10-in. solenoid is 25.4 cm. When the ampere-turn gradient is 400 amp.-turns per centimeter, the total number of ampere-turns is equal to 10,160, or 1,016 amp.-turns per inch. If a plunger 1 sq. in. in cross-section is used, its cross-section is 6.452 sq. cm. The maximum net pull is then equal to 10.16 lb. per square inch, or to 1.575 lb. per square centimeter. One pound = 0.4536 kg. Hence, the maximum net pull in kilograms per square centimeter of plunger is equal to $1.575 \times 0.4536 = 0.715$.

One joule = 10.198 kg.-cm.; 1 kg.-cm. = 0.098 06 joule. Therefore, to convert the work in kilogram-centimeters into joules, multiply the work in kilogram-centimeters by 0.098 06. Then proceed as previously explained.

135. Ratio of the Mechanical Force to the Self-induced Voltage. When there are no (assumed) internal losses, the time-rate at which mechanical energy is expended, as during the movement of a plunger at constant velocity in overcoming friction or moving

against the reaction of a dash pot, is equal to the time-rate at which magnetic energy is simultaneously being stored in the magnetic circuit. The electric power expended exclusively in supplying the mechanical power and the magnetic power is double the mechanical power, since one-half of the total electric power is equal to the time-rate of storage of magnetic energy.

The electric power in watts is equal to the product of the self-induced voltage and the amperage. The mechanical power is equal to the product of the mechanical force and the velocity of the plunger or armature.

One joule = 10.198 kg.-cm. = 8.8513 in.-lb. Therefore, 1 watt = 10.198 kg.-cm. per second = 8.8513 in.-lb. per second, and for each watt (self-induced voltage × amperage) of electric power, ½ watt is the time-rate of storage of magnetic energy and 5.099 kg.-cm. per second, or 4.426 lb.-in. per second, is the maximum mechanical power developed.

If the force is 5.099 kg., the maximum velocity per watt of electric power is 1 cm. per second. If the force is 4.426 lb., the maximum velocity per watt of electric power is 1 in. per second. These are the greatest constant velocities ever developed per watt of electric power with the above respective forces. If the forces are 1 kg. and 1 lb., respectively, then the maximum velocities will be 5.099 cm. per second and 4.426 in. per second, respectively, per watt of electric power. Therefore (page 8) kilograms × centimeters per second = 5.099 × self-induced volts × amperes, and pounds × inches per second = 4.426 × self-induced volts × amperes. Likewise, self-induced volts × amperes = 0.1961 × kilograms × centimeters per second = 0.226 × pounds × inches per second.

From the foregoing:

1. The number of kilograms per volt is equal to 5.099 times the ratio of the amperage to the velocity in centimeters per second.

2. The number of pounds per volt is equal to 4.426 times the ratio of the amperage to the velocity in inches per second.

3. The number of volts per kilogram is equal to 0.1961 times the ratio of the velocity in centimeters per second to the amperage.

4. The number of volts per pound is equal to 0.226 times the ratio of the velocity in inches per second to the amperage.

In connection with the foregoing, an additional voltage must be added to provide for losses. In the case of a direct-current electromagnet, the additional voltage required to overcome resistance reaction is equal to the product of the amperage and the resistance in ohms.

Let the resistance of a direct-current solenoid be 100 ohms and let the constant current strength be 1 amp. during the movement of its plunger. Then the active voltage will be 100 volts and the additional electric power will be 100 watts. If the line voltage is 110 volts, the voltage available for doing work is equal to 110 − 100 = 10 volts and, if the desired plunger velocity is 2 in. per second, the number of pounds pull per self-induced volt is equal to 4.426 × ½ = 2.213 when the entire 10 volts are exerted in moving the plunger at the constant velocity of 2 in. per second, and the total force is equal to 2.213 × 10 = 22.13 lb. After the armature has completed its movement, the current strength will be equal to $^{110}/_{100}$ = 1.1 amp.

The number of self-induced volts per pound pull is (4) equal to 0.226 times the ratio of the velocity of the plunger or armature in inches per second to the amperage. In the case of the 10-in. solenoid (page 209) the plunger velocity is 100 in. per second and the current strength is 8 amp. Therefore, the number of self-induced volts per pound pull is equal to 0.226 × ($^{100}/_{8}$) = 0.226 × 12.5 = 2.82. When the pull is 111 lb., the self-induced voltage is equal to 111 × 2.82 = 313 volts.

136. The Maximum Velocity at Which the Plunger Can Move While Overcoming Friction.—The maximum constant velocity is herein meant, as in Art. 135. The greater the internal losses the slower is the movement of the plunger or armature for a given electric power input, because the mechanical force is thereby decreased.

The rules on page 214 may be interpreted as below for finding the maximum velocity.

1. The velocity in centimeters per second is equal to 5.099 times the number of watts per kilogram.

2. The velocity in inches per second is equal to 4.426 times the number of watts per pound.

In the case of the 10-in. solenoid referred to in Art. 135, the self-induced voltage is 313 volts and the current strength is 8 amp. Therefore, the electric power consumed in moving the plunger or armature under the influence of the force of 111 lb. is

equal to $313 \times 8 = 2{,}500$ watts and the plunger velocity is equal to $4.426 \times \left(\frac{2{,}500}{111}\right) = 4.426 \times 22.6 = 100$ in. per second, as already known.

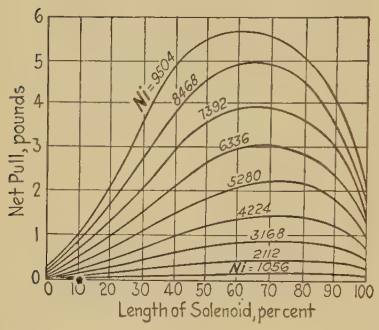


FIG. 240.—Test characteristics with 0.518-sq. in. plunger.

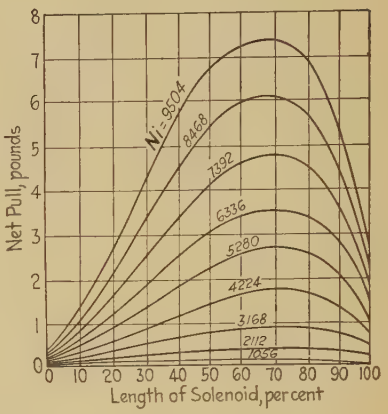


FIG. 241.—Test characteristics with 0.785-sq. in. plunger.

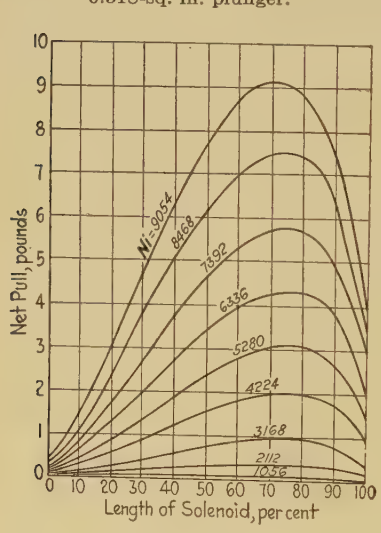


FIG. 242.—Test characteristics with 1-sq. in. plunger.

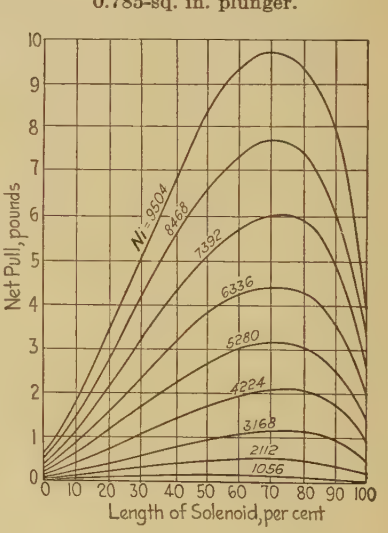


FIG. 243.—Test characteristics with 1.23-sq. in. plunger.

137. The Effects of Using Various Cross-sections of Plunger.—The solenoid used in the tests about to be described had the following dimensions and turns: outside diameter, $3\frac{1}{16}$ in.; inside diameter, $1\frac{3}{8}$ in.; length, $7\frac{1}{4}$ in.; average radius, 1.05

in.; number of sections, 6.9; cross-section of core hole, 1.49 sq. in.; number of layers, 12; number of turns per layer, 88; total number of turns, 1,056. The plungers were of cold-rolled steel 8⅛ in. long. The cross-sections of the plungers in square inches were respectively 0.518, 0.7854, 1, and 1.23. The direct-current test characteristics are given in Figs. 240 to 243.

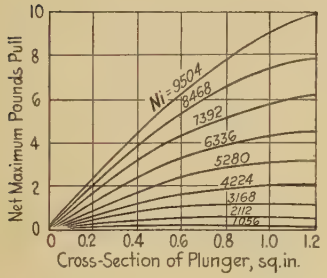


Fig. 244.—Net maximum pulls.

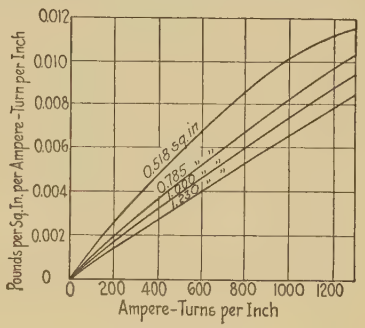


Fig. 245.—Maximum net pressures per maximum ampere-turn gradients.

Figure 244 shows the maximum net pulls for each current strength used and for various cross-sections of plunger. In Fig. 245 the maximum net pounds per square inch of plunger per ampere-turn per inch of solenoid are plotted against the ampere-turn gradients for the various plungers used. As the cross-section of the plunger approaches the maximum, as limited by the cross-section of the core hole, the pressure per ampere-turn gradient is reduced. For this reason, the pull of a solenoid cannot be increased in direct proportion to the cross-section of the plunger by increasing the cross-section of the core hole and using a larger plunger. Figure 245 is a good guide for such changes. It is to be observed that the plunger is only 12 per cent longer than the solenoid in each of the above cases.

The work in inch-pounds for the solenoid with the 1-sq. in. plunger is plotted against the various positions of the plunger inside of the solenoid in Fig. 246.

138. Approximate Rules.—These apply to relatively long direct-current solenoids in general. The maximum net pounds pull per unit ampere-turn gradient is herein somewhat reduced.

1. The maximum net pull in pounds is equal to 0.009 times the product of (*a*) the number of ampere-turns and (*b*) the

ratio of the cross-section of the plunger in square inches to the length of the solenoid in inches.

2. The product of the maximum net pull in pounds and the length of the solenoid in inches is equal to 0.009 times the product of the number of ampere-turns and the cross-section of the plunger in square inches.

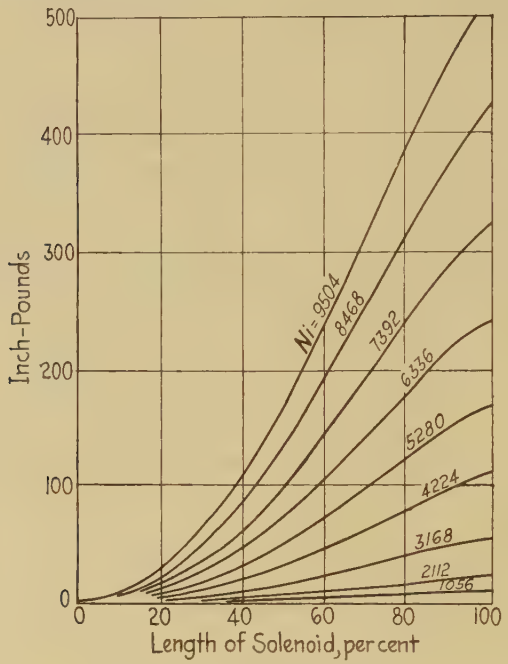


Fig. 246.—Work characteristics.

3. The maximum net pull varies directly as the number of ampere-turns, directly as the cross-section of the plunger, and inversely as the length of the solenoid.

4. The maximum net pull may be approximately doubled by (*a*) doubling the number of ampere-turns, (*b*) doubling the cross-section of the plunger, or (*c*) halving the length of the solenoid.

5. The number of ampere-turns for constant active voltage may be doubled by doubling the cross-section of the coil conductor, and *vice versa*. This is equivalent to changing the A.w.g. gage number by three sizes.

6. Since the pull varies as the cross-section of the plunger, with a given number of ampere-turns, if the solenoid will not lift its plunger it cannot be made to do so by increasing the cross-section of the plunger, since the weight also varies directly with the cross-section for a given plunger length. Furthermore, the pull does not increase directly with the cross-section of the plunger in solenoids having the usual lengths, because the number of sections is reduced as the coil radius is increased, thus reducing the magnetizing force for a given number of ampere-turns.

139. General Statements.—When a solenoid is in the vertical position with the plunger approaching from the bottom, the weight of the plunger must be added to the desired pull in order to obtain the net pull. When the plunger approaches the solenoid from the top, the weight of the plunger must be subtracted from the desired pull in order to obtain the net pull. When the solenoid is in the horizontal position, or at an angle thereto other than 90 deg., the friction must be added to the desired pull in order to obtain the net pull. In all other positions, the weight to be added or subtracted varies as the product of the weight of the plunger and the sine of the angle of inclination with the horizontal. In all cases, the weight of the plunger may be counterbalanced when convenient or practicable. For instance, if a pull of 10 lb. is desired and the plunger weighs 1 lb., the plunger must be capable of pulling 11 lb. upward, 9 lb. downward, and 10 lb. plus friction horizontally. However, plungers that approach solenoids from above are usually more than counterbalanced.

When a plunger is suspended above the core hole of a solenoid, its axis tends to coincide with the axis of the solenoid and to keep away from the wall of the core hole. This is because of the tendency of magnetic materials to get into the densest part of a magnetic field. However, when the same end of the plunger has passed through the core hole and approaches the opposite end of the solenoid, that end of the plunger is deflected away from the axis of the solenoid towards the wall of the core hole because, by so moving, it increases the stored magnetic energy of the system.

A solenoid sometimes is caused to pull its plunger upward into a position where the pull just balances the force of gravity, where the plunger is said to float. When the current is switched off, the plunger falls and either opens or closes a switch, strikes

a gong, or otherwise does work. This application is chiefly found in alternating-current apparatus.

Solenoids sometimes are caused to lift their plungers and then the current is switched off at the critical instant so that the momentum of the plunger shall carry it still further upwards. As the plunger falls, the current is automatically switched on again as it enters the solenoid from its opposite end, so that this time the solenoid pulls the plunger downwards. The total downward force is then equal to the pull of the solenoid plus the pull due to gravity.

As an analogy, a solenoid-and-plunger behaves as a very large body of matter in space would behave, which had a hole through it into which a huge core of matter might be attracted through the mutual attraction of gravitation. When their centers of gravity coincided there would be two balanced forces, each tending to pull each half core towards the middle of the body, but the inertia of the core would cause it to oscillate to and fro, the mutual attraction tending to cause it to stop with the two centers of gravity coincident. When a solenoid is securely fastened to a table in a horizontal position and a plunger is placed with one end near the core hole, the former quickly moves within the latter and continues moving until its magnetic center is considerably beyond the magnetic center of the solenoid. Movement in the reverse direction then begins and the plunger oscillates to and fro until the kinetic energy is damped out by friction, which usually occurs in a very short time.

CHAPTER XIII

DIRECT-CURRENT ELECTROMAGNETS

This chapter deals with some fundamental principles, after which the subjects of iron-clad solenoids, double-plunger solenoids, short external air gaps, coned plungers and stops, and commercial forms of iron-clad solenoids are described and illustrated, the dimensions, weights, and force-distance ratings of the latter being given.

140. Iron-clad Solenoids.—The general term *solenoid* is applied to many forms of electromagnets with plungers or movable cores. The term *iron-clad solenoid* may refer to some particular form of plunger electromagnet. Plunger electromagnets or iron-clad solenoids are used extensively with brakes and, therefore, often are called brake magnets. In general, the plungers of plunger electromagnets coact with stops or plugs, or else with the ends of frames or shells. A frame or shell completely surrounding a solenoid increases the magnetic leakage from the plunger and, therefore, increases the pull due to the greater linkage of ampere-weber-turns and the rate at which they will increase with a corresponding movement of the plunger with respect to the coil and shell. The effect of the plug or stop is to produce a good sealing or holding pull at or near the end of the stroke, as well as to increase the pull when the air gap is relatively short.

141. Double-plunger Solenoids.—In Fig. 247 the solenoid is in a balanced position with respect to the two plungers and it tends to float in that position. The least unbalance will cause the solenoid to move to the middle of one or the other of the plungers, if the latter are fixed in position and the solenoid is free to move. When one of the plungers is fixed in position with respect to the solenoid, as in Fig. 248, it will be found that the pull between the solenoid and the plunger is about the same as that between the solenoid and the plunger in Fig. 249, or between the two plungers (with the solenoid linked with them) in Fig. 247. The reason is that the air gap is so long that the attraction

between the plungers is scarcely noticeable in addition to the solenoid pull, and in the case of Fig. 247 the solenoid equally attracts both plungers.

However, as the free plunger approaches the fixed plunger or stop in Fig. 248, the attraction between them is very much in evidence.

FIG. 247.—Solenoid with two plungers.

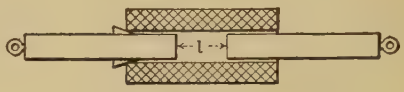


FIG. 248.—Solenoid with one plunger fixed in position.

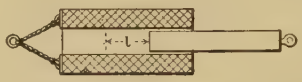


FIG. 249.—One of the plungers is omitted.

In order to obtain double the force, the range must be halved, as in Fig. 250.[1]

Each stationary plunger forms a stop or plug. The pull between the solenoid and the stationary plunger or stop exists

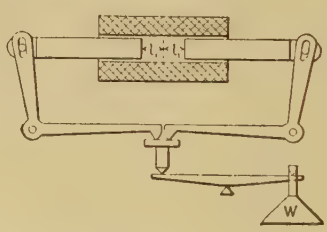


FIG. 250.—Both plungers working together.

just as though the latter was movable, but it cannot be utilized although magnetic energy is stored in it. Only the mechanical construction prevents its movement. Whenever a core is fastened with respect to the coil, some of the ability to do work is taken away from the electromagnet.

An interesting study of the plunger electromagnet is given in Figs. 251 to 253. Figure 254 shows the external magnetic field of a bar electromagnet or of a solenoid with the plunger in the balanced position. Figure 255 shows the external ferromagnetic fields of the solenoid in Fig. 254 with the plunger

[1] UNDERHILL, C. R., "Solenoids," *Trans.* A.I.E.E., 1914. Figures 247 to 249, 254, and 255 also are taken from this paper by permission.

separated at its middle to form a two-plunger solenoid of the general type of that in Figs. 247 or 251. Some interesting experiments[1] were performed with the electromagnet in Fig. 255. The

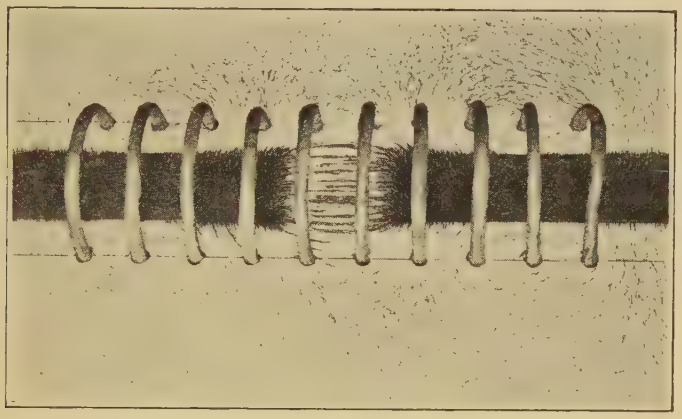


FIG. 251.—Magnetic fields of solenoid with two plungers.

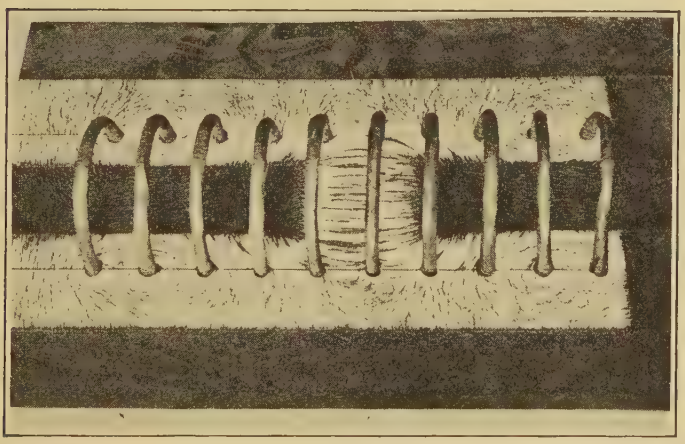


FIG. 252.—Magnetic fields of plunger electromagnet with open frame.

net pull was about the same whether it was measured between the solenoid and one plunger, between the two plungers through a 1.25-in. air gap, or between the two plungers when the solenoid was surrounded by a massive steel frame. Small pins, not

[1] One of many performed by the author in the course of his lectures in the American colleges. Some of the readers may recall pulling on the spring scale at the opposite end.

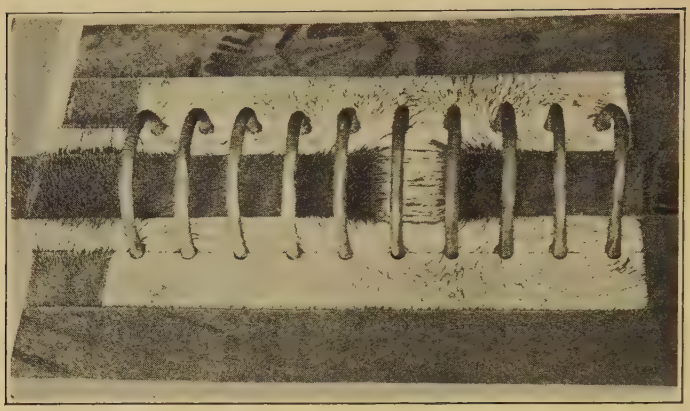


FIG. 253.—Magnetic fields of plunger electromagne.

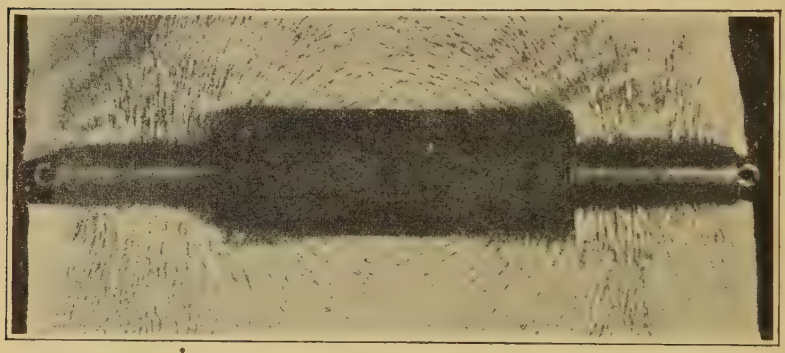


FIG. 254.—External magnetic field of solenoid-and-plunger.

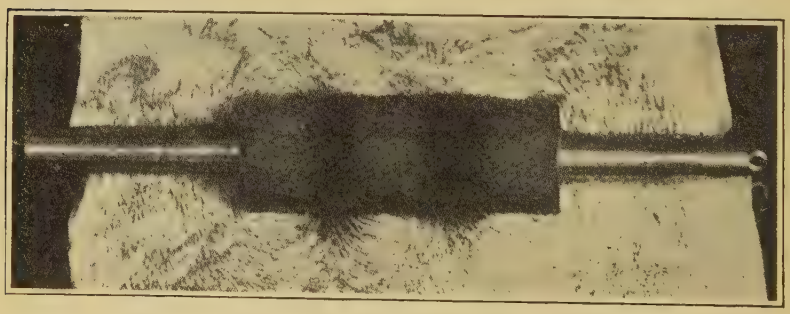


FIG. 255.—Magnetic fields when an air gap is introduced.

visible in Fig. 255, prevented the plungers from passing into the solenoid. When a non-magnetic spacer filled the air gap between the two freely mounted plungers (page 180), one or the other of the plungers (the pins having been removed) was violently ejected from the solenoid, the remaining plunger occupying the core hole.

142. Total Pull of a Plunger Electromagnet.—This is due to the usual solenoid effect *plus* any improvement which the shell or frame may produce due to magnetic leakage and the magnetic energy stored in the air gap between the ends of the plunger

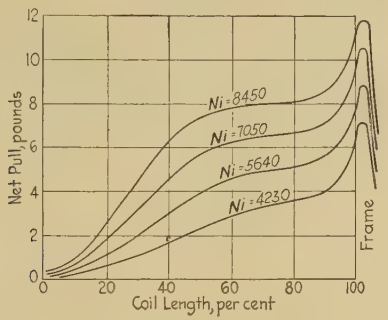


FIG. 256.—Test characteristics of magnetic-cushion type of plunger electromagnet.

and the stop. Consequently, the solenoid pull exists just as though the air gap was not present, and the air-gap pull is added to the solenoid pull. At some points along the axis the solenoid pull is greater than the air-gap pull, while the latter is usually greater than the solenoid pull at points near the end of the stop, under the condition that the air gap is not very short. When the air gap is relatively small, however, the air-gap pull may always exceed the solenoid pull, particularly when the magnetizing force is small (page 63).

Figure 256 shows the test characteristics of a plunger electromagnet or iron-clad solenoid of the magnetic-cushion type having a hole through the lower end of the frame. When the plunger passes into the opposite end of the frame through this hole, the ferromagnetic circuit is made about as complete as it can be made under the circumstances and the pull in the direction of the axis falls to, or approaches, zero, since there is then practically no pull except the side pull. The coil contained 4,700 turns. The direct-current strengths were 0.9, 1.2, 1.5, and 1.8 amp.,

respectively. The resistance was 34 ohms. The cross-section of the plunger was 0.6 sq. in. The coil length was 7 in.

When the current strength was 1.8 amp. the number of ampere-turns was equal to $1.8 \times 4{,}700 = 8{,}450$, and the ampere-turn gradient was equal to $\frac{8{,}450}{7} = 1{,}200$. Hence, the maximum

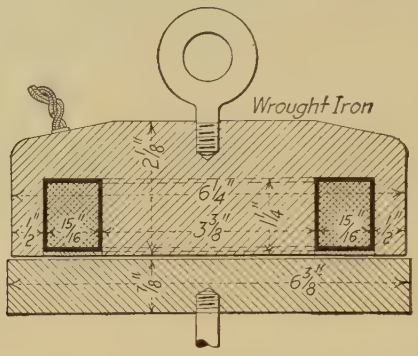


Fig. 257.—Hazeltine's lifting magnet.

net pull due to the solenoid and plunger without the iron frame would be equal to about $0.009 \times 1{,}200 \times 0.6 = 6.5$ lb. The presence of the steel frame increases the solenoid pull by about

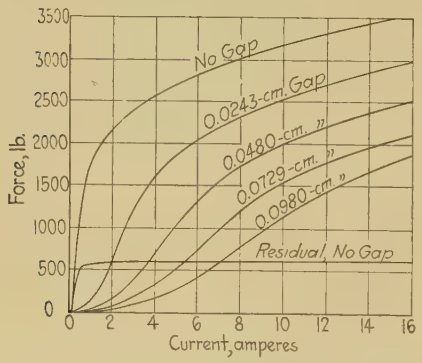


Fig. 258.—Force-current curves of Hazeltine's lifting magnet.

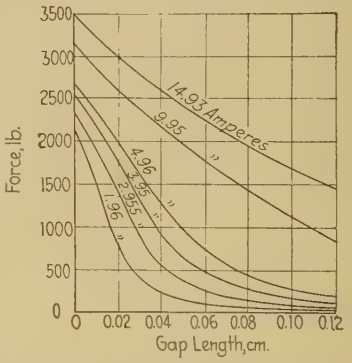


Fig. 259.—Force-distance curves of Hazeltine's lifting magnet.

1 lb. The rapidly rising pull curve is due to the mutual attraction between the end of the plunger and the surface of the frame which it approaches. The difference between the total pull and the solenoid pull is the pull through the air gap.

143. Pulls through Short External Air Gaps.—The small lifting magnet in Fig. 257 was designed by Prof. L. A. Hazeltine, who kindly furnished the drawing and test data. This little magnet will lift 1 ton attached to its armature when its coil is connected to a single dry cell. The coil contained 180 turns of No. 14 (A.w.g.) d.c.c. magnet wire. The resistance was 0.586 ohm at 40° C.

Referring to the test characteristics in Figs. 258 and 259, while the magnet will lift more than 2,000 lb. through a zero

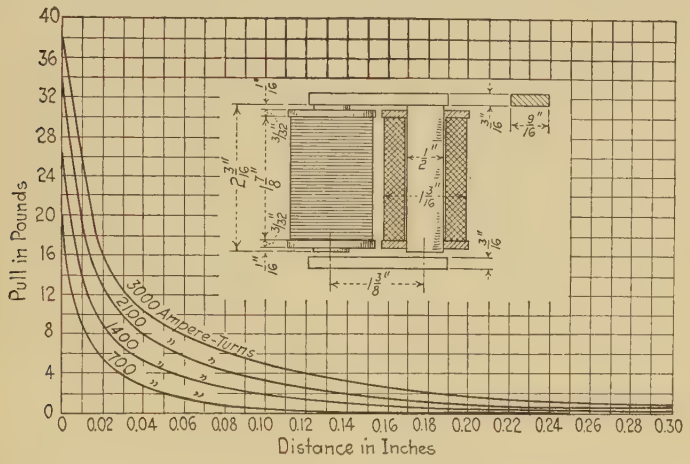


FIG. 260.—Tests characteristics of a small horseshoe electromagnet.

air gap with a current strength of 1.96 amp. it will not lift more than 50 lb. through an air gap 0.1 cm. (1 mm. or 0.0394 in.) long with the same current strength. This is characteristic of electromagnets with small coil lengths and external armatures (page 38).

Figure 260[1] shows the dimensions and test characteristics of a small horseshoe electromagnet. In this case the effects of saturation are not so apparent and there is a greater solenoid pull on the armature due to the greater lengths of the coils.

144. Coned Plungers and Stops.—The arrangement shown in Fig. 261 may be called a magnetic lever, since what is gained in travel is lost in force. This electromagnet contained 3,179 turns of No. 20 enameled magnet wire having a resistance of

[1] UNDERHILL, C. R., "The Performance of Different Types of Electromagnet," *Am. Electrician*, June, 1905.

32.5 ohms at 40° C. The test characteristics are respectively given in Figs. 262 and 263. The electromagnet was designed

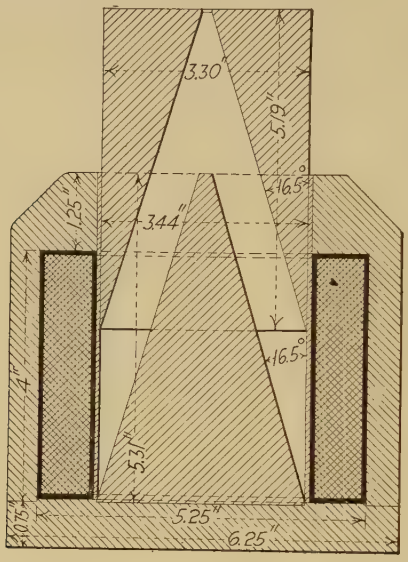


Fig. 261.—Hazeltine's coned-plunger electromagnet.

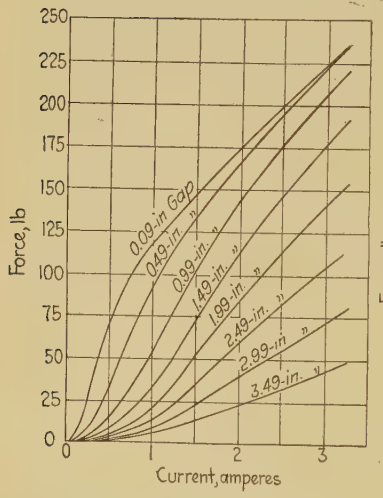


Fig. 262.—Force-current curves of Hazeltine's coned-plunger magnet.

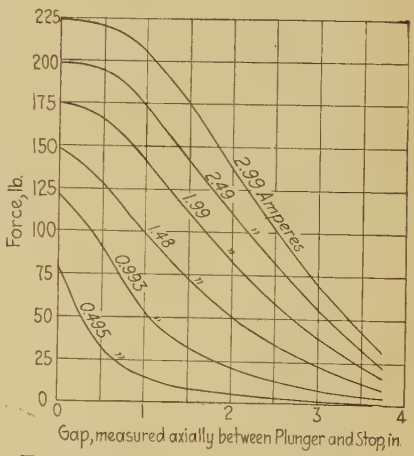


Fig. 263.—Force-distance curves of Hazeltine's coned-plunger magnet.

and tested by Prof. L. A. Hazeltine, who kindly furnished the data.

The effects of coning the plungers of the usual types of plunger electromagnets are illustrated in the following data supplied by H. L. Blood. Figure 264 shows the construction of the frame,

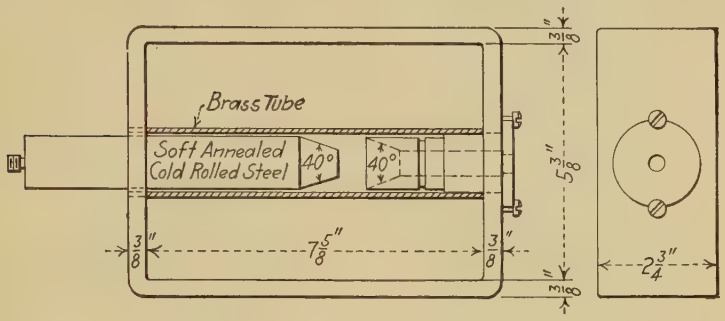


FIG. 264.—Coned plunger and stop.

stop, and plunger. Figure 265 shows the test characteristics with flat-faced plungers and stops, while Fig. 266 shows the test

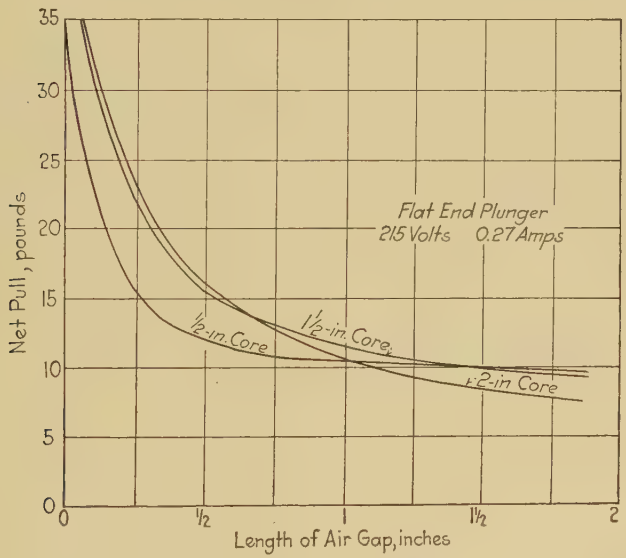


FIG. 265.—Test characteristics with different flat-faced plungers.

characteristics of the electromagnet as constructed, but with various angles of taper. The stops projected about 20 per cent inside the coil.

Coned plungers should always meet their complimentary stops at their centers rather than at their circumferences. The constant hammering at the circumferences causes hardening of the steel and the resulting remanent magnetism extended over so great an area eventually causes sticking, so that the electromagnet will not release. When the mechanical and magnetic contact is at or near

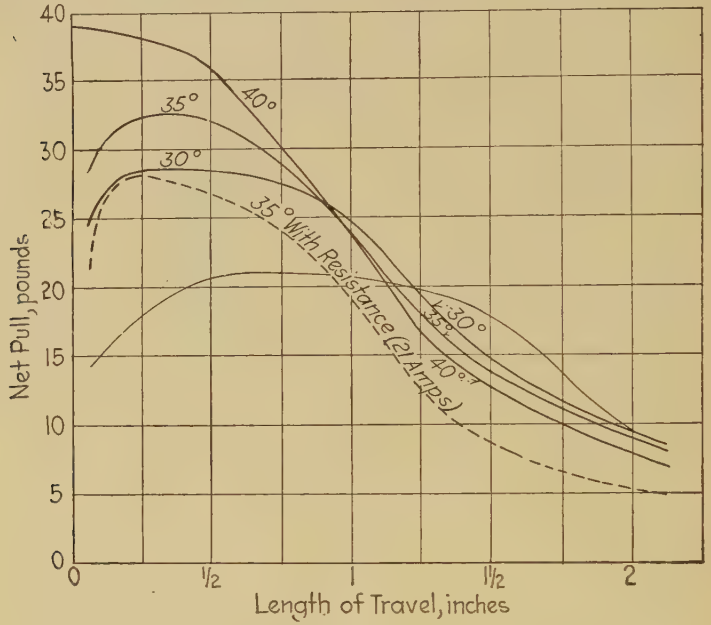


Fig. 266.—Test characteristics with various coned plungers and stops.

the common center or axis, the area is small and the tendency to stick is not so noticeable. In this case, the equivalent of magnetizing force is small, so the reasoning on page 71 does not apply to this case.

The principle of the coned plunger and stop is that the attracting area thereby is increased. For instance, if the diameter of the plunger is 1⅛ in., its cross-section is 1 sq. in. Hence, if the plunger is flat-faced, the attracting area will be 1 sq. in. However, if the angle of taper in Fig. 264 is 45 deg., and the cone continues on to a point, there will be a cone on the end of the plunger having a base 1⅛ in. in diameter and a height (or depth) of 1.36 in. The area of the cone useful for attraction with a very short air gap is equal to $0.5 \times 1.36 \times 3.1416 \times 1.125 = 2.4$ sq.

in., instead of 1 sq. in. as in the case of the flat end, and the reluctance of the air gap is more than halved (and the permeance more than doubled) for a given distance between the coacting surfaces. However, the direction of the pull is now at an angle of 67.5 deg. from the axis of the plunger and stop, so that

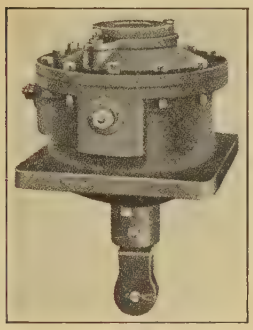


FIG. 267.—C-H series-wound solenoid.

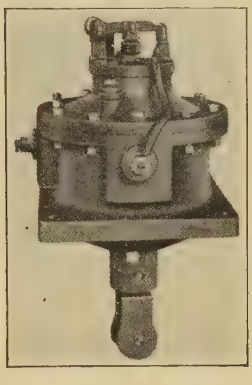


FIG. 268.—C-H shunt-wound solenoid with protecting switch.

the actual pull available in the direction of the axis is equal to the product of the pull at an angle of 67.5 deg. from the axis and the cosine of 67.5 deg. Consequently, while the area of the air gap is increased 240 per cent, the actual pull along the axis is equal to but 38.3 per cent of the actual pull across the short conical

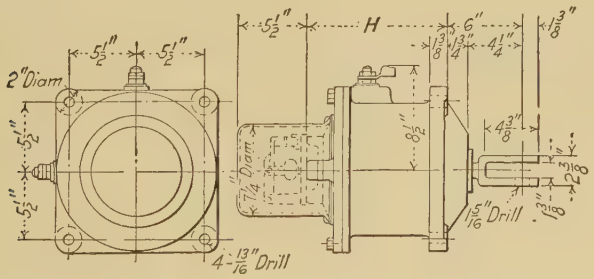


FIG. 269.—Dimensions of C-H iron-clad solenoids.

air gap at the angle of 67.5 deg. from the axis. However, the length of travel of the plunger for a given change in the air-gap length is now 38.3 per cent greater than in the case of the flat-faced plunger and stop.

145. Standard Commercial Solenoids.—The direct-current, iron-clad solenoids in Figs. 267 and 268 are similar, with the exception that the shunt-wound solenoid is equipped with a protecting switch which remains closed during the lifting stroke, but during the last ¼ in. of the plunger travel it is opened by the plunger and a resistor is inserted in series with the coil.

These solenoids are made in four sizes. The general dimensions are shown in Fig. 269, the only variable being dimension *H*. Table V gives the dimensions and weights.

TABLE V.—DIMENSIONS AND WEIGHTS OF SOLENOIDS

Size	Dimension *H*, inches	Weight of plunger, pounds	Shipping weight, pounds
A	6 3/16	18	230
B	9 3/16	26	270
C	12 5/8	33	375
D	15 5/8	38	400

The protecting switch on the shunt-wound solenoids greatly increases the lifting capacity of the solenoid because it permits a greater current density during the lifting period. When the plunger seals against the stop or plug, the current strength is reduced by the resistor to a value which is sufficient to hold the plunger sealed continuously, but which will not cause the coil to overheat.

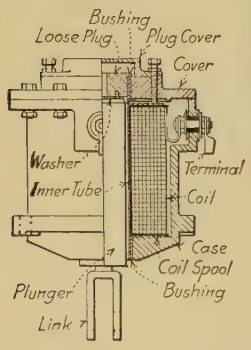


FIG. 270.—Details of C-H iron-clad construction.

The general construction is indicated in Fig. 270. Standard voltages are 115, 220, and 500 volts. The coils are impregnated by the vacuum process, and the space between the coil and the case is filled with the same compound after assembling. The maximum temperature rise of the coil is 75° C. for various duties, as follows: crane duty, current on 1 min., off 2 min.; mill duty, current on 1 min., off 1 min.; continuous duty, current on continuously.

The coil is bobbin-wound. The guide for the plunger is a heavy brass tube which passes through the coil bobbin with a tight fit in both the cover and the case. Inserted in one end of this tube is a loose-fitting floating plug or stop to eliminate the

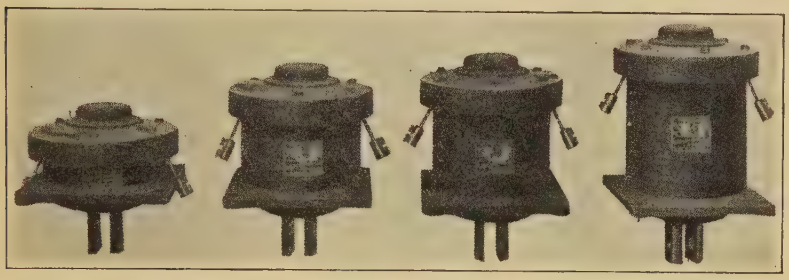


Fig. 271.—Comparative sizes of E.C.&M. solenoids.

hammer blow. The steel plunger has a loose sliding fit in the tube and engages the plug at the end of its working stroke, but it continues to rise for a fraction of an inch until the kinetic energy is spent, when both the plunger and the stop drop back again until the stop resists on its seat. The plunger is hollow at its lower end and is provided with a link or connecting clevis quite similar to that of the piston and connecting rod of a gas engine, which allows for the angular movement of the link. The equalizer bushings prevent excessive side pull.

The operating characteristics are given in Tables VI and VII, wherein the gross pulls in pounds are given. The weights of the plungers must be subtracted from the gross pulls in order to obtain the net pulls.

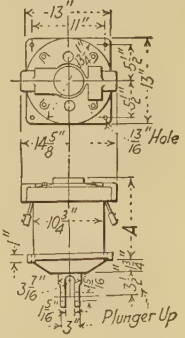


Fig. 272. — Dimensions of E.C.&M. solenoids.

The line of iron-clad solenoids in Fig. 271 are of the magnetic-cushion type. The general dimensions are given in Fig. 272. The force-distance characteristics are shown in Figs. 273 and 274. The dimension *A*, ratings, and weights for the various solenoids are given in Table VIII.

The lifts in Figs. 273 and 274 include the weights of the plungers. Shunt-wound solenoids are provided with cut-out or protecting switches for the insertion of resistance at the end of

TABLE VI.—OPERATING CHARACTERISTICS OF *C-H* SHUNT-WOUND SOLENOIDS

	Stroke in inches	Pounds gross pull				Watts on full-rated voltage		
		Size A	Size B	Size C	Size D	Size	Inrush	Sealed
CONTINUOUS DUTY with protecting switch and protecting resistance, 80 per cent of rated voltage. Not over two lifts per minute	1	275	310	360	400	A	1,250	407
	1½	255	290	345	390	B	1,600	532
	2	190	270	330	380	C	2,600	765
	2½	130	240	315	360	D	3,500	1,140
	3	80	220	290	340			Includes watts in protecting resistance
	4	...	205	275	320			
	5	...	...	260	310			
	6	...	...	...	300			
	7	...	...	...	290			
CONTINUOUS DUTY without protecting switch and resistance, 80 per cent of rated voltage	1	60	90	180	200	A	.....	135
	1½	45	60	125	155	B	.....	185
	2	35	50	95	125	C	.....	230
	2½	25	42	75	105	D	.....	280
	3	...	32	60	85			
	4	...	...	45	65			
	5	...	...	...	55			
MILL DUTY without protecting switch and resistance, 80 per cent of rated voltage	1	100	200	290	300	A	.....	270
	1½	75	145	220	240	B	.....	370
	2	52	102	180	200	C	.....	460
	2½	38	85	150	175	D	.....	560
	3	25	65	125	160			
	4	...	45	95	140			
	5	...	27	75	127			
	6	...	...	55	120			

TABLE VII.—OPERATING CHARACTERISTICS OF *C-II* SERIES-WOUND SOLENOIDS

	Stroke in inches	Per cent, full-load current	Pounds gross pull				Consumption with full-load current	
			Size A	Size B	Size C	Size D	Size	Watts
CRANE DUTY	1	100	190	340	370	400	A	405
		40	60	90	150	170	B	555
	1½	100	135	230	315	325	C	690
		40	45	60	100	130	D	840
	2	100	90	180	280	285		
		40	35	50	75	100		
	2½	100	63	145	250	240		
		40	...	45	60	80		
	3	100	45	110	225	225		
		40	...	35	50	70		
	4	100	...	70	185	210		
		40	...	...	35	50		
	5	100	...	69	53	200		
		40	...	...	...	35		
	6	100	...	32	117	190		
	7	100	...	...	...	180		
MILL DUTY	1	100	135	280	305	345	A	270
		40	50	65	105	135	B	370
	1½	100	100	200	250	275	C	460
		40	40	45	75	90	D	560
	2	100	68	160	215	245		
		40	30	40	55	75		
	2½	100	45	125	190	215		
		40	...	35	45	60		
	3	100	32	100	165	195		
		40	...	...	40	50		
	4	100	...	63	135	175		
		40	...	...	25	40		
	5	100	...	40	105	160		
		40						
	6	100	...	...	81	150		
	7	100	...	...	...	140		

TABLE VIII.—RATINGS, DIMENSIONS, AND WEIGHTS OF SOLENOIDS

Size	Rating, hp.	A, inches	Approximate net weight, pounds	Weight of plunger, pounds
1	2-10	7½	185	31
2	10–30	11⅝	250	43
3	25–75	13	295	45
4	50–150	16	325	52

the stroke. Intermittent service without cut-out is based on the current being on for one-half the time and off for one-half the time, these time intervals not to exceed 5 min. Intermittent service with cut-out implies that the solenoid can remain in circuit indefinitely with resistance inserted, but that the lifts

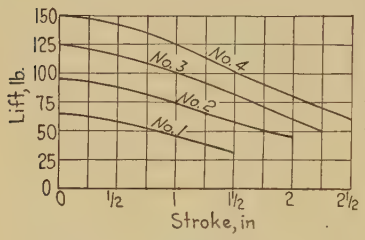


Fig. 273.—Force-distance curves of E.C.&M. series-wound solenoids; crane ratings.

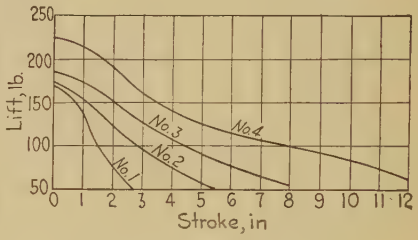


Fig. 274.—Lifts at full load.

should not be more frequent than twice per minute. Periodic service without cut-out implies a definite period of operation of one stroke every 6 min.; the current being on for 1 min.

In one form of direct-current, iron-clad solenoid the casing is ribbed to facilitate cooling.

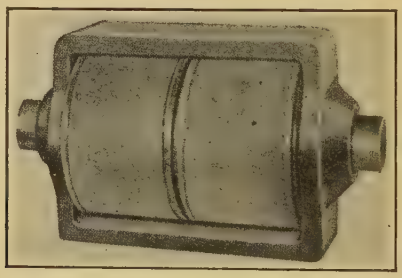


Fig. 275.—C-H short-pull, double-plunger brake solenoid.

146. Short-pull, Double-plunger Brake Solenoids.—Figure 275 shows one type. Such a magnet rated at 1,000 lb. one-quarter of an inch has an air gap between the plungers one-quarter of an inch long and the pull on each plunger is 1,000 lb. through one-eighth of an inch (page 222).

CHAPTER XIV

ELECTRIC CIRCUITS AND THEIR PHENOMENA

Direct-current circuits are principally treated in the present chapter, electric batteries being commonly shown in the illustrations to avoid confusion, although direct-current generators or dynamos may be the sources of electric energy. The effects due to connecting resistors, reactors, and condensors in various combinations are discussed in detail.

147. Classification of Electric Circuits.—Electric circuits may be generally classified as follows:

Direct-current circuit	High-voltage circuit
Alternating-current circuit	Low-voltage circuit
Constant-current circuit	Primary circuit
Constant-voltage circuit	Secondary circuit
Series circuit	Oscillating circuit
Shunt circuit	

Electric circuits are further classified with reference to other phenomena which they possess or contain.

At ordinary temperatures, all electric conductors possess electric resistance. All electric circuits which are linked with magnetic flux produced by electric currents flowing in them possess self-inductance. Whenever electric insulation is strained or otherwise affected by a voltage acting on the electricity in an electric circuit, that circuit is said to possess capacitance.

An electric conductor purposely introduced into an electric circuit to increase the resistance of the circuit is a resistor. A device purposely designed to possess self-inductance is a reactor. A device purposely designed to possess capacitance is an electrostatic condenser.

While all electric circuits possess resistance, self-inductance, and capacitance to some extent, the effects due to certain of these phenomena may be more marked in some cases than in others, so that the effects of self-inductance and capacitance may be neglected in considering electric circuits wherein the

phenomenon of electric resistance predominates, and also when the voltage and the amperage are constant and continuous.

An electromagnet is the equivalent of the combination of a reactor and a resistor connected in series. The capacitance between adjacent turns in a coil may be neglected in most cases where the frequency and the voltage are relatively low.

The resistance of an alternating-current electromagnet may be so small with respect to the inductive reactance (6.2832 $\times$ frequency $\times$ self-inductance) that only the latter need be considered.

Therefore, the cases to be considered in connection with either direct or alternating currents are circuits containing the following.

1. Resistance only.
2. Self-inductance only.
3. Capacitance only.
4. Resistance and self-inductance.
5. Resistance and capacitance.
6. Self-inductance and capacitance.
7. Resistance, self-inductance, and capacitance.

These may be arranged in series, in parallel, or in series-parallel combinations.

148. Reconversions and Transferences of Energy.—Heat is not directly convertible into electric energy in ordinary electric circuits. Therefore, resistance reaction acts in only one direction with respect to the active voltage.

Magnetic energy is reconverted into electric energy whenever the linkage of magnetic flux with electric current is decreasing. Then the self-induced voltage acts in the same direction as the impressed voltage in a direct-current circuit, thus increasing the total voltage of the circuit. This always happens when an attempt is made to open a direct-current circuit containing self-inductance and in which an electric current is flowing.

After an electric condenser is charged, that is, after the dielectric voltage is equally opposed to the charging voltage, it may be disconnected from the charging circuit and removed therefrom in a charged condition, provided the voltage of the charging circuit is applied to the condenser up to the instant when the condenser is disconnected from the circuit. If the charging voltage is reduced while the condenser is still connected in the

circuit, the condenser will discharge at least some of its stored electric energy into the electric circuit. This periodically occurs in alternating-current circuits.

A resistor cannot discharge electric energy. It can only expend the heat developed in it. A reactor can only be discharged into the electric circuit in which it is charged with magnetic energy, or in a branch of that circuit. A condenser may be charged in one electric circuit and discharged in another electric circuit.

149. Components of Power.—The time-rate of conversion of electric energy into heat, in watts, is equal to the product of the active voltage and the total amperage, or to the product of the total voltage and the heating component of the total amperage. The time-rate of conversion of electric energy into magnetic energy, in watts, is equal to the product of the e.m.f. of self-induction and the total amperage, or to the product of the total voltage and the magnetizing component of the total amperage. The time-rate of storage of electric energy in a dielectric, in watts, is equal to the product of the e.m.f. of capacitance and the total amperage, or to the product of the total voltage and the displacement component of the total amperage. These are general statements and must be carefully applied to specific cases. The self-induced voltage and the dielectric voltage respectively balance the e.m.f. of self-induction and the e.m.f. of capacitance, which, in turn, are components of the impressed voltage. Therefore, the former voltages are negative with respect to the impressed voltage and have − signs.

150. Components of the Various Voltages.—The active e.m.f. or voltage in volts is equal to the product of the resistance in ohms and the current strength in amperes. The e.m.f. of self-induction in volts (which balances the self-induced voltage) is equal to the product of the self-inductance in henries and the time-rate of change in the current strength in amperes per second, or to the product of the constant current strength in amperes and the time-rate of change in the self-inductance in henries per second. The e.m.f. of capacitance in volts (which balances the dielectric voltage) is equal to the product of the elastance in darafs and the quantity of electricity in coulombs.

151. Graphic Electric Symbols.—Figure 276 is self-explanatory. The devices may be connected in series, in parallel, or in series-parallel combinations.

152. Series Connections.—When two or more sources of voltage are connected in series, the total voltage is equal to the sum of the individual voltages when polarities and phase differences are taken into consideration.

When two or more resistors are connected in series, the total resistance is equal to the sum of the individual resistances. When two or more reactors are connected in series, the total self-

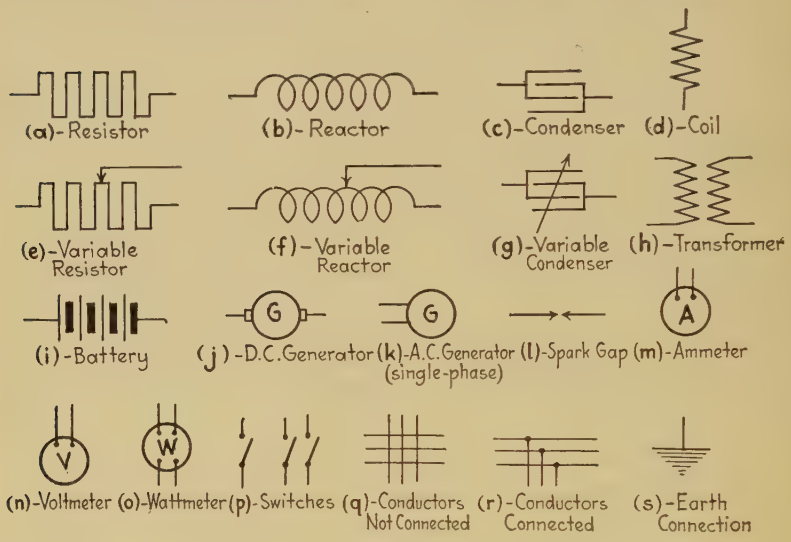


FIG. 276.—Graphic electric symbols.

inductance is equal to the sum of the individual self-inductances. When two or more condensers are connected in series, the total elastance (reciprocal of capacitance) is equal to the sum of the individual elastances. If the capacitances are 1, 2, and 4 mf., respectively, the elastances are 1, 0.5, and 0.25 megadarafs (million darafs) respectively. Hence, the total elastance is equal to 1.75 megadarafs. The joint capacitance is equal to the reciprocal of the total elastance, or to $\frac{1}{1.75} = 0.571$ mf.

153. Direct-current Circuit Containing Only Resistance.—In Fig. 277 a resistor, a switch, and a battery are connected in series. All of the resistance is assumed to be contained in the resistor. Assuming that the self-inductance is negligible and that the voltage is constant, when the switch is closed the current will instantly attain, and will continue to maintain, its maximum

strength as limited by the resistance. The chemical energy of the battery is converted into electric energy and thence into heat.

The amount of electric energy converted into heat in the resistor at a constant time-rate is equal to (1) the product of the active voltage and the quantity of electricity; or (2) the continued product of (*a*) the resistance in ohms, (*b*) the elapsed time in seconds, and (*c*) the square of the amperage, or (3) the product of (*a*) the ratio of the time in seconds to the resistance in ohms and (*b*) the square of the active voltage.

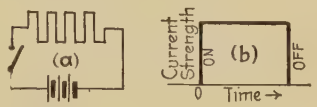


FIG. 277.—Direct-current circuit containing only resistance.

When the switch is opened, there will be very little sparking at the contacts, unless the voltage exceeds about 28 volts, and the current will tend to instantly increase to zero, as indicated in Fig. 277*b*.

154. Direct-current Circuit Containing Only Self-inductance. With a reactor connected in a circuit of negligible resistance, as in Fig. 278, when the switch is closed the current strength will start from zero and increase at a uniform time-rate dependent upon the ratio of the constant impressed voltage to the self-inductance in henries of the reactor. The self-induced voltage will exactly balance the impressed voltage. If the impressed voltage is 8 volts, the self-induced voltage of the reactor will be −8 volts, meaning that electric energy is being abstracted from the electric circuit. Since the self-induced voltage is equal to the product of the self-inductance and the time-rate of change in the current strength, the time-rate of change in the current strength in amperes per second is equal to the ratio of the self-induced voltage to the self-inductance in henries. If the self-inductance of the reactor is 80 millihenries (0.08 henry), the time-rate of increase in the current strength in amperes per second will be equal to $\frac{8 \text{ volts}}{0.08 \text{ henry}} = 100$ amp. per second. Hence, at the instant the switch is closed the current strength will be equal to zero, at the

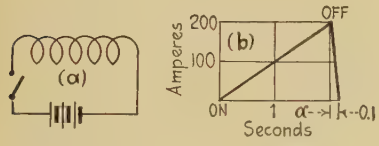


FIG. 278.—Direct-current circuit containing only self-inductance.

end of the first second the current strength will be equal to 100 amp., and at the end of 2 sec. it will be 200 amp.

The amount of magnetic energy stored in the reactor is equal to the product of the constant self-induced voltage and the quantity of electricity moved through any plane in the cross-section of the coiled conductor under the influence of the equal but opposed e.m.f. of self-induction (equal to the impressed voltage in this case). The self-induced voltage is equal to the product of the constant self-inductance and the time-rate of change in the current strength. The quantity of electricity is equal to the product of the average current strength and the time required to store the magnetic energy. Therefore, the magnitude of the magnetic energy stored in the reactor is equal to the continued product of (1) the self-inductance in henries, (2) the ratio $\frac{\text{current strength}}{\text{time}}$ and (3) 0.5 $\times$ current strength $\times$ time. Hence, the time cancels out and the magnitude of the magnetic energy stored in the reactor is equal to one-half the product of the self-inductance in henries and the square of the current strength in amperes.

When the current strength is 100 amp., the amount of stored magnetic energy is equal to $0.5 \times 0.08 \times 10{,}000 = 400$ joules. When the current strength is 200 amp., the number of joules is equal to $0.5 \times 0.08 \times 40{,}000 = 1{,}600$, the amount of magnetic energy stored with constant voltage and self-inductance varying directly as the square of the elapsed time. Since 1 joule = 0.7376 ft.-lb., 1,600 joules = 1,180 ft.-lb. Therefore, when 8 volts are impressed upon a reactor having a self-inductance of 0.08 henry for a period of 2 sec., the energy stored therein is sufficient to lift 1,180 lb. vertically against the force of gravity for a distance of 1 ft. if it could thus be utilized. In order actually to expend this amount of mechanical energy in 2 sec. while storing an equal amount of magnetic energy, the square of the current strength must be doubled, which means that the ratio of the self-induced voltage to the self-inductance in henries must be increased 1.414 (the square root of 2) times. Thus, the self-induced voltage could be increased to $8 \times 1.414 = 11.31$ volts, or the self-inductance might be reduced to $\frac{0.08}{1.414} = 0.0566$ henry.

If, at the instant the current strength becomes 200 amp., the switch could be opened in such a manner (as by uniformly intro-

ducing resistance into the circuit) that the current strength would be uniformly reduced from 200 to 0 amp. in 0.1 sec., as in Fig. 278*b*, the time-rate of decrease in the current strength during the 0.1 sec. would be equal to $\frac{200}{0.1} = 2{,}000$ amp. per second, and the self-induced voltage would be equal to 0.08 henry $\times$ 2,000 amp. per second = 160 volts. Since the self-induced voltage would then act in the same direction as the impressed voltage, the total voltage during the 0.1 sec. would be 168 volts.

The quicker the electric circuit is opened the greater is the time-rate of conversion of magnetic energy into electric energy, that is, the greater is the power and the greater is the self-induced voltage. The current strength can only fall from a given value to zero, but the self-induced voltage is proportional to the rapidity with which the circuit is opened. Hence, switches designed to open such circuits very quickly must open widely in order that the arc shall not persist for too long a period of time; the switch may be provided with a magnetic blow-out or be opened under oil, or else means must be provided to absorb the exhaust of the stored magnetic energy.

155. Direct-current Circuit Containing Only Capacitance.—Assuming that there is no resistance or self-inductance in the circuit in Fig. 279, when the switch is closed the condenser will become instantly charged. The presence of a very little resistance will be assumed in order to introduce the time element. Then the current will instantly attain its maximum strength, equal to the ratio of the impressed voltage to the resistance, and will fall to zero as the dielectric voltage balances the impressed voltage (to be exact—the e.m.f. of capacitance).

FIG. 279.—Direct-current circuit containing only capacitance.

If the impressed voltage is 8 volts, the dielectric voltage will increase from 0 to 8 volts. Hence, the average dielectric voltage will be 4 volts, and this also will be the average e.m.f. of capacitance. If the capacitance is 2 mf., the quantity of electricity displaced in charging the condenser will be equal to the product of 8 volts and 0.000 002 farad, or to 0.000 016 coulomb, that is, 0.000 016 coulomb of electricity will pass through every plane in the cross-section of every conductor in series during the charg-

ing process. If the impressed voltage is 80 volts, then 0.000 16 coulomb will be displaced in the charging process. If the condenser is charged in 0.000 001 sec. with a constant impressed voltage of 80 volts, the average current strength will be 160 amp. If the impressed voltage is 8 volts, the average current strength will be 16 amp. However, while a condenser may become approximately charged in so short a time interval, theory indicates that when resistance is in series with capacitance an infinite time is required to charge the condenser entirely.

The electric energy stored in the dielectric of the above condenser at a pressure of 8 volts is equal to the product of the average dielectric voltage and the quantity of electricity, or to 4 volts $\times$ 0.000 016 coulomb = 0.000 064 joule. If the voltage is 80 volts, the electric energy that may be stored in a 2-mf. condenser is equal to 40 volts $\times$ 0.000 16 coulomb = 0.0064 joule, the number of joules per farad being equal to one-half the square of the impressed voltage. In the above example, one-half the square of 80 is 3,200, and this is equal to $\frac{0.0064 \text{ joule}}{0.000\,002 \text{ farad}} = 3{,}200.$

Assuming that a paper condenser can safely withstand a direct voltage of 500 volts, the capacitance required for the storage of 1 joule of electric energy therein would be equal to 1 joule $\times$ $\frac{2}{(500)^2} = \frac{2}{250{,}000} = 0.000\,008$ farad, or 8 mf. If the impressed voltage is 100, the condenser must be 25 times as large, that is, its capacitance must be equal to 25 $\times$ 8 = 400 mf.

There will be no sparking at the switch contacts when the switch is opened after the condenser is charged, since the dielectric voltage will then exactly balance the impressed voltage. However, if the switch is opened before the condenser is fully charged, sparking may occur because there will then be a difference between the impressed and dielectric voltages, and a current will be flowing.

156. Direct-current Circuit Containing Resistance and Self-inductance in Series.—This combination simulates the coil of an electromagnet. Although the resistance is in the turns of the conductor and the self-inductance is proportional to the number of turns of conductor, the effect is the same as though all of the resistance was in one unit and the self-inductance was contained in another unit.

When the switch in Fig. 280 is closed, the current will at first tend to increase in strength at a constant time-rate equal to the ratio of the impressed voltage to the self-inductance in henries in the reactor, as represented by the dotted line *a*. As soon as an electric current flows, however, the reaction from the resistor

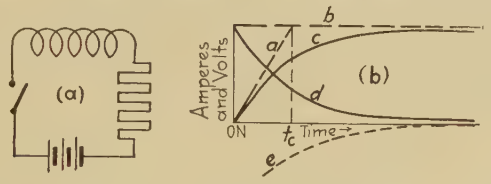


FIG. 280.—Direct-current circuit containing resistance and self-inductance in series.

will cause an active-voltage drop across the resistor, equal to the product of the amperage and the resistance in ohms of the resistor, and this voltage increases directly with the current strength.

The difference between the impressed voltage and the active voltage is the e.m.f. of self-induction, which balances this self-induced voltage of the reactor. Consequently, the time-rate of increase in the current strength is reduced as the current strength increases. Eventually, the current strength will become a maximum and then will remain constant, as limited by the ratio of the impressed voltage to the resistance in ohms of the resistor. The reactor then has no influence on the voltage or the current strength, and there will be no self-induced voltage. The active voltage will then be equal to the impressed voltage.

At the instant the switch is closed, the current strength tends to increase at a time-rate represented by the dotted line *a*. Eventually, however, the current strength is limited by the resistance, so that it cannot exceed the value represented by the dotted line *b*. Hence, the current strength increases according to the law represented by curve *c*. This curve also generally represents the active-voltage-time relations, since the active voltage is equal to the product of the current strength and the constant resistance. The dotted line *b* also represents the constant impressed voltage. Therefore, since the e.m.f. of self-induction is equal to the difference between the impressed voltage and the active voltage when the current strength is increasing, the e.m.f. of self-induction is represented by curve *d*. The self-induced voltage for the increasing current strength has the same

form as curve d but is exactly opposed to it, as shown by dotted curve e.

Line b represents the current-time relations which would exist if there were only resistance in the circuit. Line a represents the current-time relations that would result if there were only self-inductance in the circuit. t_c is the time which would be required for the current to attain its maximum strength, as represented by the dotted line b, if there was only self-inductance in the circuit.

The time t_c is the time constant, which is the time required for the current to attain 63.2 per cent of its maximum strength, while increasing, with constant resistance and constant self-inductance in series in a constant-voltage, direct-current circuit. It is equal to the ratio of the self-inductance in henries to the resistance in ohms (see also page 83). Hence, the time constant is the time in seconds for any one of the quantities following this general law to increase from 0 to 63.2 per cent of its maximum magnitude, or to decrease from a maximum to $100 - 63.2 = 36.8$ per cent of its maximum magnitude.

The time required for these quantities to attain 99.9 per cent of their maximum magnitudes while increasing, or 0.009 77 per cent of their maximum magnitudes while decreasing, is equal to 6.9315 times the time constant.

The time constant is equal to 1.4427 times the time required for any one of these quantities to increase or to decrease to one-half of its maximum magnitude.

The time constant of a circuit having a self-inductance of 0.02 henry and a resistance of 5 ohms is equal to $\frac{0.02}{5} = 0.004$ sec., that is, with a constant impressed voltage the current would increase from 0 to 63.2 per cent of its maximum strength, as limited by the resistance, in 0.004 sec., which is 1.4427 times the time required for the current to attain one-half of its maximum strength. The active voltage would also increase from 0 to 63.2 per cent of the value of the impressed voltage in 0.004 sec., and the e.m.f. of self-induction would decrease from a value equal to the impressed voltage to $100 - 63.2 = 36.8$ per cent of the value of the constant impressed voltage in 0.004 sec. The time required for the current strength and the active voltage to attain 99.9 per cent of their maximum values, or for the e.m.f. of self-induction to decrease from the value of the constant impressed

voltage to 0.009 77 per cent of that value, is equal to 0.004 × 6.9315 = 0.027 73 sec.

When the switch is opened, the self-induced voltage reenforces the impressed voltage. However, some of the electric energy reconverted from the stored magnetic energy in the reactor is converted into heat in the resistor when the switch is opened.

157. Direct-current Circuit Containing Resistance and Capacitance in Series.—Referring to Fig. 281, at the instant the switch is closed the current attains a strength in amperes equal to the

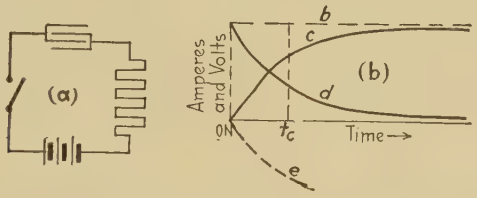


FIG. 281.—Direct-current circuit containing resistance and capacitance in series.

ratio of the impressed voltage to the resistance in ohms, since the electricity in the dielectric has not yet been displaced and the dielectric voltage and the e.m.f. of capacitance are equal to zero The active voltage is then equal to the impressed voltage. However, the electricity in the circuit is displaced as time elapses, the resistance limiting the time-rate of flow, and the dielectric is increasingly strained, the current strength in amperes being equal to the product of the capacitance in farads and the time-rate of change in the dielectric voltage in volts per second.

The impressed voltage is equal to the sum of the e.m.f. of capacitance (which balances the dielectric voltage) and the active voltage (equal to the product of the resistance in ohms and the amperage). Hence, as the current strength decreases (as the dielectric voltage increases) the active voltage decreases and the e.m.f. of capacitance increases. Eventually, the current strength becomes equal to 0, as the difference between the impressed voltage and the e.m.f. of capacitance becomes equal to 0, when the active voltage becomes equal to 0 and the e.m.f. of capacitance becomes equal to and the same as the impressed voltage. The dotted line *b* in Fig. 281 represents the maximum current strength and the constant impressed voltage. The curves in Fig. 281 are similar to those in Fig. 280, except that curve *d* now represents the current strength and the active

voltage, while curve c represents the e.m.f. of capacitance and curve e represents the dielectric voltage, all with respect to time.

In the case of a resistor in series with a condenser, the time-constant is equal to the product of the capacitance in farads and the resistance in ohms, or to the ratio of the resistance in ohms to the elastance in darafs. In a circuit having a capacitance of 10 mf., or 0.000 01 farad, and a resistance of 100 ohms, the time constant will be equal to 0.001 sec. Then the current strength and the active voltage will decrease from 100 per cent to $100 - 63.2 = 36.8$ per cent of their respective maximum values in 0.001 sec., or to one-half of their respective maximum values in $\frac{0.001}{1.4427} = 0.000\ 693$ sec., while the e.m.f. of capacitance will have increased from 0 to 63.2 per cent of the value of the impressed voltage in 0.001 sec., or to one-half of the constant impressed voltage in 0.000 693 sec. Likewise, the current strength and the active voltage will have decreased to 0.009 77 per cent of their maximum values, and the e.m.f. of capacitance will have increased to 99.9 per cent of the value of the constant impressed voltage, in $0.001 \times 6.93 = 0.006\ 93$ sec.

158. Parallel Connections.—It is to be noted that voltage acts like a pressure. Hence, the same volt will cause a current of any desired strength to flow in a circuit which is properly proportioned. Connecting sources of equal voltage in parallel does not alter the magnitude of the voltage. Parallel connections cause an increase in the current strength for a given constant voltage.

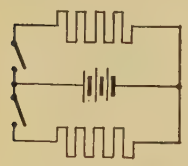


Fig. 282. — Resistors in parallel.

Referring to Fig. 282, let the resistance of the two resistors be equal. Then, when both switches are closed, the total current strength will be double that which results when only one switch is closed. If there were no internal resistance in the battery, and no resistance in the connecting wires or in the switch contacts, the number of resistors in parallel might be increased indefinitely without altering the current strength in each resistor, provided the voltage of the battery remained constant. Then the total current strength would be equal to the product of the current strength in each resistor and the number of resistors, that is, to the sum of the individual current strengths.

The total conductance of any number of resistors connected in parallel is equal to the sum of the individual conductances. Conductance is the reciprocal of resistance. Hence, the joint resistance of any number of conductors connected in parallel is equal to the reciprocal of the sum of the individual conductances.

When any number of reactors are connected in parallel, as in Fig. 283, the voltage across each reactor will be equal to the impressed voltage, but the time-rates of increase in current strength will be different if the self-inductances of the individual reactors are different. However, in each reactor, the time-rate of increase in current strength will be equal to the ratio of the e.m.f. of self-induction (the impressed voltage in this case) to the self-inductance.

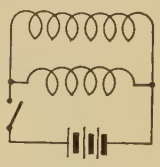


FIG. 283.—Reactors in parallel.

The joint self-inductance of several resistanceless reactors connected in parallel is found by taking the reciprocals of the self-inductances of each reactor, adding these together, and then taking the reciprocal of the sum.

In Fig. 283, let the self-inductance of one of the reactors be twice that of the other reactor. Then the time-rate of increase in the current strength in the reactor having the lesser self-inductance will be double that in the reactor having the greater self-inductance, and the amount of magnetic energy stored in the former at the end of a given time interval will be double that stored in the latter, the product of the stored magnetic energy in joules and the self-inductance always being a constant quantity when the voltage and the time interval are constant.

If the switch is now opened, the reactor possessing the lesser self-inductance will discharge its stored magnetic energy at double the time-rate of the reactor having the greater self-inductance, but the voltage across the reactors will be the same, since both reactors are assumed to be discharged during the same time interval. The above holds true for any number of reactors connected in parallel, provided there is no active voltage developed in them, the product of the stored magnetic energy in joules and the self-inductance in henries being equal to (1) the square of the product of (*a*) the constant self-induced voltage and (*b*) the time interval in seconds, or (2) one-half the square of the number of weber-turns.

The total capacitance of a group of condensers connected in parallel, as in Fig. 284, is equal to the sum of the individual capacitances. Any number of condensers thus connected act as a unit.

159. Direct-current Circuit Containing Resistance and Self-inductance in Parallel.—When the switch in Fig. 285 is closed, the current strength in the resistor will instantly attain a maxi-

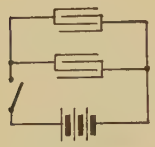


Fig. 284.—Condensers in parallel.

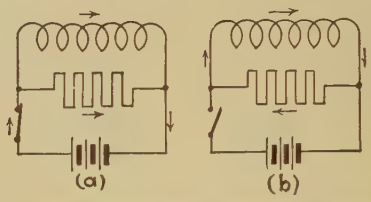


Fig. 285.—Resistor and reactor in parallel.

mum value, but the current strength in the reactor will increase from 0 as time elapses, the directions of the respective currents being as shown in Fig. 285*a*. Figure 285*b* shows the direction of the current after the switch has been opened. Since there is then no reaction from the resistor, the reactor will tend to discharge most of its energy through the path of least resistance. Although some electricity will tend to move across the switch contacts, most of this energy will be discharged into the resistor and there converted into heat. This method sometimes is employed to reduce sparking at switch contacts when circuits containing electromagnets are opened, and also to cause the electromagnet to hold its load for a brief time interval after the switch is opened and while the current is dying away.

Referring to Fig. 285*a*, when the switch is closed, the respective currents flow in the resistor and the reactor in the same general directions and at the same general time-rates as respectively depicted in Figs. 277 and 278. When the switch is opened, as in Fig. 285*b*, the direction of the current in the reactor is the same as formerly, but the direction of the current in the resistor is reversed. The current dies away as depicted in curve *d*, Fig. 281. Figure 285*b* is the equivalent of a short-circuited coil after a voltage has been induced therein and then discontinued.

160. Direct-current Circuit Containing Resistance and Capacitance in Parallel.—When the switch in Fig. 286 is closed, the condenser will become instantly charged whether or not

the resistor is present. However, when the switch is opened, the condenser will immediately begin to discharge into the resistor, thus converting its store of electric energy into heat in the resistor. There will be no sparking at the switch contacts after the condenser is charged. The current will die away as in curve *d*, Fig. 281.

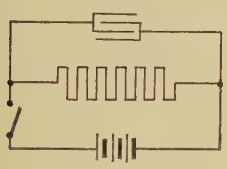


Fig. 286.—Resistor and condenser in parallel.

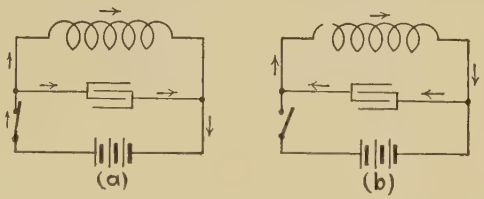


Fig. 287.—Reactor and condenser in parallel.

161. Direct-current Circuit Containing Self-inductance and Capacitance in Parallel.—The reactor and condenser in Fig. 287 are in parallel when the switch is closed but are in series when the switch is opened. When the switch is first closed, the condenser will become instantly charged and magnetic energy will be stored in the reactor at a constant time-rate. During the instant while the condenser is charging (the term *instantly* means that there is no purposely delayed action), the general directions of the respective currents are as shown in Fig. 287*a*. When the switch is opened, the self-inductance of the reactor will tend to keep the current flowing in the reactor in the same direction as when the switch was closed, but the dielectric of the condenser will tend to move the electricity in the condenser and its connections in a direction opposite to that in which it flowed when the switch was first closed. The result is an oscillating current.

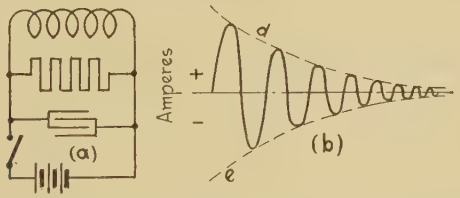


Fig. 288.—Parallel conditions for damped oscillations.

162. Direct-current Circuit with Resistance, Self-inductance, and Capacitance in Parallel.—When the switch in Fig. 288 is closed, the conditions are the same as in Fig. 287, except that a

current of constant strength will flow in the resistor. When the switch is opened, the conditions will be the same as in the case of 287*b*, except that a portion of the oscillating current will flow in the resistor, thereby soon converting the energy of the oscillating system into heat, as depicted in Fig. 288*b*, wherein curves *d* and *e* are of the same forms as those in Fig. 280.

163. Direct-current Circuit Containing Self-inductance and Capacitance in Series.—When the switch is closed, the dielectric voltage is equal to zero and the entire impressed voltage is exerted in accelerating the electricity. As the current strength increases, pressure is exerted upon the dielectric until the self-induced voltage becomes equal to zero and the e.m.f. of capacitance (which balances the dielectric voltage) becomes equal to the impressed voltage. But the kinetic energy of the moving electricity is not yet expended and the momentum of the electricity causes it to continue in motion until the stored magnetic energy is converted into potential electric energy and stored in the dielectric, thus increasing the dielectric voltage to a value numerically greater than that of the impressed voltage. When the motion of the electricity has ceased and the current strength is equal to zero, the electricity is hurled back in the opposite direction, due to the extra dielectric voltage (the difference between the total dielectric voltage and the impressed voltage) and an oscillating current results, which, however, is soon damped out if resistance (either in series or in parallel) is present, as in Fig. 288*b*. Any change in the voltage will again be productive of oscillations.

When the switch is opened with the system in a state of equilibrium, there will be no current flowing and no sparking will occur at the switch contacts. The reactor and the condenser may then be removed from the system and the combination may be used to establish an oscillating current in another circuit.

164. Series-parallel Connections.—Any of the resistors, reactors, or condensers shown in any of the illustrations may be considered as a group of units connected either in series, in parallel, or partly in series and partly in parallel. As an example, assume a group of parallel resistors connected in series with another group of series resistors. In such a case, simply add the joint resistance of the parallel resistors to the total resistance of the series resistors. The sum will be the total resistance of the series-parallel combination, and may be represented by a single resistor.

165. Condenser in Shunt with Switch.—The arrangement in Fig. 289 is designed to cause a high time-rate of decrease in current strength in a reactor or in a circuit containing self-inductance. When the switch is closed, magnetic energy is stored in the reactor at the usual time-rate. When the switch is opened, the self-induced voltage of the reactor causes the condenser to be charged at a high time-rate, since there is no time delay in a condenser. The electricity in the circuit then oscillates in the manner previously described, until its motion is damped out due to resistance, except that the total voltage is equal to the sum of the self-induced voltage and the battery voltage during one alternation, and is equal to the difference between the two voltages during the succeeding alternation.

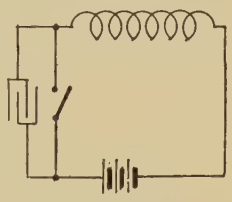


FIG. 289.—Condenser in shunt with switch.

166. Resistor in Shunt with Condenser and Switch.—If the switch in Fig. 289 is again closed while the condenser is charged, the condenser will be suddenly discharged through the switch contacts and the sudden conversion of electric energy into heat in the switch contacts may produce a high temperature therein, thus tending to weld the contacts together and cause "freezing" or sticking of the contacts. This is very troublesome unless the switch is rugged and is positively operated.

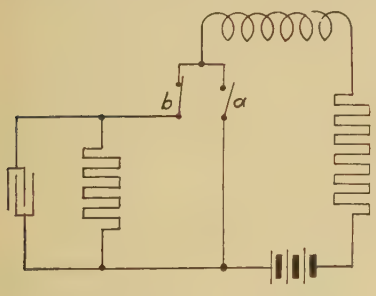


FIG. 290.—Method of discharging condenser between operations.

One method of overcoming the difficulty is to employ the arrangement in Fig. 290. A resistor shunted across a condenser is sometimes called a leakage condenser, or simply a leak. The combination is the equivalent of a "leaky" condenser, or one in which the dielectric does not possess the usual high ohmic resistance. When the switch *a* alone is operated, the results are similar to those in Fig. 289, except that some current will flow in the resistor, shunting the condenser when switch *a* is open. However, when switch *b* also is opened, shortly after switch *a* is opened, the condenser will have performed its function and will then discharge its energy into the resistor.

167. Resistor in Series with Condenser Across Switch.—In ignition and similar induction coils the resistance of the plates of the condenser and the connecting wires is not sufficient to interfere seriously with the function of the condenser, but it limits the maximum current strength during both the charging and discharging of the condenser. In speed regulators for small alternating-current series motors of the universal type, it is preferable to connect a resistor in series with the condenser. In such a case, the capacitance of the condenser is relatively very small as compared with that for a direct-current case, because a condenser is a conductor for alternating currents. If the capacitance is made too great, regulation will not be accomplished.

168. Potentiometer.—Referring to Fig. 291, the total voltage across the variable resistor is equal to the voltage of the source of electric energy. However, the active-voltage drop, as measured between any two points along the resistor, is equal to the product of the current strength in amperes and the resistance in ohms between said points. The smaller resistor and the reactor may represent the coil of an electromagnet. By this arrangement, any desired voltage up to that of the source of electric energy may be impressed upon the circuit in shunt with it.

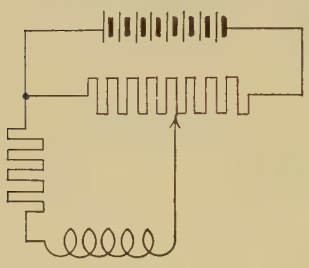


FIG. 291.—Potentiometer.

169. Coupled Circuits.—Two or more electric circuits are said to be coupled when at least some of their parts are mutually linked by magnetic flux produced by an electric current in at least one of the circuits. Electric circuits may be electrostatically coupled and also coupled through resistance, but only electromagnetic couplings are treated in this volume. Figure 292 illustrates the general principle, wherein P is the primary circuit, M is the mutual magnetic circuit, and S is the secondary circuit. The arrows indicate the relative directions of the primary current, the magnetic flux, and the secondary current while the primary-current strength and the magnitude of the magnetic flux are changing.

FIG. 292.—Inductively coupled circuits.

Devices for electromagnetically coupling two or more circuits are called static transformers. An induction coil is one type of transformer. Figure 276*h* symbolically represents a transformer in which the coils may be wound concentrically or else end to end. The ratio of the primary voltage to the secondary voltage is proportional to the number of turns in the primary and secondary, respectively. A step-up transformer has more turns in the secondary than in the primary. A step-down transformer has more turns in the primary than in the secondary.

Figure 293 shows the connections of one form of induction coil. The primary is wound over a bundle of soft irons, which forms the core. The secondary consists of thousands of turns of fine wire. When the vibrator is attracted towards the core as the primary-current strength increases as time elapses after the primary circuit is closed, the primary circuit is suddenly opened with a condenser across the contacts, with the result that a voltage of, say, 10,000 volts is momentarily induced in the secondary. This voltage is sufficient to rupture the gas in a spark gap in the secondary circuit, with the result that the electricity oscillates to and fro across the spark gap until the energy is sufficiently damped out by resistance to reduce the voltage to such a value that it cannot force the electricity across the spark gap. The presence of the condenser makes both the primary and secondary currents oscillatory, and both die away logarithmically, but the secondary current is interrupted by the resistance of the spark gap before the primary current is completely damped out.

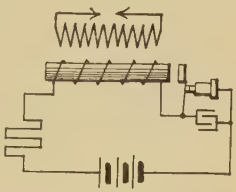


FIG. 293.—Induction coil.

All secondary currents are alternating in nature because the primary current first grows from 0 to a definite strength and then falls to 0 again. The time-rate of change in a growing positive quantity, as a current, or in a decreasing negative quantity, as a current, is positive. The time-rate of change in a decreasing positive quantity, or in an increasing negative quantity, is negative. Since the magnetic flux linking both the primary and the secondary circuits changes with the primary current, and since the induced voltage is proportional to the time-rate of change in the magnetic flux, the direction of the secondary voltage and current while the primary current is

decreasing is opposite to that in which they act when the primary current is increasing. Transformers are connected in alternating-current circuits.

Figure 294 represents an autotransformer with the equivalent of a coil connected thereto, wherein a tap is brought out from some portion of the single coil of the autotransformer so that one part of the coil shall act as a primary (P) and the other portion of the same coil shall act as a secondary (S).

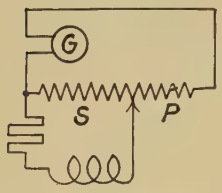


FIG. 294.—Autotransformer.

170. Effects of Short-circuited Turns.—Any short-circuited turn or group of turns becomes a secondary. Any short-circuited turn, the core of an electromagnet, a metal tube placed over the core of an electromagnet or over a coil, or any metal or other conductor linked with varying magnetic flux has secondary currents induced therein which heat the conducting materials in which they flow, thus causing energy losses, and they also tend to prevent the production of magnetic flux by the primary or magnetizing current.

Whenever secondary currents occur, mechanical forces are in evidence which tend to produce explosive effects, as in the short-circuited turns of power transformers. The effect of a short-circuited secondary is to take the inertia out of the electricity, so to speak, so that the current strength increases more rapidly than the magnitude of the magnetic flux. Therefore, the current strength quickly rises to abnormal values with resulting abnormal m.m.fs. and the tendency to produce an abnormal magnitude of magnetic flux. But these m.m.fs. tend to neutralize one another. Hence, if the primary and secondary coils could be separated, the abnormal m.m.fs. could produce abnormal amounts of magnetic flux.

171. Electromagnetic Induction and the Induced Voltage.—A simple method of inducing a voltage in a turn of electric conductor, or in a coil, is to thrust one end of a bar permanent magnet into the turn, or into the core hole of a coil. The direction of the resulting induced voltage is such that if an electric current flows under the action of the induced voltage it will tend to demagnetize the permanent magnet, just as the secondary current tends to demagnetize the core of a transformer or induction coil. This is but one instance of the general law (Lenz's)

that whenever there is a change in the linkage of electric current and magnetic flux, from whatever cause, the direction of the induced voltage is such as to resist the change. This is perfectly natural since magnetic energy is converted into electric energy, and *vice versa*, whenever there is a change in the linkage of electric current and magnetic flux.

The effect of thrusting one end of a bar permanent magnet into the core hole of a coil is to tend to cause the electricity in the turns of the coil conductor to circulate around the core hole in the turns and, therefore, around the permanent magnet. However, no appreciable force is required to move the magnetized steel into the core hole unless the two ends of the wire of the coil are electrically connected together. Then the energy expended in forcing the steel into the core hole of the coil is imparted to the electricity in the electric conductor constituting the coil. The force exerted in pushing the magnetized steel into the core hole of the coil is transmitted to the electricity to cause it to flow in the turns of the coil. A part of this force also is transmitted to the electricity bound in the dielectrics present, but that electricity cannot be moved very far, unless the voltage is sufficient to rupture the dielectric.

While the magnetized steel is being withdrawn, the motion of the electricity is first arrested, if it has not already been stopped by the frictional effects of electric resistance, and it is then forced in an opposite direction in the turns of conductor. The more rapidly a permanent magnet, a solenoid, or an electrically energized bar electromagnet is thrust into or withdrawn from the core hole of a short-circuited coil or a hollow cylinder of solid electric conductor, the greater is the mechanical force required.

It is a remarkable fact that the induced voltage is independent of the material in which it is produced. The material may be any conductor or dielectric. The general law for all materials is that the induced voltage is equal to the product of the length of the material and the time-rate of change in the number of units of magnetic flux linked with each unit of length of said material. Matter is made of electricity, and the reactions between magnetic flux and the electricity tend either to set the electricity in motion or to arrest its motion. The induced voltage is the same for all materials of a given length and for a given time-rate of change in the linkage of magnetic flux with the electricity contained within

them, but is independent of the shapes or cross-sections of said materials. Voltage is a length phenomenon independent of the physical state, so far as the self-induced voltage is concerned. Current strength is a cross-section phenomenon dependent upon the material and its physical state, so far as the active voltage is concerned.

CHAPTER XV

CONSTANT IMPRESSED DIRECT-VOLTAGE CURRENT-TIME CHARACTERISTICS

The logarithmic current-time and voltage-time characteristics of direct-current phenomena are herein described in detail, tables having been calculated for the easy solution of problems usually requiring rather complicated mathematical treatment. A full explanation of the method is given, and a number of examples are worked out.

172. Logarithmic Curves.—All of the curves in Figs. 280 and 281, except the straight lines, are logarithmic curves. These are sometimes referred to as exponential curves. In the equation $100 = 10^2$, 2 is the exponent of 10 but it is also the logarithm of

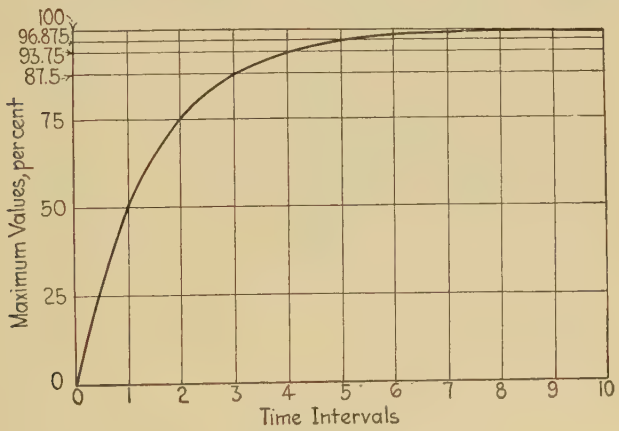


FIG. 295.—Ascending logarithmic curve.

100 to the base 10. The sum of the values of the ascending and descending voltage curves is a constant quantity, equal to the constant impressed voltage in each case. Hence, when the maximum value of any quantity that varies logarithmically as time elapses is taken as 1, the value of the quantity at any instant may be represented as a decimal fraction or as a percentage of its maximum value. All ascending curves will then coincide, and

all descending curves will also coincide. Consequently, there are only two curves to be considered, and each one when subtracted from 1, or 100 per cent, produces the other.

The ascending logarithmic curve is based upon the principle that, if a quantity attains one-half its maximum magnitude during a given time interval under uniform conditions, it will gain one-quarter more of its maximum magnitude during a second succeeding time interval equal to the first, one-eighth more during the third succeeding time interval equal to each of the two preceding time intervals, and so on as illustrated in Fig. 295, the quantity always increasing during each succeeding time interval by one-half the increase during the preceding time interval.

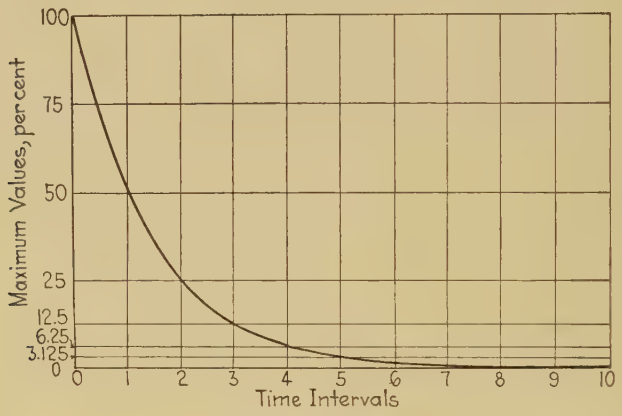


FIG. 296.—Descending logarithmic curve.

The descending logarithmic curve, also known as the die-away curve, is illustrated in Fig. 296 in terms of percentages of its maximum value.

Figure 295 illustrates, in percentages of maximum values, how the current strength and the active voltage increase as time elapses when a constant, continuous voltage is impressed upon a reactor and a resistor in series, or upon an electromagnetic coil; or how the e.m.f. of capacitance (equally opposed to the dielectric voltage) increases as time elapses when a constant, continuous voltage is impressed upon a condenser and a resistor in series.

Figure 296 shows how the current strength and the active voltage decreases as time elapses when a constant, continuous voltage is impressed upon a condenser and a resistor in series, or

when a condenser is discharging into a resistor; and how the e.m.f. of self-induction (equally opposed to the self-induced voltage) decreases as time elapses when a constant, continuous voltage is impressed upon a reactor and a resistor in series. Figure 296 further illustrates how the current strength and the active voltage decrease as time elapses in a short-circuited coil containing both self-inductance and resistance, after a voltage has been induced therein and then discontinued so that the electricity shall at first be set in motion and then allowed to flow on its momentum until its motion is arrested, due to the conversion of the stored magnetic energy into electric energy and thence into heat.

Theoretically, no varying quantity represented by a logarithmic curve ever attains its maximum magnitude when increasing, or its minimum magnitude (zero) when decreasing. However, a quantity may increase to 99.9 per cent of its maximum magnitude, or it may decrease to 0.0977 per cent of its maximum magnitude, at the end of the tenth consecutive time interval, that is, after ten times the time interval required for the phenomenon to increase from 0 to one-half its maximum magnitude, or to decrease from its maximum magnitude to one-half its maximum magnitude.

173. Relation of the Time Constant to the Logarithmic Values. The maximum current strength is equal to the ratio of the impressed voltage to the resistance. Hence, if the impressed voltage is 12 volts and the resistance is 10 ohms, the maximum current strength will be 1.2 amp. This holds for the case of a reactor in series with a resistor and also for the case of a condenser in series with a resistor.

If the self-inductance of a circuit is 0.01 henry and its resistance is 10 ohms, the time constant will be equal to $\frac{0.01}{10} = 0.001$ sec. Then, when a constant, continuous voltage is impressed upon the circuit, the current strength and the active voltage will increase from 0 to 63.2 per cent of their respective maximum magnitudes in 0.001 sec., or to one-half their respective maximum magnitudes in $\frac{0.001}{1.4427} = 0.000\ 693\ 15$ sec. At the end of $0.000\ 693\ 15 \times 2 = 0.001\ 386\ 3$ sec., they will have attained 75 per cent of their respective maximum values; at the end of $0.000\ 693\ 15 \times 3 = 0.002\ 079\ 45$ sec., they will have attained 87.5 per cent of their maximum values; and so on, according to

Table IX, wherein $T = 6.9315$ times the time constant or 10 times the time required for a quantity to increase or decrease to one-half its maximum magnitude, and t is any other time interval

TABLE IX.—INCREASING LOGARITHMIC VALUES

$\frac{t}{T}$	$\frac{\text{Instaneous value}}{\text{Maximum value}}$	Fraction
0.0	0	0
0.1	0.5	$\frac{1}{2}$
0.2	0.75	$\frac{3}{4}$
0.3	0.875	$\frac{7}{8}$
0.4	0.937 5	$\frac{15}{16}$
0.5	0.968 75	$\frac{31}{32}$
0.6	0.984 375	$\frac{63}{64}$
0.7	0.992 187 5	$\frac{127}{128}$
0.8	0.996 093 75	$\frac{255}{256}$
0.9	0.998 046 875	$\frac{511}{512}$
1.0	0.999 023 437 5	$\frac{1,023}{1,024}$

In the practical application of the principle, when the time, as measured from the beginning of a phenomenon, is equal to $T = 6.9315$ times the time constant, the value of the phenomenon may be assumed as constant.

In a circuit having a capacitance of 0.000 01 farad in series with a resistance of 100 ohms, the time constant will be equal to 0.001 sec. and the current strength and the active voltage will decrease from 100 to 36.8 per cent of their respective maximum values in 0.001 sec., or to one-half the respective values in $\frac{0.001}{1.4427} = 0.000\ 693\ 15$ sec. At the end of $0.000\ 693\ 15 \times 2 = 0.001\ 386\ 3$ sec. they will have decreased to $100 - 75 = 25$ per cent of their respective maximum values, and so on, according to Table X, wherein $T = 6.9315$ times the time constant and t is any other time interval, as in the previous case.

Table X clearly shows that the decrease in the magnitude of a phenomenon during any constant time interval is equal to one-half the decrease during the preceding equal time interval, as measured from the beginning of the phenomenon. The sum of

the values in Tables IX and X, for equal values of $\frac{t}{T}$, is equal to 1 in all cases.

Tables XI to XIII were calculated in a similar manner, only they are more complete.

TABLE X.—DECREASING LOGARITHMIC VALUES

$\frac{t}{T}$	$\frac{\text{Instantaneous value}}{\text{Maximum value}}$	Fraction
0.0	1.0	1
0.1	0.5	$\frac{1}{2}$
0.2	0.25	$\frac{1}{4}$
0.3	0.125	$\frac{1}{8}$
0.4	0.062 5	$\frac{1}{16}$
0.5	0.031 25	$\frac{1}{32}$
0.6	0.015 625	$\frac{1}{64}$
0.7	0.007 812 5	$\frac{1}{128}$
0.8	0.003 906 25	$\frac{1}{256}$
0.9	0.001 953 125	$\frac{1}{512}$
1.0	0.000 976 562 5	$\frac{1}{1,024}$

174. Logarithmic Tables.—By adopting a time standard, T = 6.9315 times the time constant, and comparing all other time intervals with this standard, and also by taking the ratios of the instantaneous values of a quantity to its maximum value, as in Tables IX and X, the values of all phenomena which vary logarithmically as time elapses, as current strengths, voltages, and the temperatures of conductors and coils may be computed from the following tables, wherein y is the instantaneous value of a quantity, Y is the maximum value of the same quantity, and $\frac{y}{Y}$ is their ratio. $\frac{t}{T}$ is the ratio of any elapsed time to the time standard. When $t = T$, $\frac{t}{T} = 1$. t may be greater than T, but y can never be greater than Y. The values given are unnecessarily accurate, except for purposes of interpolation. Tables XI to XIII[1] have been calculated for both increasing and decreasing values of y by making 100 ratios of $\frac{t}{T}$ and then calculating $\frac{y}{Y}$, and then assigning values of $\frac{y}{Y}$ and calculating $\frac{t}{T}$.

[1] UNDERHILL, C. R., "Practical Method for Calculating Operating Characteristics of Magnet Coils," *Elec. World*, vol. 78, p. 872, 1921.

Table XI is an enlargement of Table IX. Table XII is the reverse of Table XI, that is, 100 ratios of $\frac{y}{Y}$ have been assumed and the values of $\frac{t}{T}$ calculated therefor. Table XIII is an enlargement of Table X.

Table XIII is the reverse of Table XII when $\frac{y}{Y} = 0.99$ is substituted for $\frac{y}{Y} = 0.01$, $\frac{y}{Y} = 0.98$ for $\frac{y}{Y} = 0.02$, and so forth. For instance, in Table XIII, when $\frac{t}{T} = 0.10$, $\frac{y}{Y} = 0.500\ 00$. To find the value of $\frac{t}{T}$ corresponding to $\frac{y}{Y} = 0.500\ 00$ in the decreasing series from Table XII, subtract the value of $\frac{y}{Y}$ from 1, which gives $\frac{y}{Y} = 0.5$. Referring to Table XII, the value of $\frac{t}{T}$ is found to be 0.100 000.

This method of cross-tabulation makes it possible to find values of either $\frac{t}{T}$ or $\frac{y}{Y}$ by referring to either the assumed or calculated values as a basis. The preceding paragraph refers to a specific case wherein the duplication of the values of $\frac{t}{T}$ in Table XII is avoided in a further table.

175. Interpolation.—When the calculated value of $\frac{t}{T}$ falls between the tabulated values, proceed as in the following example: Let $\frac{t}{T} = 0.214$. This value falls between $\frac{t}{T} = 0.21$ and $\frac{t}{T} = 0.22$. In Table XI, for instance, the value of $\frac{y}{Y}$ corresponding to $\frac{t}{T} = 0.21$ is 0.766 74 and that corresponding to $\frac{t}{T} = 0.22$ is 0.782 36. Taking the differences:

$\frac{t}{T}$	$\frac{y}{Y}$	
0.22	0.782 36	0.214
0.21	0.766 74	0.21
0.01	0.015 62	0.004

TABLE XI.—INCREASING LOGARITHMIC QUANTITIES

$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$
0.01	0.066 97	0.36	0.917 53	0.71	0.992 20
0.02	0.129 45	0.37	0.923 05	0.72	0.993 71
0.03	0.187 75	0.38	0.928 21	0.73	0.993 63
0.04	0.242 14	0.39	0.933 01	0.74	0.994 08
0.05	0.292 89	0.40	0.937 50	0.75	0.994 48
0.06	0.340 25	0.41	0.941 69	0.76	0.994 85
0.07	0.384 43	0.42	0.945 59	0.77	0.995 19
0.08	0.425 65	0.43	0.949 23	0.78	0.995 51
0.09	0.464 11	0.44	0.952 63	0.79	0.995 81
0.10	0.500 00	0.45	0.955 81	0.80	0.996 09
0.11	0.533 48	0.46	0.958 77	0.81	0.996 36
0.12	0.564 73	0.47	0.961 53	0.82	0.996 60
0.13	0.593 87	0.48	0.964 10	0.83	0.996 83
0.14	0.621 07	0.49	0.966 51	0.84	0.997 04
0.15	0.646 45	0.50	0.968 75	0.85	0.997 24
0.16	0.670 12	0.51	0.970 84	0.86	0.997 42
0.17	0.692 21	0.52	0.972 80	0.87	0.997 60
0.18	0.712 83	0.53	0.974 62	0.88	0.997 76
0.19	0.732 06	0.54	0.976 32	0.89	0.997 91
0.20	0.750 00	0.55	0.977 90	0.90	0.998 05
0.21	0.766 74	0.56	0.979 38	0.91	0.998 18
0.22	0.782 36	0.57	0.980 76	0.92	0.998 30
0.23	0.796 94	0.58	0.982 05	0.93	0.998 41
0.24	0.810 54	0.59	0.983 25	0.94	0.998 52
0.25	0.823 22	0.60	0.984 38	0.95	0.998 62
0.26	0.835 06	0.61	0.985 42	0.96	0.998 71
0.27	0.846 11	0.62	0.986 40	0.97	0.998 80
0.28	0.856 41	0.63	0.987 31	0.98	0.998 88
0.29	0.866 03	0.64	0.988 16	0.99	0.998 95
0.30	0.875 00	0.65	0.988 95	1.00	0.999 02
0.31	0.883 37	0.66	0.989 69		
0.32	0.891 18	0.67	0.990 32		
0.33	0.898 47	0.68	0.991 03		
0.34	0.905 27	0.69	0.991 63		
0.35	0.911 61	0.70	0.992 19		

TABLE XII.—INCREASING LOGARITHMIC QUANTITIES

$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$
0.01	0.001 450 0	0.36	0.064 386	0.71	0.178 59
0.02	0.002 914 6	0.37	0.066 658	0.72	0.183 65
0.03	0.004 394 4	0.38	0.068 966	0.73	0.188 90
0.04	0.005 889 4	0.39	0.071 312	0.74	0.194 34
0.05	0.007 400 1	0.40	0.073 697	0.75	0.200 00
0.06	0.008 926 7	0.41	0.076 121	0.76	0.205 89
0.07	0.010 470	0.42	0.078 588	0.77	0.212 03
0.08	0.012 029	0.43	0.081 097	0.78	0.218 44
0.09	0.013 606	0.44	0.083 650	0.79	0.225 15
0.10	0.015 200	0.45	0.086 250	0.80	0.232 19
0.11	0.016 812	0.46	0.088 897	0.81	0.239 59
0.12	0.018 442	0.47	0.091 594	0.82	0.247 39
0.13	0.020 091	0.48	0.094 342	0.83	0.255 64
0.14	0.021 759	0.49	0.097 143	0.84	0.264 39
0.15	0.023 447	0.50	0.100 000	0.85	0.273 70
0.16	0.025 154	0.51	0.102 92	0.86	0.283 65
0.17	0.026 882	0.52	0.105 89	0.87	0.294 34
0.18	0.028 630	0.53	0.108 93	0.88	0.305 89
0.19	0.030 401	0.54	0.112 03	0.89	0.318 44
0.20	0.032 193	0.55	0.115 20	0.90	0.332 19
0.21	0.034 008	0.56	0.118 44	0.91	0.347 39
0.22	0.035 845	0.57	0.121 76	0.92	0.364 39
0.23	0.037 707	0.58	0.125 15	0.93	0.383 65
0.24	0.039 593	0.59	0.128 63	0.94	0.405 89
0.25	0.041 504	0.60	0.132 19	0.95	0.432 19
0.26	0.043 440	0.61	0.135 85	0.96	0.464 39
0.27	0.045 404	0.62	0.139 59	0.97	0.505 89
0.28	0.047 393	0.63	0.143 44	0.98	0.564 39
0.29	0.049 411	0.64	0.147 39	0.99	0.664 39
0.30	0.051 457	0.65	0.151 46	0.9999	1.328 77
0.31	0.053 533	0.66	0.155 64		
0.32	0.055 639	0.67	0.159 95		
0.33	0.057 777	0.68	0.164 39		
0.34	0.059 946	0.69	0.168 97		
0.35	0.062 149	0.70	0.173 70		

TABLE XIII.—DECREASING LOGARITHMIC QUANTITIES

$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$	$\frac{t}{T}$	$\frac{y}{Y}$
0.00	1.000 00	0.36	0.082 47	0.71	0.007 29
0.01	0.933 03	0.37	0.076 95	0.72	0.006 80
0.02	0.870 55	0.38	0.071 79	0.73	0.006 34
0.03	0.812 25	0.39	0.066 99	0.74	0.005 92
0.04	0.757 86	0.40	0.062 50	0.75	0.005 52
0.05	0.707 11				
0.06	0.659 75	0.41	0.058 32	0.76	0.005 15
0.07	0.615 57	0.42	0.054 41	0.77	0.004 81
0.08	0.574 35	0.43	0.050 77	0.78	0.004 49
0.09	0.535 89	0.44	0.047 37	0.79	0.004 19
0.10	0.500 00	0.45	0.044 19	0.80	0.003 91
0.11	0.466 52	0.46	0.041 24	0.81	0.003 64
0.12	0.435 28	0.47	0.038 49	0.82	0.003 40
0.13	0.406 13	0.48	0.035 90	0.83	0.003 17
0.14	0.378 93	0.49	0.033 49	0.84	0.002 96
0.15	0.353 55	0.50	0.031 25	0.85	0.002 76
0.16	0.329 88	0.51	0.029 16	0.86	0.002 58
0.17	0.307 79	0.52	0.027 21	0.87	0.002 40
0.18	0.287 18	0.53	0.025 38	0.88	0.002 24
0.19	0.267 94	0.54	0.023 68	0.89	0.002 09
0.20	0.250 00	0.55	0.022 10	0.90	0.001 95
0.21	0.233 26	0.56	0.020 62	0.91	0.001 82
0.22	0.217 64	0.57	0.019 24	0.92	0.001 70
0.23	0.203 06	0.58	0.017 95	0.93	0.001 59
0.24	0.189 47	0.59	0.016 75	0.94	0.001 48
0.25	0.176 78	0.60	0.015 63	0.95	0.001 38
0.26	0.164 94	0.61	0.014 58	0.96	0.001 29
0.27	0.153 89	0.62	0.013 60	0.97	0.001 20
0.28	0.143 59	0.63	0.012 69	0.98	0.001 12
0.29	0.133 97	0.64	0.011 84	0.99	0.001 05
0.30	0.125 00	0.65	0.011 05	1.00	0.000 977
0.31	0.116 63	0.66	0.010 31		
0.32	0.108 82	0.67	0.009 68		
0.33	0.101 53	0.68	0.008 97		
0.34	0.094 73	0.69	0.008 37		
0.35	0.088 39	0.70	0.007 81		

Then the difference in $\frac{y}{Y}$ corresponding to a difference in $\frac{t}{T}$ equal to 0.004 is equal to 0.015 62 $\times \frac{0.004}{0.01} = 0.006\ 248$. Adding 0.006 248 to 0.766 74 gives 0.772 99 as the value of $\frac{y}{Y}$ corresponding to $\frac{t}{T} = 0.214$. This operation is quite unnecessary in practice.

176. Examples Which Explain the Uses of the Logarithmic Tables.—*Example* 1.—The time constant of a circuit having a self-inductance of 0.02 henry and a resistance of 5 ohms is equal to $\frac{0.02}{5} = 0.004$ sec. The time standard T will then be equal to $6.9315 \times 0.004 = 0.027\ 726$ sec. If a constant, continuous voltage of 110 volts is suddenly impressed upon the circuit, the current will increase from 0 to a final strength of $\frac{110}{5} = 22$ amp., the active voltage will increase from 0 to a final or maximum value of 110 volts, and the e.m.f. of self-induction will decrease from 110 volts to a final value 0.

With these easily calculated data and with the aid of the tables, the values of the above phenomena at the end of any time interval up to 99.9 per cent, or down to 0.009 77 per cent, of their maxima are easily obtainable. For instance, what will be the values of the various phenomena 0.005 55 sec. after the constant, continuous voltage of 110 volts is suddenly impressed upon the circuit?

Dividing the finite time, $t = 0.005\ 55$ sec., by the time standard $T = 0.027\ 726$ sec., gives $\frac{t}{T} = 0.20$. Referring to Table XI, when $\frac{t}{T} = 0.20$, $\frac{y}{Y} = 0.75$. Since the maximum current strength is 22 amp., at the end of 0.005 55 sec. the current strength will be equal to $22 \times 0.75 = 16.5$ amp., the active voltage will be equal to $110 \times 0.75 = 82.5$ volts, and the e.m.f. of self-induction will be equal to $110 - 82.5 = 27.5$ volts. The e.m.f. of self-induction also may be found from Table XIII, as follows: When $\frac{t}{T} = 0.20$, $\frac{y}{Y} = 0.25$. Therefore, the e.m.f. of self-induction is equal to $110 \times 0.25 = 27.5$ volts, as previously calculated.

Example 2.—Assuming the same time constant and the same impressed voltage as in Example 1, what will be the value of $\frac{y}{Y}$ after the impressed voltage has acted on the circuit for 0.01 sec.? $\frac{t}{T} = \frac{0.01}{0.027\ 726} = 0.360\ 67$. Hence $\frac{t}{T}$ falls between 0.36 and 0.37. Referring to Table XI and interpolating,

$\frac{t}{T}$	$\frac{y}{Y}$	
0.37	0.923 05	0.360 67
0.36	0.917 53	0.36
0.01	0.005 52	0.000 67

Then the difference in $\frac{y}{Y}$ corresponding to a difference in $\frac{t}{T}$ of 0.000 67 is equal to $0.005\ 52 \times \frac{0.000\ 67}{0.01} = 0.000\ 37$. Adding 0.000 37 to 0.917 53 gives 0.917 90, or approximately 0.918, as the value of $\frac{y}{Y}$ corresponding to $\frac{t}{T} = 0.360\ 67$. Hence, the current strength will be equal to $0.918 \times 22 = 20.2$ amp., the active voltage will be equal to $110 \times 0.918 = 101$ volts, and the e.m.f. of self-induction will be equal to $110 - 101 = 9$ volts, after 110 volts have been impressed upon the circuit for 0.01 sec.

For ordinary calculations, the nearest value of $\frac{y}{Y}$ might have been taken, that is, $\frac{y}{Y} = 0.92$ in Table XII, or $\frac{t}{T}$ might have been taken as 0.36, which would make $\frac{y}{Y} = 0.918$ (approximately) in Table XI. Both tables may be consulted in each case.

Example 3.—With the same time constant as in the previous examples, how much time will elapse after the circuit is closed before the current strength and the active voltage will attain 70 per cent of their maximum values, and for the e.m.f. of self-induction to decrease to 30 per cent of its maximum value?

Referring to Table XII, when $\frac{y}{Y} = 0.70$, $\frac{t}{T} = 0.174$. But $T = 0.027\ 726$ sec. (Example 1). Consequently, $t = 0.174\ T = 0.174 \times 0.0277 = 0.0048$ sec.

Example 4.—An oscillograph test shows that the current strength in a circuit containing constant self-inductance and

resistance in series increases from 0 to 50 per cent of its maximum in 0.0015 sec. after a constant, continuous voltage has been impressed upon the circuit. (*a*) What is the time constant? (*b*) If the resistance is 10 ohms, what is the self-inductance of the circuit?

(*a*) Referring to Table XII, when $\frac{y}{Y} = 0.50$, $\frac{t}{T} = 0.10$. Since $t = 0.0015$ sec., $T = \frac{0.0015}{0.10} = 0.015$ sec. Hence, the time constant is equal to $0.144\,27\ T = 0.144\,27 \times 0.015 = 0.002\,16$ sec.

(*b*) The self-inductance is equal to $0.002\ 16 \times 10$ ohms = 0.0216 henry, or 21.6 millihenries.

Example 5.—A 2-mf. condenser is in series with a resistance of 1,000 ohms. If a constant, continuous voltage of 110 volts is suddenly impressed upon the circuit, assuming that no self-inductance is present, what will be the current strength at the end of 0.005 sec.?

The capacitance of the condenser is 0.000 002 farad. Hence, the time constant is equal to $0.000\ 002 \times 1{,}000 = 0.002$ sec., and $T = 6.9315 \times 0.002 = 0.013\ 86$ sec. Consequently, $\frac{t}{T} = \frac{0.005}{0.0139} = 0.36$. Referring to Table XIII, when $\frac{t}{T} = 0.36$, $\frac{y}{Y} = 0.082$, that is, the current strength will have decreased from $\frac{110}{1{,}000} = 0.11$ amp. to $0.11 \times 0.082 = 0.009$ amp., or 9 milliamp., in 0.005 sec. after the voltage of 110 volts was first impressed upon the circuit.

Incidentally, the active voltage would then be equal to $1{,}000 \times 0.009 = 9$ volts and the e.m.f. of capacitance would be equal to $110 - 9 = 101$ volts.

If, after the condenser was fully charged, the impressed voltage was discontinued and the condenser was discharged in series with a resistor, the current strength and the active voltage would decrease at the same time-rates, but in a reverse direction, as in the above example, but the e.m.f. of capacitance would then always be equal, but opposed, to the active voltage, since the dielectric voltage causes the discharge of the condenser and, therefore, is equal to the active voltage at all instants.

Example 6.—For how great a time interval must a constant, continuous voltage of 110 volts be impressed upon a circuit containing only such capacitance and resistance in series that the

time constant shall be 0.004 sec.. in order to cause the voltage of the condenser to be 50 volts?

Here $\frac{y}{Y} = \frac{50}{110} = 0.456$. Inspection of Table XII shows that $1 - 0.456 = 0.544$ for $\frac{y}{Y}$ when the quantity is decreasing, and that $\frac{t}{T}$ corresponding to $\frac{y}{Y}$ is equal to about 0.113 (see also page 264). Since $T = 6.9315 \times 0.004$ sec. $= 0.027\ 73$ sec., $t = 0.113 \times 0.027\ 73 = 0.003\ 13$ (approximately).

177. The Simultaneous Conversion and Storage of Energy.— Electric energy is simultaneously converted into heat and stored magnetic energy when a voltage is impressed upon a circuit containing a resistor and a reactor in series. A circuit containing an electromagnet coil is the equivalent of the above. If the impressed voltage is constant and continuous, the total electric energy in joules simultaneously converted into magnetic energy and heat is equal to the product of the impressed voltage and the quantity of electricity in coulombs moved through any plane in the cross-section of any one of the electric conductors in series under the influence of the impressed voltage. The amount of electric energy in joules converted into heat during any given time interval is determined by subtracting the stored magnetic energy in joules from the total energy in joules converted. In the above, the self-inductance and the resistance are assumed to be constant and it is further assumed that there are no other energy conversions or storages.

Electric energy is simultaneously stored as electric energy and converted into heat when a voltage is impressed upon a circuit containing a condenser and a resistor in series. If the impressed voltage is constant and continuous, the total electric energy in joules simultaneously stored in the dielectric of the condenser and converted into heat in the resistor is equal to the product of the impressed voltage and the quantity of electricity displaced in the circuit due to the impressed voltage. The amount of electric energy in joules converted into heat in the resistor during any given time interval is determined by subtracting the amount of electric energy stored in the dielectric from the total electric energy stored and converted. In the above, the capacitance and the resistance are assumed to be constant and it is further assumed that there are no other energy conversions.

178. Time-rates of Change and Areas under Curves.—The ascending logarithmic curve (Fig. 295 and Table XI) is a natural curve expressing the law of organic growth. The descending logarithmic curve (Fig. 296 and Table XIII) is a natural curve expressing how a phenomenon dies away as time elapses. The former is obtained by subtracting the values of the latter (for corresponding $\frac{t}{T}$ values) from the maximum value -1 in the present case. Thus, the values of either curve may be obtained by subtracting the corresponding values of $\frac{y}{Y}$ in the other curve from 1.

The descending logarithmic curve (Fig. 296 and Table XIII) expresses the time-rate of change in the slope of the ascending logarithmic curve (Fig. 295 and Table XI) for corresponding values of $\frac{t}{T}$. As an example, take the ratio

$$\frac{\text{difference in } \left(\frac{y}{Y}\right)}{\text{difference in } \left(\frac{t}{T}\right)}$$

in Table XI, as follows:

	$\frac{t}{T}$	$\frac{y}{Y}$
	0.16	0.670 12
	0.14	0.621 07
Differences.....	0.02	0.049 05

Then

$$\frac{\text{difference in } \left(\frac{y}{Y}\right)}{\text{difference in } \left(\frac{t}{T}\right)} = \frac{0.049\,05}{0.02} = 2.4525.$$

The average value of $\frac{t}{T}$ is 0.15. Multiplying 2.4525 by 0.144 27 (or dividing by 6.9315) gives the time-rate of change in the ascending values of $\frac{y}{Y}$ when $\frac{t}{T} = 0.15$, and this product is equal to 0.353 82 as based on an assumed average value of 0.15 for $\frac{t}{T}$. When the exact value $\frac{t}{T} = 0.15$ is taken, the correct value of $\frac{y}{Y}$ is 0.353 55, as found in Table XIII.

Conversely, the differences in the values of the ascending logarithmic curve (Fig. 295 and Table XI) express the differences in the area $\left(\frac{yt}{YT}\right)$ under the descending logarithmic curve (Fig. 296 and Table XIII) for corresponding values of $\frac{t}{T}$. For instance, in Table XIII the average value of $\frac{y}{Y}$ between the limits $\frac{t}{T} = 0.14$ and $\frac{t}{T} = 0.16$, a difference in $\frac{t}{T}$ of 0.02, is that corresponding to $\frac{t}{T} = 0.15$, which is 0.353 55. The area considered is equal to the product (1) $\frac{y}{Y} = 0.353\ 55$ and (2) difference in $\frac{t}{T} = 0.02$, the product being equal to 0.007 071. Multiplying 0.007 071 by 6.9315 (or dividing by 0.144 27) gives 0.049 01 as the difference in $\frac{y}{Y}$ in Fig. 295 and Table XI, corresponding to $\frac{t}{T} = 0.15$ when the difference in $\frac{t}{T}$ is 0.02, as in the preceding example.

The area $\frac{yt}{YT}$ under the ascending logarithmic curve (Fig. 295 and Table XI) as measured from $\frac{t}{T} = 0$ to any finite value of $\frac{t}{T}$ up to 1, is found as follows:

1. Multiply the value of $\frac{y}{Y}$ corresponding to the given value of $\frac{t}{T}$ by 0.144 27.
2. Subtract the product obtained in (1) from the given value of $\frac{t}{T}$.

The areas thus found are given in Table XIV and are plotted in Fig. 297.

Example.—The value of $\frac{y}{Y}$ corresponding to $\frac{t}{T} = 0.10$ in Table XIII is 0.500 00. Following the above rule:

1. $0.500\ 00 \times 0.144\ 27 = 0.072\ 135$ (see Table XV).
2. $0.10 - 0.072\ 135 = 0.027\ 865$ (see Table XIV).

Figure 298 and Table XV show the areas $\left(\frac{yt}{YT}\right)$ under the descending logarithmic curve (Fig. 296 and Table XIII). These are obtained by multiplying the values of $\frac{y}{Y}$ in Table XI by 0.144 27, as in the above example.

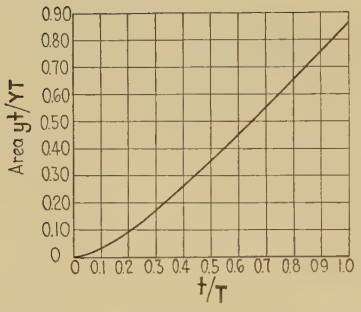


Fig. 297.—Areas under ascending logarithmic curve.

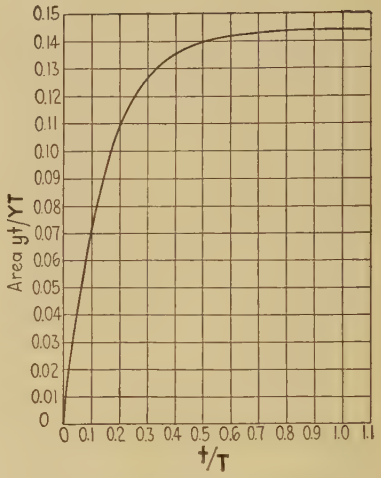


Fig. 298.—Areas under descending logarithmic curve.

To summarize:

1. The die-away curve (Fig. 296) is the basis.

2. The ascending curve representing the law of organic growth (Fig. 295), is obtained by subtracting the values of Fig. 296 or Table XIII from 1.

3. The area $\frac{yt}{YT}$ (Fig. 298 and Table XV) under the die-away curve (Fig. 296 and Table XIII) is found by multiplying the value of $\frac{y}{Y}$ corresponding to the given value of $\frac{t}{T}$ by 0.144 27.

4. The area $\frac{yt}{YT}$ (Fig. 297 and Table XIV) under the ascending logarithmic curve (Fig. 295 and Table XI) is found by subtracting the values of $\frac{y}{Y}$ in Table XV from the corresponding values of $\frac{t}{T}$.

The e.m.f. of self-induction is equal to the product of the constant self-inductance in henries and the time-rate of change in the amperage, when the proper sign is used. Hence, the de-

scending curve (Fig. 296 and Table XIII) represents the value of the e.m.f. of self-induction at any instant when the current strength is represented by the ascending logarithmic curve.

TABLE XIV.—AREAS UNDER THE ASCENDING LOGARITHMIC CURVE

$\frac{t}{T}$	Area $\frac{yt}{YT}$	$\frac{t}{T}$	Area $\frac{yt}{YT}$	$\frac{t}{T}$	Area $\frac{yt}{YT}$
0.01	0.000 338 7	0.36	0.227 63	0.71	0.566 78
0.02	0.001 324 5	0.37	0.236 83	0.72	0.576 71
0.03	0.002 913 7	0.38	0.246 09	0.73	0.586 65
0.04	0.005 066 3	0.39	0.255 40	0.74	0.596 59
0.05	0.007 744 5	0.40	0.264 75	0.75	0.606 53
0.06	0.010 913	0.41	0.274 14	0.76	0.616 47
0.07	0.014 539	0.42	0.283 58	0.77	0.626 42
0.08	0.018 592	0.43	0.293 05	0.78	0.636 38
0.09	0.023 043	0.44	0.302 56	0.79	0.646 34
0.10	0.027 865	0.45	0.312 11	0.80	0.656 29
0.11	0.033 035	0.46	0.321 68	0.81	0.666 26
0.12	0.038 527	0.47	0.331 28	0.82	0.676 22
0.13	0.044 322	0.48	0.340 91	0.83	0.686 19
0.14	0.050 398	0.49	0.350 56	0.84	0.696 16
0.15	0.056 737	0.50	0.360 24	0.85	0.706 13
0.16	0.063 322	0.51	0.369 94	0.86	0.716 10
0.17	0.070 135	0.52	0.379 65	0.87	0.726 08
0.18	0.077 161	0.53	0.389 40	0.88	0.736 06
0.19	0.084 386	0.54	0.399 15	0.89	0.746 03
0.20	0.091 798	0.55	0.408 92	0.90	0.756 01
0.21	0.099 383	0.56	0.418 71	0.91	0.765 99
0.22	0.107 13	0.57	0.428 51	0.92	0.775 98
0.23	0.115 03	0.58	0.438 32	0.93	0.785 96
0.24	0.123 07	0.59	0.448 15	0.94	0.795 95
0.25	0.131 23	0.60	0.457 99	0.95	0.805 93
0.26	0.139 53	0.61	0.467 83	0.96	0.815 92
0.27	0.147 93	0.62	0.477 69	0.97	0.825 90
0.28	0.156 45	0.63	0.487 56	0.98	0.835 89
0.29	0.165 06	0.64	0.497 44	0.99	0.845 88
0.30	0.173 76	0.65	0.507 33	1.00	0.855 87
0.31	0.182 56	0.66	0.517 22		
0.32	0.191 43	0.67	0.527 13		
0.33	0.200 38	0.68	0.537 03		
0.34	0.209 40	0.69	0.546 94		
0.35	0.218 48	0.70	0.556 86		

Note: These areas under the curve in Fig. 295 are obtained by subtracting the values of $\frac{yt}{YT}$ in Table XV from the corresponding values of $\frac{t}{T}$.

Since the quantity of electricity in coulombs is equal to the product of the average current strength in amperes and the time

in seconds during which the current flows, the area under the current-time curve (either ascending or descending) expresses the quantity of electricity. Figure 297 and Table XIV are for finding

TABLE XV.—AREAS UNDER THE DESCENDING LOGARITHMIC CURVE

$\frac{t}{T}$	Area $\frac{yt}{YT}$	$\frac{t}{T}$	Area $\frac{yt}{YT}$	$\frac{t}{T}$	Area $\frac{yt}{YT}$
0.01	0.009 661	0.36	0.132 37	0.71	0.143 22
0.02	0.018 676	0.37	0.133 17	0.72	0.143 29
0.03	0.027 086	0.38	0.133 91	0.73	0.143 36
0.04	0.034 934	0.39	0.134 61	0.74	0.143 42
0.05	0.042 256	0.40	0.135 25	0.75	0.143 47
0.06	0.049 087	0.41	0.135 86	0.76	0.143 53
0.07	0.055 461	0.42	0.136 42	0.77	0.143 58
0.08	0.061 408	0.43	0.136 95	0.78	0.143 62
0.09	0.066 957	0.44	0.137 44	0.79	0.143 67
0.10	0.072 135	0.45	0.137 89	0.80	0.143 71
0.11	0.076 966	0.46	0.138 32	0.81	0.143 74
0.12	0.081 473	0.47	0.138 72	0.82	0.143 78
0.13	0.085 678	0.48	0.139 09	0.83	0.143 81
0.14	0.089 602	0.49	0.139 44	0.84	0.143 84
0.15	0.093 263	0.50	0.139 76	0.85	0.143 87
0.16	0.096 678	0.51	0.140 06	0.86	0.143 90
0.17	0.099 865	0.52	0.140 35	0.87	0.143 92
0.18	0.102 84	0.53	0.140 60	0.88	0.143 95
0.19	0.105 61	0.54	0.140 85	0.89	0.143 97
0.20	0.108 20	0.55	0.141 08	0.90	0.143 99
0.21	0.110 62	0.56	0.141 30	0.91	0.144 01
0.22	0.112 87	0.57	0.141 49	0.92	0.144 02
0.23	0.114 97	0.58	0.141 68	0.93	0.144 04
0.24	0.116 94	0.59	0.141 85	0.94	0.144 06
0.25	0.118 77	0.60	0.142 02	0.95	0.144 07
0.26	0.120 47	0.61	0.142 17	0.96	0.144 08
0.27	0.122 07	0.62	0.142 31	0.97	0.144 10
0.28	0.123 55	0.63	0.142 44	0.98	0.144 11
0.29	0.124 94	0.64	0.142 55	0.99	0.144 12
0.30	0.126 24	0.65	0.142 67	1.00	0.144 13
0.31	0.127 44	0.66	0.142 78		
0.32	0.128 57	0.67	0.142 87		
0.33	0.129 62	0.68	0.142 97		
0.34	0.130 60	0.69	0.143 06		
0.35	0.131 52	0.70	0.143 14		

Note: These areas under the curve in Fig. 296 are obtained by multiplying the values in Table XI by 0.144 27.

the quantity of electricity in coulombs at any instant when the current strength is increasing logarithmically as time elapses. Figure 298 and Table XV are for finding the quantity of electricity

at any instant when the current strength is decreasing logarithmically with respect to time.

179. Examples Which Show the Uses of Tables XIV and XV.—While the tables are more accurate than the curves in Figs. 297 and 298, the latter may be used for rough calculations.

In using the curves or tables, first find the time constant. Next find $T = 6.9315$ times the time constant. Then divide the given time interval t by T, thus obtaining $\frac{t}{T}$. From the curves or tables find the value of $\frac{yt}{YT}$ corresponding to $\frac{t}{T}$. Then divide the constant, continuous impressed voltage by the resistance in ohms to obtain the maximum current strength in amperes. The quantity of electricity is equal to the continued product of (1) the area $\frac{yt}{YT}$, (2) the maximum current strength in amperes, and (3) 6.9315 times the time constant. Article 177 should be read in connection with what follows.

Example 1.—In a circuit containing only a self-inductance of 0.1 henry and a resistance of 10 ohms in series, (*a*) what will be the quantity of electricity 0.02 sec. ($= t$) after a constant, continuous voltage of 50 volts is suddenly impressed upon the circuit? (*b*) What will be the total electric energy converted? (*c*) How much electric energy will be converted into magnetic energy? (*d*) How much electric energy will be converted into heat? (*e*) What is the quantity of electricity utilized in storing the magnetic energy? (*f*) What is the quantity of electricity untilized in converting electric energy into heat? (*g*) What is the average current strength?

(*a*) The time constant is equal to $\frac{0.1 \text{ henry}}{10 \text{ ohms}} = 0.01$ sec. Hence, the time standard T is equal to 6.9315×0.01 sec. $= 0.069\ 315$ sec. The definite time is 0.02 sec. ($= t$). Therefore, $\frac{t}{T} = \frac{0.02}{0.069\ 315} = 0.289$. Referring to Fig. 297 or Table XIV, when $\frac{t}{T} = 0.289$, $\frac{yt}{YT} = 0.164$. The maximum current strength is equal to $\frac{50 \text{ volts}}{10 \text{ ohms}} = 5$ amp. Hence, the quantity of electricity is equal to $0.164 \times 5 \times 0.069\ 315 = 0.0568$ coulomb.

(*b*) The electric energy converted into both magnetic energy and heat is equal to 50 volts $\times$ 0.0568 coulomb = 2.84 joules or volt-coulombs.

(*c*) The electric energy converted into magnetic energy is equal to one-half the product of the self-inductance in henries and the square of the current strength, or to 0.5 $\times$ 0.1 henry $\times$ 25 = 1.25 joules.

(*d*) The electric energy converted into heat is equal to the difference between the total energy converted and that converted into magnetic energy, or to 2.84 joules $-$ 1.25 joules = 1.59 joules.

(*e*) The quantity of electricity utilized in storing the magnetic energy is equal to $\frac{1.25 \text{ joules}}{50 \text{ volts}} = 0.025$ coulomb.

(*f*) The quantity of electricity utilized in converting electric energy into heat is equal to $\frac{1.59 \text{ joules}}{50 \text{ volts}} = 0.0318$ coulomb.

(*g*) The average current strength is equal to $\frac{0.0568 \text{ coulomb}}{0.02 \text{ sec.}}$ = 2.84 amp.

Example 2.—In a circuit containing only a capacitance of 0.000 002 farad (2 mf.) and a resistance of 1,000 ohms in series, what will be the quantity of electricity 0.0015 sec. after a constant, continuous voltage of 100 volts is suddenly impressed upon the circuit?

The time constant is equal to 0.000 002 $\times$ 1,000 = 0.002 sec. The time standard T = 6.9315 $\times$ 0.002 = 0.013 86 sec., and $\frac{t}{T} = \frac{0.0015}{0.013\ 86} = 0.108$. Referring to Table XV,

$\frac{t}{T}$	$\frac{yt}{YT}$	
0.11	0.076 966	0.108
0.10	0.072 135	0.10
0.01	0.004 861	0.008

Then the difference in $\frac{yt}{YT}$ corresponding to a difference in $\frac{t}{T}$ of 0.008 is equal to 0.004 861 $\times \frac{0.008}{0.01} = 0.003\ 889$. Adding this to $\frac{yt}{YT} = 0.072\ 135$ gives 0.076 024, or 0.076, as the value of $\frac{yt}{YT}$ corresponding to $\frac{t}{T} = 0.108$. The maximum current strength

when the voltage is first impressed upon the circuit is equal to $\frac{100 \text{ volts}}{1{,}000 \text{ ohms}} = 0.10$ amp. Hence, after the voltage has been impressed upon the circuit for 0.0015 sec., the quantity of electricity is equal to $0.076\ 02 \times 0.10 \times 0.013\ 86 = 0.000\ 105\ 4$ coulomb. The maximum quantity of electricity is equal to the product of the capacitance in farads and the impressed voltage, or to $0.000\ 002 \times 100 = 0.0002$ coulomb. Similar results may be obtained by using Table XI, as in the following example.

Example 3.—The maximum quantity of electricity is 0.0002 coulomb and the constants are the same as in Example 2. The value of $\frac{y}{Y}$, in Table XI, corresponding to $\frac{t}{T} = 0.108$, is found as follows:

$\frac{t}{T}$	$\frac{y}{Y}$	
0.11	0.533 48	0.108
0.10	0.500 00	0.10
0.01	0.033 48	0.008

Then the difference in $\frac{y}{Y}$ corresponding to a difference in $\frac{t}{T}$ of 0.008 is equal to $0.033\ 48 \times \frac{0.008}{0.01} = 0.027\ 84$, and $0.500\ 00 + 0.027\ 84 = 0.527\ 84$, as the value of $\frac{y}{Y}$ corresponding to $\frac{t}{T} = 0.108$. The quantity of electricity at the end of 0.0015 sec. is equal to $0.0002 \times 0.527\ 84 = 0.000\ 105\ 5$ coulomb.

Example 4.—In Example 2, what are the amounts of electric energy respectively stored in the dielectric of the condenser and converted into heat in the resistor?

The total electric energy is equal to 100 volts $\times$ $0.000\ 150\ 4$ $= 0.015\ 04$ joule. At the end of 0.0015 sec., the e.m.f. of capacitance is found from Table XI, as follows:

$\frac{t}{T}$	$\frac{y}{Y}$	
0.11	0.533 48	0.108
0.10	0.500 00	0.10
0.01	0.033 48	0.008

Then the difference in $\frac{y}{Y}$ corresponding to a difference in $\frac{t}{T}$ of 0.008 is equal to $0.033\ 48 \times \frac{0.008}{0.01} = 0.026\ 78$. Adding 0.02678

to 0.500 00 gives 0.526 78 as the value of $\frac{y}{Y}$ corresponding to $\frac{t}{T}$ = 0.108. The maximum voltage is 100 volts. Therefore, the e.m.f. of capacitance is equal to 0.526 78 × 100 = 52.678 volts 0.0015 sec. after the e.m.f. of 100 volts is impressed upon the circuit.

The electric energy stored in the dielectric of the condenser is equal to 0.5 × 52.678 volts × 0.000 105 4 coulomb = 0.002 776 joule. The total energy is 0.015 04 joule. Consequently, the electric energy converted into heat in the resistor is equal to 0.015 04 − 0.002 776 = 0.012 26 joule.

Example 5.—A closed coil containing a self-inductance of 0.1 henry and a resistance of 10 ohms has a current of 5 amp. therein at the instant the voltage (either impressed or induced) is discontinued. How much stored magnetic energy is converted into heat 0.01 sec. after the voltage is discontinued?

The magnitude of the stored magnetic energy at the instant the voltage is discontinued is equal to 0.5 × 0.1 henry × 25 = 1.25 joules. The time constant is equal to $\frac{0.1}{10}$ = 0.01 sec. T = 6.9315 × 0.01 = 0.069 315 sec., and $\frac{t}{T} = \frac{0.01}{0.069\,315} = 0.144\ 27$. Referring to Table XIII, when $\frac{t}{T}$ = 0.144 27, $\frac{y}{Y}$ = 0.367 88. Then the current strength will be equal to 0.367 88 × 5 = 1.8394 amp., and the magnitude of the magnetic energy still unconverted is equal to 0.5 × 0.1 × 3.38 = 0.169 joule. Therefore, the total energy converted into heat in 0.01 sec. is equal to 1.25 joules − 0.169 joule = 1.081 joule, or 86.5 per cent of the original amount of stored magnetic energy.

In this example, the elapsed time t is the same as the time constant (t_c), but the example has been worked out in the same manner as though some other elapsed time was used.

CHAPTER XVI

MAGNETIC CIRCUITS AND THEIR PHENOMENA

Simple magnetic circuits are herein treated and a number of magnetic and electromagnetic phenomena explained. The effects of changes in permeability on various phenomena with respect to time are discussed in detail.

180. The Vortex Ring.—In the rings so familiar to smokers each particle of smoke describes a circle or orbit, all of the orbits are in parallel, and the axes of the orbits form a closed loop. In still air these rings retain their forms for some time and are held together by some force which appears to resist the tendency of the particles to fly from the ring due to centrifugal forces. As the rotation slows down, the rings become larger. While there is much more to be said about the vortex ring, it is here used as an analogy to introduce what follows.

181. The Toroidal Coil.—This may be wound around a torus ring of paraffin and the latter may then be removed by melting it and allowing it to run out between the turns of insulated conductor. The turns may be connected in series or in parallel, so long as the number of ampere-turns is the same in both cases. When an electric current flows in the turns of a toroidal coil the electricity moves around the air core like the smoke in the vortex ring and the magnetic flux established in the core hole is linked with every turn and, therefore, with all of the ampere-turns, and this magnetic flux tends to contract and pull the turns of the conductor containing the moving electricity closer together.

A toroidal coil may even consist of a single turn of solid electric conductor so bent as to form a hollow torus ring, although the ends of the tube need not necessarily touch, and an electric current may be induced therein by means of a primary coil wound over it. Then the electricity will flow like the smoke in the vortex ring, even after the induced voltage has been discontinued, and the field of magnetic flux within the air core will be linked with it until the current is damped out by resistance.

182. Simple Magnetic Circuit.—All of the magnetic flux produced by an electric current in a toroidal coil is generally

assumed to be in the air core, although this is not strictly the case, since some of the magnetic flux linked with the outer turns does not link with the inner turns, and *vice versa*. This is the only case where there is no magnetic flux outside of an electromagnetic system due to the electric current in that system. Since this is the simplest form of magnetic circuit, it may be easily studied and understood.

The average length of coil (as if cut across and made straight) is the same as the average length of the magnetic circuit. The term *average* must be used because the inner and outer lengths are unequal. The cross-section of the magnetic circuit is the same as the cross-section of the air core of the coil. The direction of the magnetic flux with respect to the direction of the current is such that, if the clockwise direction in which a corkscrew is turned in screwing it into a cork represents the direction of the current, the direction of the magnetic flux is from the corkscrew downward through the cork.

183. Phenomena in Simple Magnetic Circuits.—Many of these have been previously described and are here briefly defined and their practical units given, as follows:

Magnetic flux is a phenomenon existing in space by virtue of an electric current linked with it, said electric current possessing an inertia by reason of its linkage with said magnetic flux.

Magnetic flux also is termed electromagnetic momentum. It can exist in air as readily as in free space, because air is neutral with respect to it, possessing unit permeability. Practical units are the maxwell and the weber (100,000,000 maxwells).

Magnetomotive force is that which produces or tends to produce magnetic flux when an electric current flows. The practical unit is the gilbert, which is equal to 1.257 amp.-turns.

Magnetic reluctance is the reciprocal of permeance and is that which limits the magnitude of the magnetic flux produced under the action of the m.m.f. The practical unit is the oersted or gilbert per maxwell. It is equal to the product of the reluctivity and the ratio of length to cross-section of a uniform magnetic circuit from which there is no magnetic leakage. Reluctance expresses the degree of difficulty with which magnetic flux is produced.

Permeance is the reciprocal of reluctance. The units used in this treatise are the maxwell per gilbert and the weber per gilbert. It is equal to the product of the permeability and the ratio of

cross-section to length of a uniform magnetic circuit from which there is no magnetic leakage. Permeance expresses the degree of ease with which magnetic flux is produced.

Magnetic flux density is the number of maxwells or of webers per unit cross-section of magnetic circuit. The practical unit is the gauss or maxwell per square centimeter. Another unit is the maxwell per square inch.

Magnetomotive force gradient is the number of gilberts per unit length of magnetic circuit. Units are the gilbert per centimeter and the gilbert per inch.

Reluctivity is the reciprocal of permeability and is the ratio of the m.m.f. gradient to the magnetic flux density. It is the reluctance of a unit cube of space or of substance. It also may be regarded as the cross-section of magnetic circuit required for the production of one maxwell of magnetic flux when the m.m.f. gradient is one gilbert per unit length. The reluctivity of vacuo is taken as unity when the centimeter is the unit of length used, and the unit may then be called the oersted-centimeter. If the inch is taken as the unit of length, the reluctivity of air or vacuo is the oersted-inch (equal to 0.3937 oersted-cm.). The oersted-centimeter is equal to 2.54 oersted-in.

Permeability is the reciprocal of reluctivity and is the ratio of the magnetic flux density to the m.m.f. gradient. It is the permeance of a unit cube of space or of substance. It also may be regarded as the length of magnetic circuit per gilbert when the magnetic flux density is one maxwell per unit cross-section. The permeability of vacuo is taken as unity when the unit of length is the centimeter, and the unit may be then described as the permeance per centimeter. When the inch is the unit of length used, the unit of permeance is equal to 2.54 times the permeance per centimeter.

184. Simple Magnetic-circuit Relations.—From what was said in Art. 183, the magnetic-flux density is equal to the m.m.f. gradient when the reluctivity and the permeability are equal to unity. When the presence of a magnetic material increases the magnetic-flux density through the phenomenon of magnetic induction, the permeability of the space occupied by the magnetic material is increased, due to the joint magnetic flux then in that space.

Referring to the toroidal coil in Fig. 299, let the average length of the air core be 25.4 cm. and let its cross-section be 2.54 sq. cm.

Then the ratio of the length to cross-section is equal to $\frac{25.4}{2.54} = 10$. Multiplying this ratio by the reluctivity of air or vacuo gives $10 \times 1 = 10$ oersteds as the reluctance of the air core. If there are, say, 1,000 turns of wire in the coil and the current strength is 0.5 amp., the number of ampere-turns will be equal to $1{,}000 \times 0.5 = 500$, and the number of gilberts of m.m.f. will be equal to $1.257 \times 500 = 628.3$. To find the magnitude of the magnetic flux in maxwells, divide the m.m.f. in gilberts by the reluctance in oersteds, which gives $\frac{628.3}{10} = 62.83$ maxwells. The magnetizing force is then equal to the number of maxwells per square centimeter, or to $\frac{62.83}{2.54} = 24.8$ gausses. But when the reluctivity is unity, the magnetizing force (electromagnetic-flux density) in gausses is equal to the m.m.f. gradient in gilberts per centimeter, that is, to the number of gilberts of m.m.f. per centimeter length of magnetic circuit, or to $\frac{628.3}{25.4} = 24.8$ gilberts per centimeter in the present case, which checks with the former calculation. The net result is that 500 amp.-turns in the coil produces an electromagnetic-flux density or magnetizing force of 24.8 gausses in the air core.

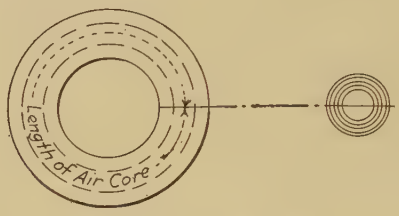


FIG. 299.—Toroidal coil.

The above example can be figured out with the inch as the unit of length: 1 in. = 2.54 cm. 1 sq. in. = 6.452 sq. cm. 1 cm. = 0.3937 in. 1 sq. cm. = 0.155 sq. in. Then the length of the magnetic circuit is equal to $25.4 \times 0.3937 = 10$ in., and its cross-section is equal to $2.54 \times 0.155 = 0.394$ sq. in. The ratio of its length to cross-section is equal to $\frac{10}{0.394} = 25.4$ and the reluctance is equal to 25.4 times the reluctivity in oersted-inches, or to 25.4×0.3957 oersted-cm. = 10 oersteds. The remaining calculations are just as tedious, so it is best to reduce

inches and square inches to centimeters and square centimeters in the first place and, after finding the flux density in gausses, reduce to maxwells per square inch, if desired, by multiplying the number of gausses by 6.452. Thus, 24.8 gausses = 24.8 × 6.452 = 160 maxwells per square inch.

Suppose it is desired to obtain a flux density of 50 gausses in the air core, and the required number of ampere-turns is to be found. The m.m.f. gradient in gilberts per centimeter is equal to the flux density in gausses, and the length of the magnetic circuit is 25.4 cm. Therefore, the number of gilberts required is equal to 50 × 25.4 = 1,270, the number of ampere-turns is equal to 1.257 × 1,270 = 1,595, and the number of ampere-turns per centimeter length, or the ampere-turn gradient, is equal to $\frac{1,595}{25.4} = 62.8$.

It is to be noted that the magnetizing force or electromagnetic-flux density in gausses in a toroidal coil is always equal to 1.257 times the ampere-turn gradient in ampere-turns per centimeter of coil length. Thus, to increase the magnitude of the magnetic flux in the air core without increasing the current strength or the number of turns, simply increase the cross-section of the air core, which will cause a corresponding increase in the length of wire per turn, thereby utilizing more electric energy, since the length of the wire in the coil is increased.

185. Magnetizing Force Around a Current in a Straight Wire. In order to make clear the foregoing statement, the subject will be temporarily treated from another angle. As stated in Art. 10, when an electric current flows in a straight conductor the electromagnetic flux produced thereby exists about and linked with it in space in the form of a cylinder of limitless diameter. The magnetizing force in gausses decreases as the radial distance increases, being equal to 0.2 times the ratio of the current strength in amperes to the radial distance in centimeters from the center of the conductor. However, the m.m.f. in gilberts is equal to the product of the magnetizing force in gausses and the length of the magnetic circuit in centimeters.

While the radial extent of the electromagnetic field is limitless, any portion of it can be considered as a separate section belonging to the rest of the field by taking a portion of the total cross-section at any distance from the center of the conductor. The average length of the portion of the magnetic circuit under cou-

sideration is 6.2832 times the radial distance and 0.2 times the current strength in amperes must also be multiplied by 6.2832 in order to keep the ratio of current strength to radius constant for a given magnetizing force. Then the magnetizing force in gausses is equal to 1.257 times the ratio of the current strength in amperes to the average length in centimeters of the portion of the magnetic circuit under consideration. But the total electromagnetic flux in maxwells linked with the current in the conductor is equal to the product of the average magnetizing force or electromagnetic-flux density in gausses and the length in centimeters of the conductor or portion of conductor under consideration. Hence, the greater the length of conductor available for a given amperage, the greater is the total amount of available magnetic flux. In such a case, the mass of conductor varies directly with the magnitude of the electromagnetic flux, and the total amount of electricity moving within the structure of the conductor at a constant average velocity increases directly with the magnitude of the electromagnetic flux which it produces.

186. General Classification of Magnetic Circuits.—Magnetic circuits may be broadly classified as series magnetic circuits and shunt magnetic circuits, after the manner in which electric circuits are broadly classified. The magnetic circuit of a coil and steel core, as in the case of a toroidal electromagnet with a core of magnetic material, is one form of joint magnetic circuit, since the induction in the core consists of the magnetizing force which would exist there if the magnetic material was not present, and the structural flux which springs from the magnetic material under the influence of the magnetizing force. It is further convenient to refer to the components of the joint magnetic circuit as the electromagnetic circuit and the ferromagnetic circuit, the former being the magnetic circuit of an air-cored solenoid, while an example of the latter is the magnetic circuit of a permanent magnet.

187. The Bothersome Constant 12.5664.—In the early mathematical development of magnetic quantities it was assumed that a hypothetical magnetic pole having very small dimensions would, when placed at the center of a sphere of 1 cm. radius, radiate one unit of magnetic flux for each square centimeter of surface of the sphere. Since the surface of the sphere of 1 cm. radius is equal to 12.5664 sq. cm., 12.5664 units of magnetic flux would be radiated from the unit pole.

Likewise, when a current strength of 10 amp. (which is the absolute unit of electric current strength, called the absampere) flows in a conductor 1 cm. long bent into an arc of 1 cm. radius, the electromagnetic-flux density at the center of the arc will be 1 gauss. But there are 6.2832 cm. of conductor in a complete turn having a radius of 1 cm., so when the current strength in a complete turn is 20 amps., the electromagnetic-flux density or magnetizing force at the center of the turn is 12.5664 gausses. But the practical unit of current strength is the ampere or 0.1 absamp. Hence, when 2 amps. flow in one turn of conductor, thus producing 2 amp.-turns, the magnetizing force in gausses at the center of the turn is equal to 1.256 64 $\times$ the radius in centimeters.

188. Magnetic Susceptibility.—This is usually referred to merely as susceptibility. It is the ratio of the intensity of magnetization to the magnetizing force. Susceptibility is of importance in connection with solenoids, permanent magnets, and certain electrical instruments.

189. Permeability and Susceptibility.—In the case of a toroidal electromagnet, the induction may be 3,000 times greater than the magnetizing force, that is, the permeability may be 3,000. In the case of an annealed wrought-iron ring, the induction may be 6,000 gausses when the magnetizing force is 2 gausses. Under these conditions, the intensity of magnetization would be equal to 0.079 58 $\times$ (6,000 $-$ 2), or about 477. The susceptibility would be equal to $\frac{477}{2}$ = 238.5. Hence, the ratio of the permeability to the susceptibility would be equal to $\frac{3{,}000}{238.5}$ = 12.6.

In any case, the permeability is equal to 1 plus 12.57 times the susceptibility, and the susceptibility is equal to 1 less than 0.079 58 times the permeability. When the permeability is 1, as in the case of air, the susceptibility is equal to 0, because there is no magnetic material present.

To find the susceptibility from the usual permeability tables or curves, multiply by 0.08. The result will usually be sufficiently accurate and within the limits of variation in the susceptibility of the material.

190. Summary of the Flux Densities.—This includes Table XVI, which shows the values based on given magnetizing forces and permeabilities in different cases, and also gives the values of

reluctivity and susceptibility corresponding to the permeability. When magnetic materials are present, the flux densities refer to the magnetic materials, that is, to the cross-sections of said magnetic materials. The following numerals refer to the columns in Table XVI:

I. Magnetizing Force: M.m.f. gradient; electromagnetic-flux density.

II. Magnetic Induction: Total flux density in a magnetic material placed in a magnetic field; electromagnetic-flux density plus ferromagnetic-flux density; magnetizing force plus 12.5664 times the intensity of magnetization.

III. Intensity of Magnetization: 0.079 58 times the ferromagnetic-flux density alone; 0.079 58 times the flux density in a permanent magnet.

IV. Ferromagnetic-flux Density: 12.5664 times the intensity of magnetization. The difference between the induction and the magnetizing force.

V. Permeability: Ratio of the induction to the magnetizing force.

VI. Reluctivity: Ratio of the magnetizing force to the induction.

VII. Susceptibility: (0.079 58 times the permeability) − 1.

TABLE XVI.—RELATIONS OF SOME MAGNETIC QUANTITIES

I	II	III	IV	V	VI	VII
100	100	0.0	0	1	1.0	0.0
100	200	7.958	100	2	0.5	0.079 58
100	1,000	71.6	900	10	0.1	0.716
100	15,000	1,186.0	14,900	150	0.006 7	11.86
8	14,000	1,113.0	13,992	1,750	0.000 571	139.0
4	10,000	795.0	9,996	2,500	0.000 4	198.8

The unit of magnetic-flux density in all cases is the gauss. There is no unit assigned for intensity of magnetization, but it is herein called the 0.079 58-gauss, meaning that the unit is equal to 0.079 58 times 1 gauss.

The susceptibility of space, air, and absolutely non-magnetic substances is equal to zero, while the permeability and reluctivity of space, air, and so forth are equal to unity.

While the induction in a piece of copper, for instance, which is not a magnetic material would, according to theory, be the same as the magnetizing force, the *increase* in flux density due to the presence of the material in a magnetic field would be equal to zero. Under these conditions, the magnetizing force or electromagnetic-flux density is infinitely greater than the increase in the strength of the magnetic field due to the placing therein of a non-magnetic material.

191. Unlimited Pull of a Solenoid.—After all of the structural flux has been coordinated in a piece of steel, the intensity of magnetization becomes a maximum. But there is no limit to the magnetizing force of a solenoid, which is dependent only upon the ampere-turn gradient in any given case. The only limit to the magnitude of the magnetizing force is the time interval during which the current can flow in the turns of conductor without developing sufficient heat therein to destroy the insulation or to fuse the conductor. The pull between a solenoid and its plunger is directly proportional to the product of the magnetizing force of the solenoid and the intensity of magnetization of the plunger. Hence, even after the plunger has become magnetized to its maximum capacity, the pull between the solenoid and the plunger will increase with increases in the current strength, if the coil does not burn out in the meantime.

192. Permeability and Saturation.—Before a piece of uniformly demagnetized magnetic material is placed in a magnetic field, its intensity of magnetization and its induction are equal to zero.

Let such a demagnetized magnetic material form the core of a toroidal electromagnet, and let a magnetizing force be produced in the space occupied by the magnetic material. As the magnetizing force is either continuously or intermittently increased, the intensity of magnetization and the induction also will increase. Eventually, the intensity of magnetization will attain a maximum, when the material is said to be saturated, but the induction will continue to increase with increases in the magnetizing force, the rate of increase in the induction being the same as the rate of increase in the magnetizing force, since the intensity of magnetization is constant when a magnetic material is saturated.

As the induction increases from O, the permeability will at first increase but will reach a maximum and thereafter will decrease as the induction continues to increase.

In Figs. 300 and 302 are shown typical B-H curves, called normal saturation curves, for various magnetic materials. B is the symbol for the induction. One kilomaxwell = 1,000 maxwells and one kilogauss = 1,000 gausses. Here B is in kilogausses, while H is

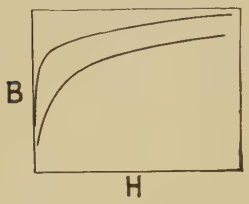


FIG. 300.—B-H curves.

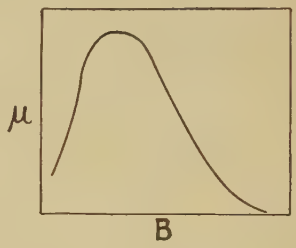


FIG. 301.—μ-B curve, commonly known as the permeability curve.

in gausses. For convenience, the number of ampere-turns per centimeter and per inch of length of steel required to produce the magnetizing force H are often given.

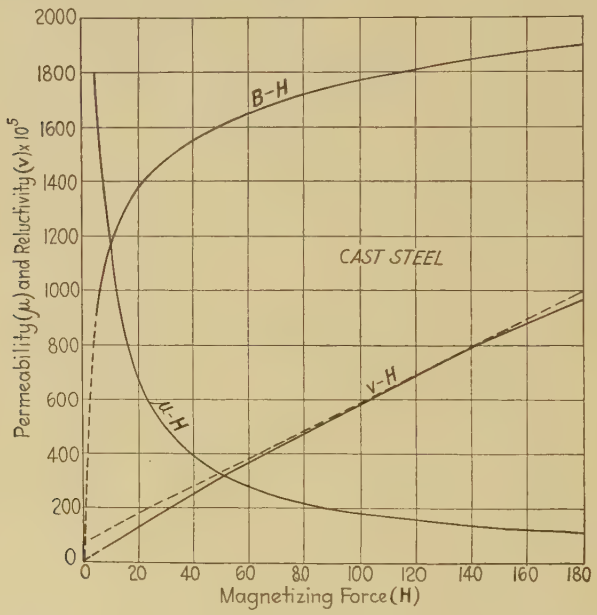


FIG. 302.—Various phenomena compared with magnetizing force.

During the increase in the induction B, the permeability μ (Greek letter *mu*) will change as in Fig. 301. However, permeability μ may be plotted against magnetizing force H, as in Fig. 302.

The μ-H curve closely resembles a rectangular hyperbola over the range shown.

193. Reluctivity and Saturation.—The reciprocal of the μ-H curve is the v-H curve, wherein v is the symbol for reluctivity. This curve is almost a straight line. $v = (H +$ a constant$) \times$ a constant. No great error will be committed in most practical work if the average ratio of the magnetizing force H to the reluctivity v of this cast steel is taken as $\frac{(H + 10)}{v} = 18{,}000$ between the limits $H = 40$ and $H = 150$, in Fig. 302, wherein $v \times 10^5$ means that 600 is to be read 0.006 and $B \times 10^{-1}$ means that 1,500 is to be read 15,000. B is in gausses in Fig. 302.

There are two conditions where the effects of permeability are constant or nearly constant. One of them is where no magnetic material is present and the permeability is constant. The other is where magnetic material is present but is saturated so that either increasing or decreasing the magnetizing force over a limited range has no appreciable effect upon the permeability of the magnetic material.

194. Remanent Magnetism.—When a torus ring of even the softest steel is magnetized, some of the magnetization remains or is retained by the steel after the magnetizing force has been discontinued. For instance, let the ring first be demagnetized. If it is composed of ordinary transformer steel and the induction is increased from 0 to about 15,000 gausses (including about 12 gausses of magnetizing force necessary to magnetize the steel), it will be found upon discontinuing the magnetizing force that the retentivity or the residual structural flux density (equal to 12.5664 times the intensity of magnetization) will be about 9,900 gausses, or about 66 per cent of the induction. However, only a trifle over 1 gauss of reversed magnetizing force will reduce the structural flux to zero and thus demagnetize the steel.

An average permanent magnet has a retentivity of about 10,000 gausses and a coercive force of approximately 65 gausses, or about 52 amp.-turns per centimeter (132 amp.-turns per inch). The retentivity is the structural flux density retained by a permanent magnet. The coercive force is the reversed magnetizing force required to demagnetize the steel completely.

In the soft steel of electromagnets, the apparent retentiveness may not be very great because the ferromagnetic circuits are not completely closed. However, the armatures will stick to the core

ends or polepieces after the current has been discontinued when the ferromagnetic circuit is approximately closed, but after an armature has once been pulled away from the core ends or polepieces the remanent magnetism is greatly reduced.

As a general proposition, the maximum permeability is directly proportional to the softness, while the coercive force is directly proportional to the hardness, of a magnetic material.

A short piece of magnetic material will retain very little remanent magnetism as compared with a solid ring of the same kind of material magnetized under similar circumstances. For similar materials, the greater the length the greater will be the remanent magnetism. Consequently, it is said that a short piece of magnetic material tends to demagnetize itself more readily than a long piece and, therefore, possesses a greater "demagnetizing force."

195. Magnetic Hysteresis.—Assuming that the steel is wholly demagnetized, let a magnetizing force H increase from 0, in a positive direction, until the steel is saturated. Then reduce the magnetizing force to zero again. Next, reverse the direction of the magnetizing force and increase its magnitude from 0 until the steel again is saturated, but with the direction of the structural flux reversed. Then reduce the magnetizing force to 0, reverse it, and again increase it in a positive direction until the steel again is saturated.

The results can be illustrated by the hysteresis loop in Fig. 303, wherein curve a is the regular B-H curve, called the virgin curve, because, having once become magnetized, the same B-H relations never again hold unless the steel or iron is again demagnetized. After the usual B-H curve (from 0 to d, curve a) has been plotted, the reduction of the magnetizing force curve to 0 will make the B-H relations as represented by curve b between d and f. At point f, H is equal to 0 but the induction B, which now is equal to 12.5664 times the intensity of magnetization and, therefore, represents only the structural flux density, has a definite value which makes the steel the equivalent of a permanent magnet with a closed magnetic circuit.

Reversing the direction of the magnetizing force H and then increasing it, forces this remanent magnetism to 0 at point g on curve b. The reversed magnetizing force $-H$ required to reduce the induction B (really the structural flux density) to zero is the coercive force. Continuing curve b to point e, where satura-

tion again occurs, and continuing the process which produces curve *c* and again causes the steel to become saturated, completes the hysteresis loop. Thereafter, if the magnetizing force is alternately decreased to 0, increased to its − maximum, reduced to 0, and again increased to its + maximum, the *B-H* relations will follow the outer curve of the hysteresis loop, regardless of the number of times this cycle is repeated, unless the steel eventually becomes aged.

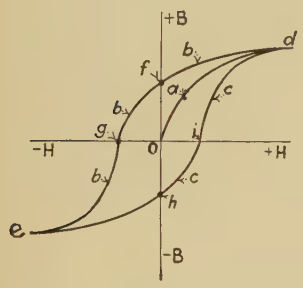


FIG. 303.

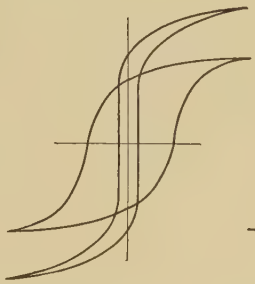


FIG. 304.

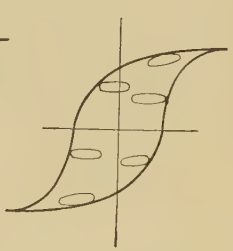


FIG. 305.

FIG. 303.—Hysteresis loop.
FIG. 304.—Hysteresis loops for soft iron and hard steel compared.
FIG. 305.—Auxiliary hysteresis loops.

The magnetic energy expended in reversing the molecules and, therefore, converted into heat each time the cycle is completed, is proportional to the area of the hysteresis loop. Thus, in the case of a 60-cycle alternating current, the electric energy converted into magnetic energy and thence into heat in the steel every second, due entirely to magnetic hysteresis, would be equal to 60 times the area of the hysteresis loop as measured in terms of $B \times H$.

For equal magnetizing forces, the shapes and areas of the hysteresis loops vary for different magnetic materials and their physical states. Figure 304 shows the relative shapes and areas for soft iron and magnet steel.

The effects of increasing and decreasing the magnetizing force in various parts of the principal hysteresis loop is illustrated in Fig. 305. The sum of the total area of the main loop and the areas of the enclosed loops is the total area representing the amount of magnetic energy converted into heat per cycle in cases where the form of the alternating-current wave is unsymmetrical, as when harmonics are present.

196. Effects of Changes in Permeability on Current-time, Flux-time, and Voltage-time Relations.—Allowing for changes in permeability, the magnitude of the magnetic flux is directly proportional to the strength of the current producing it. When the core hole of a coil contains no magnetic material, the magnitude of the magnetic flux therein is directly proportional to the current strength in the coil. Whether or not a magnetic material is present, the magnitude of the magnetic flux is constant when the current strength is constant. This relation is illustrated in Fig. 306.

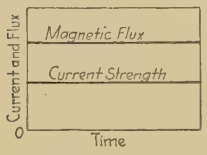


FIG. 306.

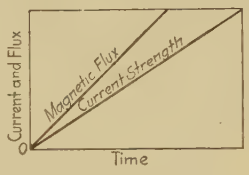


FIG. 307.

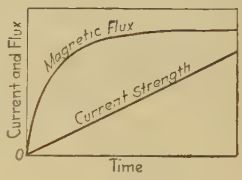


FIG. 308.

FIG. 306.—Constant current and flux.
FIG. 307.—Constant time-rates of change in current and flux.
FIG. 308.—Constant time-rate of increase in current with changing permeability.

When a constant, continuous voltage is impressed upon a coil containing only constant self-inductance (resistance negligible) the current strength and the magnitude of the magnetic flux increase as time elapses. In the case of a coil with an air core only, the relations are as in Fig. 307.

When a steel core is in the core hole and the time-rate of increase in the current strength is constant, the relations are as illustrated in Fig. 308. However, with a constant, continuous voltage impressed upon the coil, the current-time relation would not be represented by a straight line, and the magnetic-flux-time curve would not have the regular shape of the normal saturation curve.

The rate of change in the magnetic flux or the induction with respect to the change in the current strength producing it may be determined from the usual normal saturation curve by taking tangents to the curve, as in Fig. 309. The rate of change in the induction B with respect to the corresponding change in the magnetizing force H, when the induction is B_1 and the magnetizing force is H_1, is equal to $\frac{B_1}{H_1+H_a}$, since $B_1 = B_1 - 0$. $H_a + H_1$ is the difference in the magnetizing force, or the total magnetizing force, that would have been required to make the

induction equal to B_1 if B had increased from 0 at the same rate with respect to H that it is changing at point B_1, that is, the magnetizing force would have had to be applied at time a instead of at time 0. The same reasoning applies to $\frac{B_2-0}{H_b+H_2}$.

Any change in the magnitude of the magnetic flux and, therefore, in the induction causes the self-induced voltage and its balancing e.m.f. of self-induction to change in proportion to the

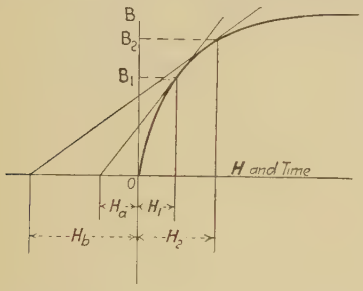


FIG. 309.—Rate of change in induction B with respect to corresponding changes in magnetizing force H.

FIG. 310.—Effects of change in permeability on current-time and voltage-time relations.

time-rate of change in the magnitude of the magnetic flux. If the current strength is constant, the magnitude of the magnetic flux is constant and the self-induced voltage is equal to zero. When a constant, continuous voltage is impressed upon a toroidal electromagnet, the current strength tends to increase logarithmically, but the increased time-rate of growth of the magnetic flux, due to the increase in permeability, causes an additional self-induced voltage to oppose the constant, continuous voltage, so that the current strength does not increase so rapidly as though the permeability remained constant. Figure 110 shows the current-time relations for an electromagnet with its ferromagnetic circuit closed. However, when there is an air gap in the ferromagnetic circuit, the current-time characteristics vary from the distorted form in Fig. 310 to one approaching the true logarithmic form, depending upon the length of the air gap and the magnitude of the magnetizing force. For this reason, the nearest value of $\frac{y}{Y}$ corresponding to the value of $\frac{t}{T}$, or *vice versa*, as given in Chap. XV, may be taken for electromagnets with the usual air gaps and magnetizing forces, but they

are not suitable for electromagnets with closed air gaps or with high magnetizing forces.

The distortion of an alternating-current-time curve or wave may be easily shown by making the magnetic-induction curve sinusoidal and plotting it with the corresponding values of the

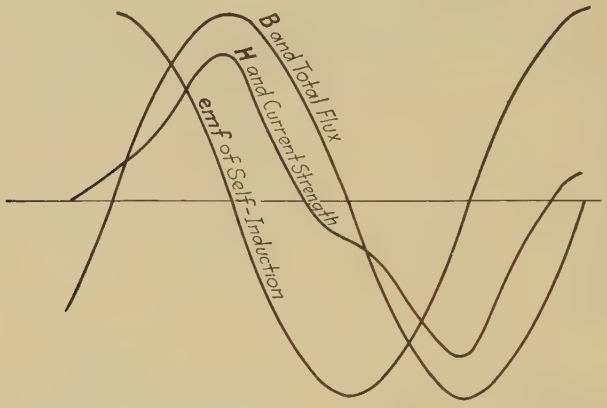


Fig. 311.—Distortion of current wave with sinusoidal magnetic flux.

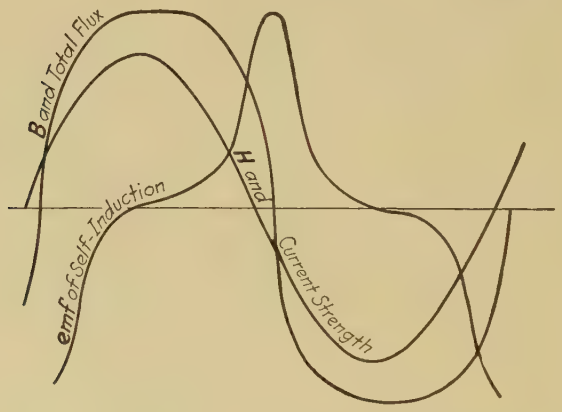


Fig. 312.—Distortion of voltage wave with sinusoidal current strength.

magnetizing force against elapsed time, as in Fig. 311, by comparing their relative values on the hysteresis loop. In this case, the alternating-voltage curve is sinusoidal. Figure 312 shows the relations obtained by making the magnetizing force sinusoidal and transferring the values from the hysteresis loop in a manner similar to that above described.

CHAPTER XVII

SERIES AND SHUNT MAGNETIC CIRCUITS

The core hole of the magnetizing coil may be thought of as the source of magnetic energy in a magnetic circuit, since that is the place where most of the energy is delivered to the magnetic circuit. Magnetic circuits are treated similarly to electric circuits. Various devices operating on the principle of the magnetic shunt are described, but these do not include those treated in Chap. III.

197. Series Magnetic Circuit.—A toroidal electromagnet has the simplest form of magnetic circuit because there is no magnetic

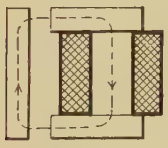


FIG. 313.—Series magnetic circuit.

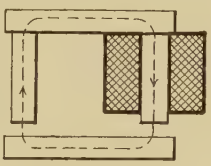


FIG. 314.—Series magnetic circuit.

leakage, due to the fact that the coil completely encompasses the magnetic circuit, that is, the magnetic circuit is entirely within the source of magnetic energy. Some forms of commercial series magnetic circuits are shown in Figs. 313 and 314, each consisting of four pieces of steel and two air gaps.

Without considering magnetic leakage, the number of ampere-turns in the coil required to produce a given magnitude of magnetic flux in the magnetic circuit is determined by multiplying the desired magnitude of magnetic flux in maxwells by 0.7958 times the total reluctance in oersteds of the magnetic circuit. Magnetic circuit calculations are preferably performed with the centimeter as the unit of length.

Find the reluctance of each iron or steel portion by dividing its length by its cross-section and multiply this quotient by the reluctivity. To find the reluctivity, divide the desired number of maxwells by the cross-section of the magnetic material

and thus obtain the induction. The reluctivity is then found from a table, or from a chart like Fig. 302. Such a chart also shows the magnetizing force required for that portion. The number of ampere-turns per centimeter length of steel is equal to 0.7958 times the magnetizing force in gausses. The reluctance of any air gap is found by dividing its length by its cross-section. The reluctivity of air is 1. When the opposite sides of the air gap have different areas, take the average cross-section of the air gap.

Add together all of the separate reluctances to obtain the total reluctance of the magnetic circuit. Then the required number of ampere-turns is equal to 0.7958 times the product of the desired magnetic flux in maxwells and the total reluctance in oersteds.

Actually, about twice the number of ampere-turns thus calculated may be required when the air gaps are relatively long, on account of the leakage of both the electromagnetic and ferromagnetic fluxes. To begin with, the magnetizing force is only about one-half as great at the end of the coil as at its middle; then much of the ferromagnetic flux does not pass through the air gap at all, but leaks back to the portion of opposite polarity at all points.

As an example, let the total magnetic flux be 30,000 maxwells, and let the parts in Fig. 313 be tabulated as in Table XVII, the centimeter being the unit of length.

TABLE XVII.—MAGNETIC-CIRCUIT DATA

Part	Length, L	Cross-section, S	$\frac{L}{S}$	Reluctivity	Reluctance (oersteds)
Core	8.0	2	4.0	0.002	0.008
Top polepiece	6.0	2	3.0	0.002	0.006
Bottom polepiece	6.0	2	3.0	0.002	0.006
Armature	10.0	2	5.0	0.002	0.010
Top air gap	0.3	2	0.15	1.00	0.150
Bottom air gap	0.3	2	0.15	1.00	0.150
Total reluctance in oersteds	...	..	....	.....	0.33
Reluctance in air gaps, per cent	...	..	....	.....	90.9

The number of ampere-turns required is equal to $0.7958 \times 0.33 \times 30{,}000 = 7{,}880$, or about 1,000 amp.-turns per centimeter length of coil. Hence, this electromagnet would belong to the momentary-duty class, as the coil soon would become very hot. However, with magnetic leakage neglected or else compensated for by additional ampere-turns, the pull in each air gap would be equal to 18.25 kg., or 40.3 lb. Actually, so great a pull would not be obtained for the reasons mentioned above.

In general, it may be assumed that all of the reluctance is in the air gaps (unless the latter are very short) and that from about 50 to 100 per cent more than the calculated number of ampere-turns for the air-gap are actually required, for electromagnets with external armatures, depending upon the ratio of the length of the air gap to the length of the core. This allows for the use of a non-magnetic stop to prevent sticking, as well as for changes in resistance due to heating, adjustments for wear, and slight tolerances in range. The factor of safety will vary with the length of the air gap.

198. Shunt Magnetic Circuit.—All magnetic flux that leaks so as not to pass through a cross-section of some part of an electromagnetic system where it is desired that it shall pass forms a branch of a shunt magnetic circuit.

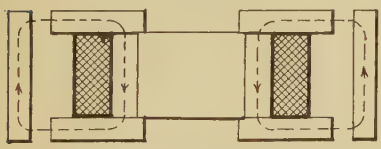


FIG. 315.—Two series magnetic circuits with one common coil.

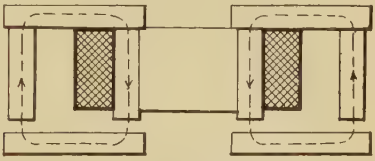


FIG. 316.—Two series magnetic circuits with one common coil.

Figure 315 shows two of the magnetic circuits in Fig. 313 magnetized from a single coil which links them both. It is obvious that the parts within the core hole mutually repel each other. Figure 316 shows two of the magnetic circuits in Fig. 314 magnetized from a single coil. These are simple illustrations of shunt magnetic circuits.

Figure 317 is a contraction of Fig. 315, and Fig. 318 is a contraction of Fig. 316. Either may have holes through their cores. Figure 317 resolves itself into the toroidal electromagnet, while Fig. 318 is of the annular type, which is the basis of the magnetic clutch and one form of lifting magnet.

The individual reluctances of the individual magnetic circuits are calculated as in the cases of series magnetic circuits, the reciprocals of the reluctances being taken to obtain the permeances. The permeances are all added together to obtain the total permeance, and the reciprocal of the total permeance is the joint reluctance.

Fig. 317.—Modification of Fig. 315.

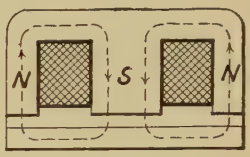


Fig. 318.—Modification of Fig. 316.

199. The Neck of the Bottle.—An isthmus is a piece of magnetic material of relatively small cross-section connecting two pieces of magnetic material of greater cross-section, the whole being integral if desired. Hence, the ratio of the magnitude of the magnetic flux that may be contained in the isthmus to that which may be contained in the similar pieces of magnetic material, but of greater cross-section, and which it connects, is equal to the ratio of (1) the product of the cross-section and the permeability of the isthmus to (2) the product of the cross-section and permeability of the similar larger pieces which it joins together. Although the isthmus may be saturated, the pieces which it connects cannot be saturated when they have cross-sections greater than that of the isthmus unless the magnetizing force is so great that the presence of the isthmus makes very little difference. For instance, let an isthmus be formed at the middle of the core of an electromagnet, as in Fig. 319. Then the portions of the core within the core hole of the coil could be saturated by making the magnetizing force sufficiently great, even with an air gap in series. However, if the isthmus is placed as in Fig. 320, the magnetic material outside of the core hole and the isthmus cannot become saturated unless the magnetizing force is enormously increased. However, the isthmus may be saturated with a relatively small applied magnetizing force.

The C-shaped or U-shaped permanent magnets of head telephone receivers are usually magnetized after assembly by placing the permanent magnets in a strong magnetic field. The ends of the soft-steel cores of the electromagnets of the receivers are generally placed on the polepieces of large electromagnets, the

intention being to "force magnetic flux through" the small soft-steel cores and thereby magnetize the permanent magnets. But each small core is "the neck of the bottle" and the polepieces are practically worthless. Where large electromagnets are employed for magnetizing, the receivers can be more quickly magnetized

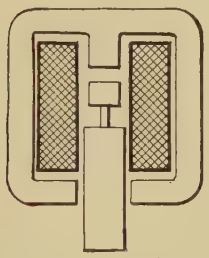


FIG. 319.—Isthmus in plunger.

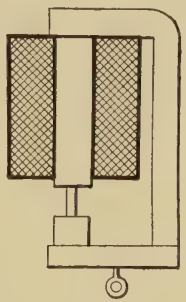


FIG. 320.—Isthmus near core end.

from a production standpoint by removing the polepieces altogether and placing the receivers between the coils near the free core ends. The permanent magnets can be magnetized by relatively small electromagnets when they are brought in direct contact with the core ends of the latter. The above method of magnetizing head sets gives the best results when large electromagnets with opposing V-shaped massive polepieces are so arranged that the head sets can be moved between the polepieces.

In the calculation of the magnitude of the magnetic flux in each branch of a shunt magnetic circuit it should be remembered that the core inside of the coil is "the neck of the bottle." In any case, no more magnetic flux can be gotten in an air gap outside of the core hole with a constant magnetizing force than exists at the middle of the core hole. The same holds for iron or steel parts between the core and the air gaps. The logical place for an air gap is inside the core hole of the coil when conditions will permit, but this is obviously impracticable in a great many cases.

200. Examples of Shunt Magnetic Circuits.—Electromagnets with shunt magnetic circuits have many uses. For instance, a counter e.m.f. motor controller may have a single coil and a number of armatures so positioned and restrained by the force of gravity or by springs that each will be attracted to the magnet and operate the contacts controlled by it to cut out resistance as

the motor accelerates and the voltage across the armature of the motor increases.

One form of frequency meter consists of an electromagnet with a number of mechanically tuned armatures having their masses and spring tensions so proportioned that they will vibrate freely only at a given frequency. Hence, an armature tuned to vibrate at a frequency of 60 cycles per second will vibrate at its maximum amplitude when the frequency of the alternating current is 60 cycles per second, while those tuned to adjacent frequencies scarcely vibrate at all, and the others appear to remain at rest.

When an iron or steel core has a cross-section sufficiently great to contain more than the required amount of magnetic flux, the magnetic circuits in Figs. 317 and 318 may be separately calculated, remembering that the amount of magnetic flux in the core must be equal to at least the sum of the magnetic flux in the two branches. The same rule applies to any number of branches. The magnetizing force is not reduced by closing any branch or any number of branches of a ferromagnetic circuit in shunt with one another. It is the saturation of the steel or iron core or some other similar part, the local shunting of a branch, or the presence of an air gap, which limits the amount of magnetic flux in the branches.

The general principles outlined above may be illustrated by two cases; namely, the case of the shunt magnetic lock-out switch and the series magnetic lock-out switch. The former is employed in connection with counter e.m.f. motorstarters and the latter with current-limit motorstarters. Figure 321 illustrates the principle of the shunt type. One of the branches is purposely restricted so that only a predetermined amount of magnetic flux can be contained in it. When the coil is energized, the induction is greater in the restricted branch than in the larger branch and the pull on the armature to hold it in the position shown is greater than the pull through the larger or main air gap. As the current strength increases, the amount of magnetic flux in the main air gap at the right increases while the flux in the restricted section cannot further increase. Hence, at a predetermined current strength the pull through the main air gap is greater than the lock-out pull and the air gap at the right is closed. The switch usually functions to cut out resistance as the motor accelerates.

In the case of the series magnetic lock-out switch, the principle of which is illustrated in Fig. 322, there are two branches in shunt with each other and in series with the core and the main air gap. In this case, the current strength is greatest at the start and the function of the magnetic contact or small air gap in one of the branches is to keep the switch from closing until

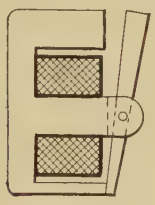


Fig. 321.

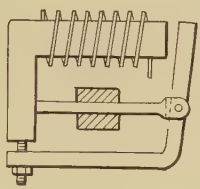


Fig. 322.

Fig. 323.

Fig. 321.—Principle of shunt magnetic lock-out switch or contactor.
Fig. 322.—Principle of series magnetic lock-out switch or contactor.
Fig. 323.—Principle of protective device.

the current strength has decreased to a predetermined value. The branch containing the small air gap is in shunt with the branch surrounding the damping coil *c*. The latter branch is saturated from the instant the current commences to flow in the coil until, as the current strength decreases as the motor accelerates, less magnetic flux is contained in the branch with the small air gap and the pull on the armature to hold it open decreases until the pull through the main air gap overcomes the pull through the small air gap, when the main air gap closes. As in the former case, the switch usually operates to cut out resistance. The damping coil *c* consists of a hollow cylinder of solid copper, its function being to prevent the operation of the switch due to sudden variations in the current strength and magnetic flux.

The device in Fig. 323 is employed to protect ammeters from short-circuit currents, particularly on automobiles. Normally, the leakage flux holds the armature against polepiece *A*, but an abnormal current strength will cause the armature to tilt to polepiece *B* and thus introduce a protecting resistor or else cut out the ammeter. In another device designed for a similar purpose, a form of buzzer connected in series with the ammeter rapidly opens and closes the circuit when the current strength becomes abnormal, thus introducing the equivalent of inductive reactance (really impedance) into the circuit.

201. Reversal of Direction of Magnetic Flux Due to Magnetic Shunt.—The phenomenon illustrated in Fig. 324 is due to the tendency of nearly closed magnetic circuits to retain some of their magnetism.

202. Adjustable Magnetic Shunts.—The cross-pieces in Figs. 325 and 326 may be moved along the polepieces to shunt or screen

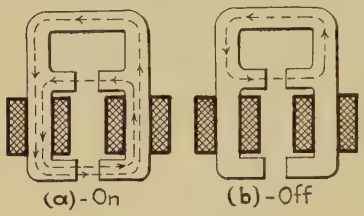


FIG. 324.

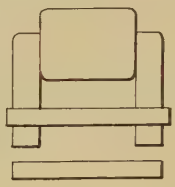


FIG. 325.

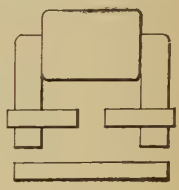


FIG. 326.

FIG. 324.—Reversal of magnetic flux due to magnetic shunt and remanent magnetism.
FIG. 325.—Adjustable magnetic shunt.
FIG. 326.—Adjustable magnetic shunt.

ferromagnetic flux from the air gaps by varying the magnetic leakage through them. The air gap in the cross-piece in Fig. 326 may be made adjustable. The direction of the magnetic flux is not reversed as in Fig. 324.

203. Magnetic Screens and Spacers.—These are often incorrectly called "magnetic insulators." The magnetic screen

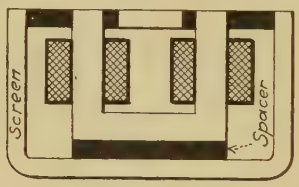


FIG. 327.—Magnetic screen and magnetic spacer.

absorbs and stores the magnetic energy due to the linkage of the leakage ferromagnetic flux with the ampere-turns, while the spacers decrease the leakage to the magnetic screens, the object being to confine the ferromagnetic flux to the electromagnetic device. These principles are used in magnetic chucks to prevent "stray flux," which magnetizes parts of grinders, shapers, and the like, and generally where electromagnetic devices are placed adjacent to one another to prevent undue interference. An example is shown in Fig. 327.

When magnetic flux is changing in magnitude, the magnetic energy may be converted into electric energy in copper sheaths, shells, and the like, as those placed over certain forms of telephone relays to prevent cross-talk between adjacent circuits in which the relays are connected.

Examples of magnetic screens are the iron casing within which a watch may be placed to prevent its becoming magnetized, and the iron or steel shell of an annular electromagnet.

204. Series-parallel Magnetic Circuits.—The chains of steel filings forming magnetic spectra are examples. There are filings in series to form chains, and the chains are in shunt or in parallel with each other. While the filings are mutually attracted together to form chains, the chains mutually repel one another.

Any number of equally magnetized closed torus rings may be placed side by side without mutual attraction resulting between them. The effect, or the lack of magnetic effect, is easily demonstrated by stringing a number of uniform and similar soft-steel rings on a stout conductor connected across the terminals of a storage battery. Even when the rings are separated, as in Fig. 328 (provided the steel is smoothly faced where the two halves come together), any number of such rings may be placed side by side around an electrically energized conductor, without their having any noticeable effect upon each other, so long as there are no appreciable air gaps in the ferromagnetic circuits. Each ring will act as an independent magnet. When any rings are separated, as in Fig. 328, however, they will attach themselves to their neighbors, using the latter to help close their air gaps and thus help to maintain the maximum linkage of ampere-turns and magnetic flux.

The general principle of the annular electromagnet is illustrated by means of a large conductor, but preferably by means of a cable consisting of parallel magnet wires. Call the respective ends of the cable A and B. The wires of the cable may be connected in parallel by electrically joining together the ends of the wires at A and B, respectively. To connect them in series, connect one end of a wire at A with the end of another wire at B. Next, connect the other end of the latter wire at A with the end of another wire at B, and continue the process until only two ends remain, when a coil will have been formed. Conversely, a coil may be wound on a form of large diameter and the coil cut across and then straightened out to form a cable.

This method is sometimes used in making soft iron-wire cores for ignition coils, the individual cores being cut from the cable. Hence, a coil may be defined as a form of cable with its insulated conductors connected in series.

Let such a coil be made so that it forms a relatively long looped cable with its turns in series. Let the coil consist of, say, 30 turns of conductor and let the diameter of the cable be slightly less than 2 cm. Furthermore, let the inside diameter

Fig. 328.—Separable steel ring.

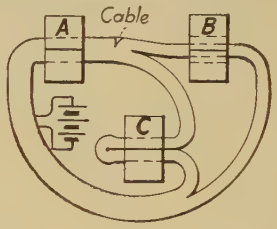


Fig. 329.—Separable steel rings on cable.

of the assembled half rings (Fig. 328) be 2 cm. Then any number of such half rings may be placed around the cable in pairs to form closed ferromagnetic circuits when the cable is energized, until the cable is encompassed by these rings throughout its entire length.

If the cross-section of each completed steel ring is 1 sq. cm., the total number of square centimeters of attracting surface will be equal to the length of the cable, if only one air gap per ring is counted. If the pull required to separate a single ring is 9 kg. (19.8 lb.), then the pull in kilograms required to separate all of the rings simultaneously will be equal to nine times the number of rings. The magnetic circuit of each of the closed rings is a series magnetic circuit, and these are all in shunt or in parallel with each other. Hence, they can all be fastened together to form an annular electromagnet.

Each ring has a certain flux density or induction which, when squared, exerts a certain mechanical force between the parts of the rings. Putting on two rings necessitates twice the length of conductor; doubles the resistance of the conductor; doubles the mass of the conductor; doubles the mass of the steel; doubles the self-inductance; doubles the permeance; doubles the total magnetic flux for a given current strength in the conductor; doubles the voltage drop in the conductor for a given constant, direct-current strength; doubles the power required; and doubles

the mechanical force. All of the above factors are multiplied in direct proportion to the number of unit rings or steel sections.

Assume that 10 such sections or rings have been placed over the conductor or cable, and it is then bent into the form of a ring. The shape of each steel section will be changed slightly, but this can be compensated for by increasing the thickness of the ring on the inside and decreasing it on the outside. If each section exerts a mechanical force of 20 lb., then the 10 sections will exert a force of 200 lb.

The radial distance from the center of the cable described on page 306 to the center of the face of any half ring is 1.5 cm. At this distance the magnetizing force in gausses is equal to 0.2 times the ratio of the total current strength in the cable to the radial distance in centimeters. Likewise, the current strength is equal to 5 times the product of the magnetizing force in gausses and the radial distance in centimeters. Hence, in this case, the magnetizing force is equal to $\frac{0.2}{1.5} = 0.133$ times the total current strength in amperes, and the total current strength in amperes is equal to 7.5 times the magnetizing force in gausses.

The total current strength in the cable is equal to the product of the current strength in each conductor and the number of conductors, which is the same as the number of ampere-turns in this case. In order to make the pressure 9 kg. per square centimeter, the induction must be 15,000 gausses and, with the permeability then about 500, the magnetizing force should be in the neighborhood of $\frac{15{,}000}{500} = 30$ gausses. The number of ampere-conductors required would be equal to $7.5 \times 30 = 225$ for each centimeter of length. Since it is assumed that there are 30 conductors in the cable, the current strength (by coincidence) in each conductor is equal to $\frac{225}{30} = 7.5$ amp. About 10 amp. or 300 amp.-conductors per centimeter might be required because of the small air gaps between the half rings.

Figure 329 illustrates such a cable split into two branches, so that but one-half as much magnetizing force exists around each branch as exists around the entire cable. One of the branches is bent back upon itself so as to be nearly non-inductive. Ring *B* can be pulled apart with approximately one-quarter the force required to separate ring *A*, while ring *C* will not even stick together.

205. Standard of Field Strength.—A form of compensated electromagnet illustrated in Fig. 330, and described by Dr. R. Beattie,[1] has coils so arranged near the air gaps that m.m.fs. may be imposed upon the main m.m.f. and the compensating m.m.f. will increase or decrease the main magnetic flux as the current strength is changed. The characteristics are shown in Fig. 331.

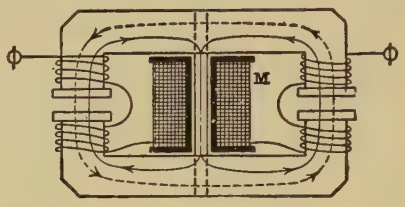


FIG. 330.—Electromagnet for maintaining constant magnetic flux.

Referring to Fig. 330, it is to be noted that, while the flux in the core divides as in Figs. 317 and 318, the compensating flux does not pass through the core. If the compensating coil at the left was not used, the flux due to the compensating coil at the right would pass through the core.

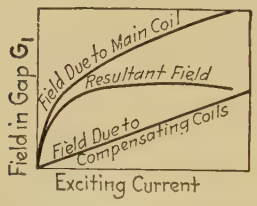


FIG. 331.—Components of flux across constant-field gap.

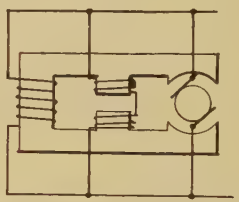


FIG. 332.—Constant-voltage generator.

206. Constant-voltage Generator.—The schematic arrangement of a variable-speed, constant-current or constant-voltage generator, disclosed in U. S. Patent No. 1202752, issued to Frederick Newton, is shown in Fig. 332.

[1] *Electrician* (London), Sept. 18, 1914; Abstracted in *Elec. World*.

CHAPTER XVIII

MAGNETIC MATERIALS

Of all the known elements, iron possesses the superior magnetic properties. But alloys of some of the non-magnetic elements are decidedly magnetic, while the magnetic properties of certain iron alloys are superior to those of the purest iron obtainable. However, sacrifices in any given property must often be made in order to obtain another desired property or to overcome some objectionable phenomenon.

207. Ferromagnetic Substances.—These come under the general class of paramagnetic substances which have positive magnetic susceptibilities and, therefore, are attracted by magnets, that is, they tend to move into the densest parts of magnetic fields. They include the ferromagnetic elements iron, cobalt, and nickel; the ferromagnetic compounds, Fe_3O_4 (magnetite), Fe_7S_8 (magnetic pyrites), and Fe_2Sb_3 (ferric antimonide); and the various ferromagnetic alloys.

208. Diamagnetic Substances.—These have negative magnetic susceptibilities and they are repelled by a magnet, that is, they tend to move away from magnetic fields. The effect of diamagnetism is most noticeable in bismuth.

209. Commercial Magnetic Materials.—The magnetic and electric properties of magnetic materials used in practical engineering work depend upon the uses for which they are required, as follows:

Electromagnets.—High permeability, low remanence, small coercive force, and high resistivity.

Permanent Magnets.—High remanence and great coercive force.

In other cases, as where a hard steel must be used to shunt a portion of a magnetic circuit to protect the coil and withstand wear, as in lifting magnets, a very low permeability is desired.

In electromagnets, a high permeability is desired so that the maximum amount of magnetic flux may be obtained for a given current strength; a low remanence permits the shortening of air gaps and the lengthening of cores without sticking of armatures;

a small coercive force permits the use of feeble alternating currents and reduces the hysteresis loss, while a high resistivity reduces the secondary-current or eddy-current losses.

In permanent magnets, a high remanence is desired so as to cause a magnet to be as strong as possible, and a great coercive force also is desired so that the magnet shall not readily become demagnetized.

For the solid cores in telegraph and other instruments of the better class, annealed Swedish charcoal iron is extensively used.

From the standpoint of low core loss, silicon steel is the best material now obtainable for the cores of alternating-current electromagnets, or for quick-acting electromagnets in general. High-silicon steel (about 3.5 per cent silicon) is brittle, but suitable for the cores of static transformers. Low-silicon steel (up to 1 per cent silicon) is the silicon steel commonly used.

The best alloy steels for permanent magnets are cobalt-chrome steel, molybdenum steel, tungsten steel, and chromium steel. Plain high-carbon steel, and even surface-hardened cast iron, also are used. Chromium steel (chrome steel) is extensively used because it is cheaper than tungsten steel and it is better than high-carbon steel.

The magnetic materials used in electromagnets preferably should be carefully annealed, while those used for permanent magnets should be oil-hardened, or else quenched in cold water, although the cobalt steels may be air-hardened.

While wrought iron is used for large electromagnets, cast steel, as dynamo steel, is the material commonly used in such cases.

Active magnetic material in any kind of magnet should be as free from manganese as possible, since manganese tends to make the best magnetic materials non-magnetic. When the manganese content becomes 12 per cent or more, the alloy steel may be considered as non-magnetic, the permeability being slightly greater than 1.

The moving parts of quick-acting electromagnets preferably are made of steel, because of its mechanical (as well as magnetic) properties.

The curves in Fig. 333[1] show the values of 12.5664 times the intensity of magnetization $(B - H)$ for some magnetic materials suitable for electromagnets. That for the alloy Fe_2Co is unusual.

[1] YENSEN, T. D., "The Magnetic Properties of Some Iron Alloys Melted in Vacuo" *Trans.* A.I.E.E., 1915.

Some of the best electrical sheets show a maximum core loss of from 1 to 4 watts per pound at a frequency of 60 cycles and maximum induction of 10,000 gausses, depending upon the thickness of the sheet and the material. Manufacturers usually furnish such data regarding their products.

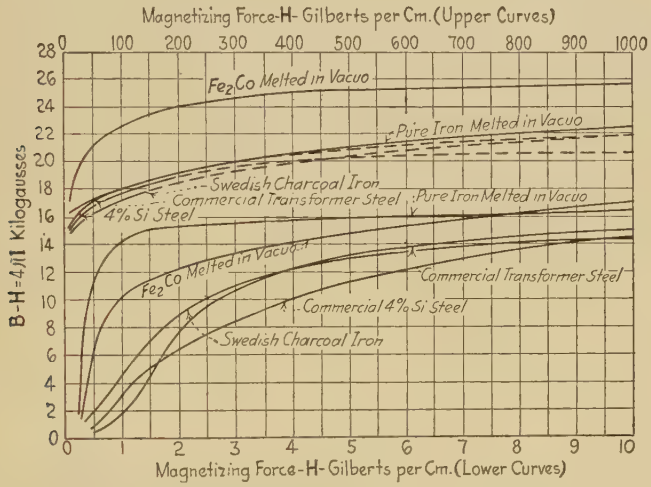

Magnetizing Force-H- Gilberts per Cm.(Upper Curves)
0 100 200 300 400 500 600 700 800 900 1000
B-H=4π I Kilogausses
28 26 24 22 20 18 16 14 12 10 8 6 4 2 0
Fe2Co Melted in Vacuo
Pure Iron Melted in Vacuo
Swedish Charcoal Iron
Commercial Transformer Steel
Pure Iron Melted in Vacuo
4% Si Steel
Fe2Co Melted in Vacuo
Commercial Transformer Steel
Commercial 4% Si Steel
Swedish Charcoal Iron
0 1 2 3 4 5 6 7 8 9 10
Magnetizing Force-H- Gilberts per Cm.(Lower Curves)

FIG. 333.—Magnetization curves for various grades of iron and iron alloys—annealed.

While electrical sheets need not be annealed after punching, it is important that the solid cores of electromagnets for use in relays and the like are not machined or hammered after annealing. Permanent magnets tend to lose their magnetism when jarred or heated.

A laminated steel structure is comparable to the structure of a coil inasmuch as the amount of active magnetic material per unit volume of core is less than as though the structure was solid. The space factor of such a structure is the ratio of the actual volume of active magnetic material to that occupied by the structure. The net permeability is the true permeability of such a structure, that is, it is the permeability of a single solid portion of the structure.

210. Examples of High Permeability.—Table XVIII[1] shows some unusually high permeabilities and low coercive forces.

[1] YENSEN, T. D., "The Magnetic Properties of Some Iron Alloys Melted in Vacuo," *Trans.* A.I.E.E., 1915.

TABLE XVIII.—PROPERTIES OF SOME MAGNETIC MATERIALS

Year	Investigator	Kind of material used	Maximum permeability	Coercive force gilberts per cm.		Hysteresis loss, ergs per cm.^{3} per $\sim$	
				for B_{max} = 10,000	for B_{max} = 15,000	for B_{max} = 10,000	for B_{max} = 15,000
1900	Hadfield	Swedish charcoal iron	4,000	0.92	1.00	about 2,700	about 5,500
1900	Hadfield	2½ per cent silicon iron	5,100	0.72	0.79	about 2,200	about 4,700
1901	Gumlich and Schmidt	Wrought iron	8,350	....	0.60		
1903	Baker	4.9 per cent silicon iron	.....	....	1.20	.....	6,200
1910	Terry	Electrolytic iron	11,000				
1912	Gumlich and Goerens	0.4 per cent silicon sheets	11,600	....	0.54		
1912	Gumlich and Goerens	4.0 per cent silicon sheets	9,400				
1912	Paglianti	1.75 per cent silicon iron	.....	0.60	0.75	1,650	3,500
1914	Yensen	Pure vacuum iron	19,000	....	0.29	813	1,640
1915	Yensen	0.15 per cent silicon vacuum iron	66,500	0.09	0.16	286	916
1915	Yensen	3.40 per cent silicon vacuum iron	63,300	0.08	0.15	280	1,025

A remarkable alloy, known as Permalloy,[1] consists of approximately 80 per cent nickel and 20 per cent iron. This alloy, which approaches saturation in the earth's magnetic field, has an initial permeability of about 13,000, while a maximum permeability, approaching 100,000 has been observed. The initial permeability is about 30 times that for soft iron, and the maximum permeability is greater than that found for silicon vacuum iron. The area of the hysteresis loop for low magnetizing forces for which it is suitable is about one-sixteenth of that for soft iron. This alloy is applicable to the field of communication, where great changes in permeability are required for correspondingly small changes in the magnetizing force.

211. The Properties of Some Magnet Steels.—An alloy steel discovered by Takagi and Honda, known as K. S. Magnet Steel, is described in *Science Abstracts—A*, 1920, vol. 22, as follows:

A favourable range for the constituents is: C, 0.4 to 0.8 per cent; Co, 30 to 40 per cent; W, 5 to 9 per cent; Cr, 1.5 to 3 per cent. Its best quenching temperature is 950° C., and the best quenching bath a heavy oil. The steel requires almost no heat treatment in order to be used as a permanent magnet in electrical instruments. The values found for the coercive force varied from 215 to 257, or about three times greater than that of the best tungsten steel hitherto known. Its residual magnetism is also greater than that of tungsten steel. The Brinell hardness numbers for the alloy were 444 (annealed), and 652 (quenched); the corresponding Shore numbers were 38 and 55. The hysteresis loss was 909,000 ergs (best tungsten steel 209,000). When artificially aged by heating in boiling water for 40 hr., there was a slight increase in residual magnetism, this having taken place in the first three hours' heating. Repeated shocks by fall caused the residual magnetism to decrease from 854 to 800; after 850 falls 1 meter onto a concrete floor. The residual magnetism increases rapidly with the dimension ratio up to the ratio 20, after which it is unaffected by the ratio. The steel when quenched is mechanically very hard, and has a fine microstructure. It is specially suited for short bar magnets.—G. E. A.

Tungsten magnet steel contains about W, 5 to 6 per cent; Cr, 0.2 to 0.4 per cent; C, 0.7 per cent. Residual magnetism, 10,500 gausses. Coercive force, 65. Chromium magnet steel (Cr, 2 to 3 per cent; C, 0.8 to 0.9 per cent) has a residual magnetism of about 9,500 gausses and a coercive force of 63 negative gausses. The coercive force of molybdenum magnet steel (Mb, 4 per cent) is about 85. The values vary with the heat

[1] Arnold, H. D., and Elmen, G. W., *J. Franklin Inst.*, 1923.

treatment and quenching temperature, and the manner in which the quenching is accomplished.

212. Ageing.—When magnetic materials other than sliicon steel are continually heated at maximum permissible coil temperatures for a month or more, the permeability decreases and the hysteresis increases. Permanent magnets are artificially aged by placing them in boiling water, partially demagnetizing them by means of an electric current, or submitting them to vibration or shock, in order that the residual or remanent magnetism shall remain constant thereafter.

213. Cases of Constant Permeance.—In connection with loading coils used in telephone transmission circuits for simultaneous telephone and telegraph transmission, it is required that there shall be no appreciable change in the inductance with changes in the strengths of the currents. This has been successfully accomplished by using cores consisting of hard iron in wire or sheet form. Hard iron wires, 0.004 in. in diameter, were wound and insulated to form the core. In one type of wire the permeability was 95, and in another it was 65, the wire of lower permeability being drawn through the dies a greater number of times without annealing.

TABLE XIX.—THE PERMEABILITIES OF SOME MAGNETIC MATERIALS

Magnetic material	Permeability	
	Maximum	Initial ($H = 0$)
Electrolytic iron, wrought bar	7,800.0	250.0
Poor grade of cast steel, annealed	710.0	131.5
Poor grade of cast steel, hardened	170.0	58.0
Cast iron, annealed	620.0	175.0
Cast iron, unannealed	240.0	69.4
Iron powder, annealed, uninsulated, compressed with 254,000 lb. pressure	540.0	80.0
Iron powder, unannealed, uninsulated, compressed with 254,000 lb. pressure	156.0	52.0
Grade A iron-powder cores	156.5	54.8
Grade B iron-powder cores	57.2	30.2
Grade C iron-powder cores	48.0	26.3

These cores are now made from powdered electrolytic iron. The iron is ground, sifted, rolled with flaked zinc in drums, shellacked, and compressed into rings at a pressure of 200,000 lb. per square inch. The magnetic properties of compressed iron powder are generally similar to those of solid iron rings in which air gaps have been cut. Table XIX[1] is of interest.

The permeabilities of grades B and C are practically constant for flux densities below B = 100. For grade A, this is true below B = 50.

Very complete data and references are given in the paper from which the above is abstracted.

214. Miscellaneous Properties of Magnetic Materials.—The phenomenon of magnetism is so intimately related to the structures of magnetic materials that hardly any physical phenomenon can occur without a corresponding change in the magnetic state, and *vice versa*. Experiments indicate that all metals, at least, become strongly magnetic at extremely low temperatures, while it is well known that the phenomenon of magnetism disappears from magnetic materials at high temperatures. The length of an iron bar is increased up to a certain point, but further magnetization causes the bar to shrink.

[1] Speed, B., and Elmen, G. W., "Magnetic Properties of Compressed Powdered Iron," *Trans.* A.I.E.E., 1921.

CHAPTER XIX

AIR GAPS AND MAGNETIC LEAKAGE

The directions of the magnetic flux paths through air gaps and surrounding magnets are shown by means of steel filings. The conception of magnetic flux is then discussed, using the straight-line forces acting on the filings as a basis.

215. Air Gap.—The term refers to any gap containing no magnetic material. In what follows, the term *coil* also applies to a solenoid. Examples of air gaps are:

1. The space between the ends of a coil bent so that the two ends approach each other.
2. The space between the ends of two adjacent coils.
3. The space between the ends of a bar or rod of magnetic material so bent that its ends approach each other.
4. The space between two adjacent magnetic bodies.
5. The space between the end of a coil and a magnetic body.

In general, the term air gap applies to abutting ends, but it is not restricted to this form. The term core hole has no reference to an air gap, although an air gap may exist in a core hole, the end of at least one of the magnetic bodies being inside of the core hole.

216. Magnetic Leakage.—When a toroidal solenoid is cut across so that the two ends of the solenoid shall be a slight distance apart, some of the electromagnetic flux leaks from points back of the air gap and is not linked with the end turns, since it does not enter the air gap. A similar but more pronounced effect is obtained by separating a toroidal electromagnet in the manner described. The neutral positions of steel filings indicate the absence of magnetic flux about a uniformly wound toroidal electromagnet. When all but a small portion of a toroidal coil has been removed from a solid torus ring of steel, the effect of magnetic leakage is similar to that around an air gap cut therein.

217. Magnetic Spectra.—Magnetic spectra or filing figures indicate the general directions and magnitudes of magnetic fields due to the tendency of the iron or steel filings to get into such

positions as to cause the maximum increase in the permeance and the self-inductance.

These figures are made by placing a magnetized steel bar or an electromagnet on a table and covering it with a sheet of paper. Steel filings are then uniformly sprinkled on the surface of the paper, and the paper is jarred slightly, so that the filings can assume their natural positions.[1] Each filing becomes a magnet

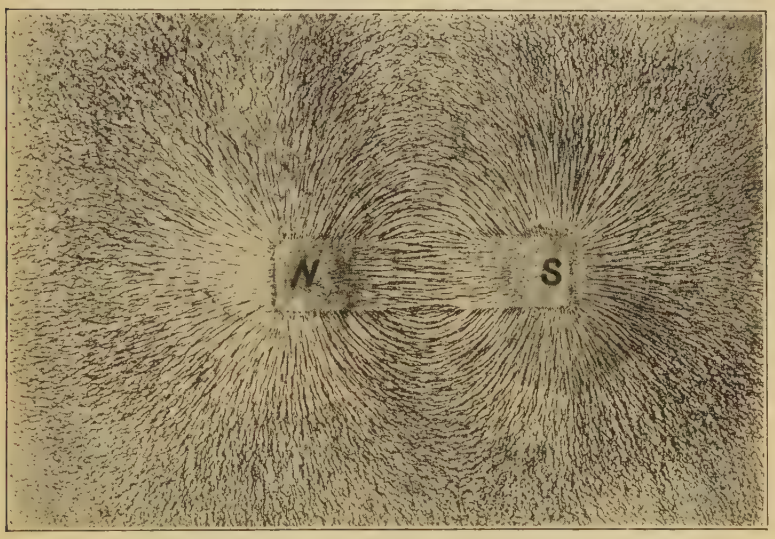


FIG. 334.—Field of a bar permanent magnet.

through magnetic induction, and each filing behaves like a compass needle. The chains of filings represent the magnetic flux paths about the magnet.

The magnetic field of a bar permanent magnet (Fig. 334) is similar to that of a bar electromagnet. Therefore, the permanent magnet is very useful in illustrating the various ferromagnetic flux paths due to leakage as well as to passage through the working air gaps. The magnetic flux that passes through the usual working air gap is in series with the source of magnetic energy, that is, with the main magnetic circuit. The leakage flux is in shunt with the main magnetic circuit or a portion thereof. The descriptions and the sequence of Figs. 335 to 343 explain the various points brought out by the positions of the filings. Such figures are very useful in estimating the reluctances or the per-

[1] This is an analogy of magnetic hysteresis

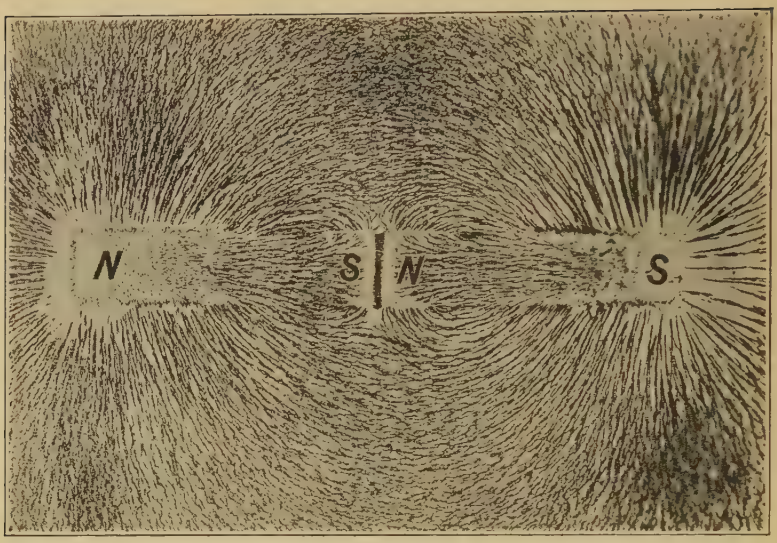


FIG. 335*a*.

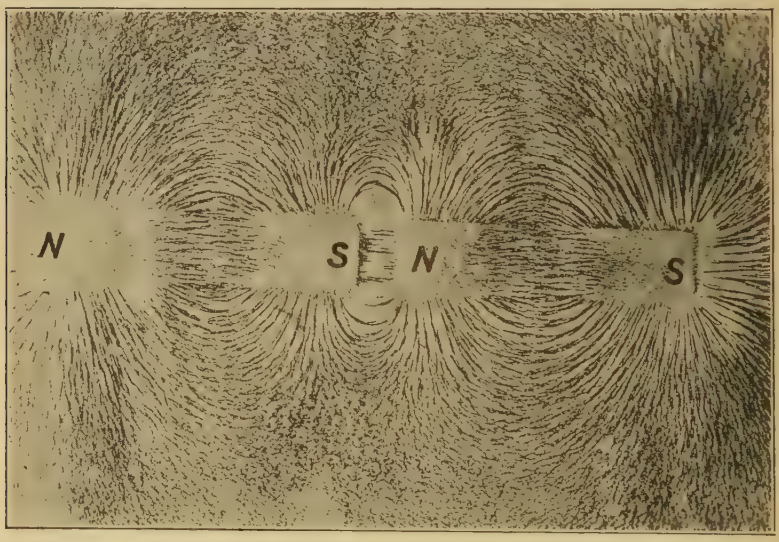


FIG. 335*b*.

FIGS. 335*a* and *b*.—Effect on fields of an air gap between the attracting ends.

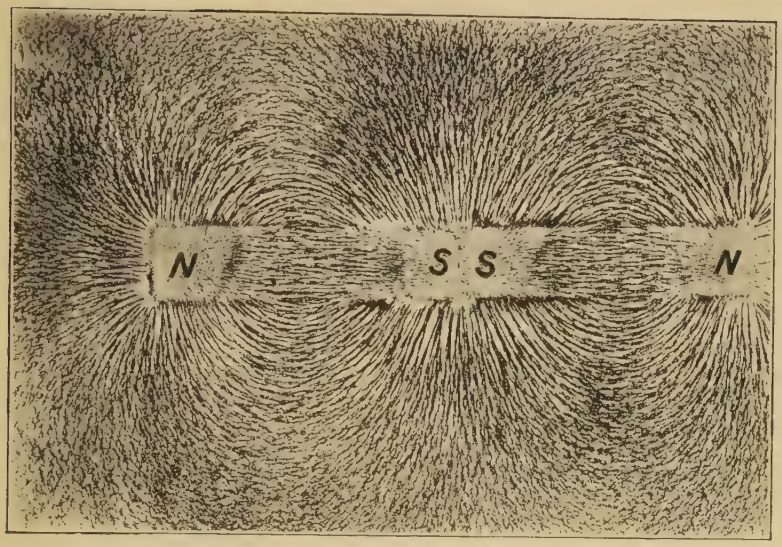


FIG. 336*a*.

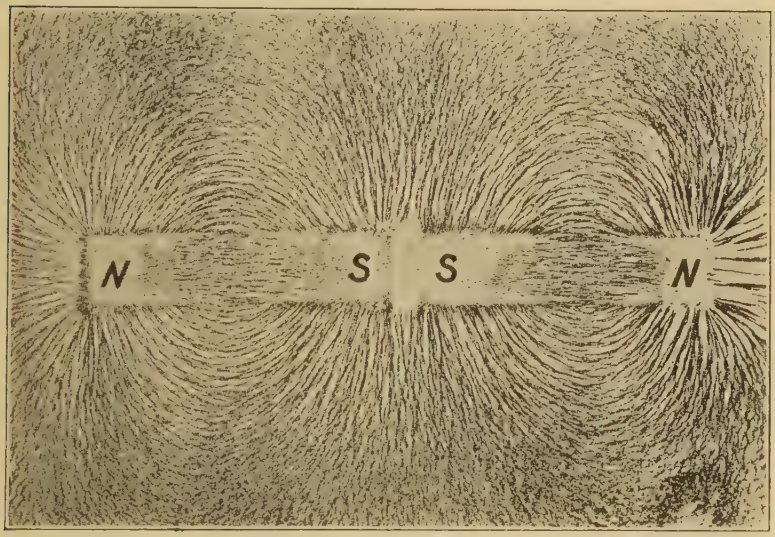


FIG. 336*b*.

FIGS. 336*a* and *b*.—Effect of fields of consequent-pole magnet by introducing an air gap.

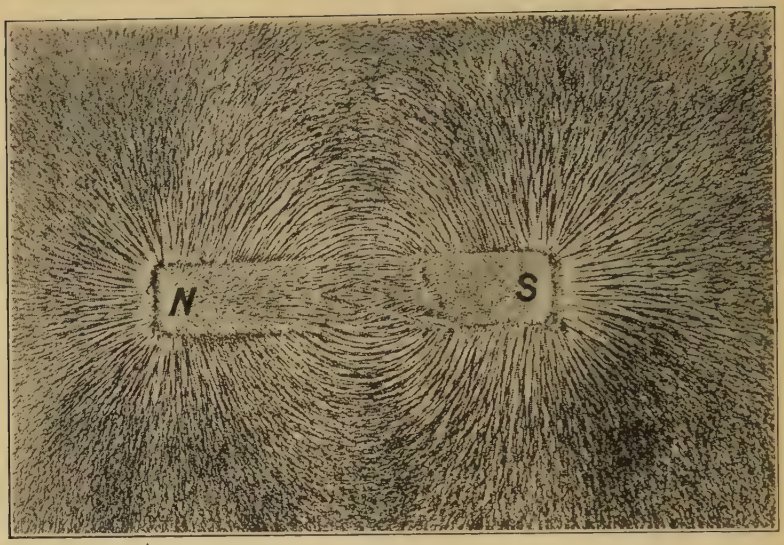


Fig. 337*a*.

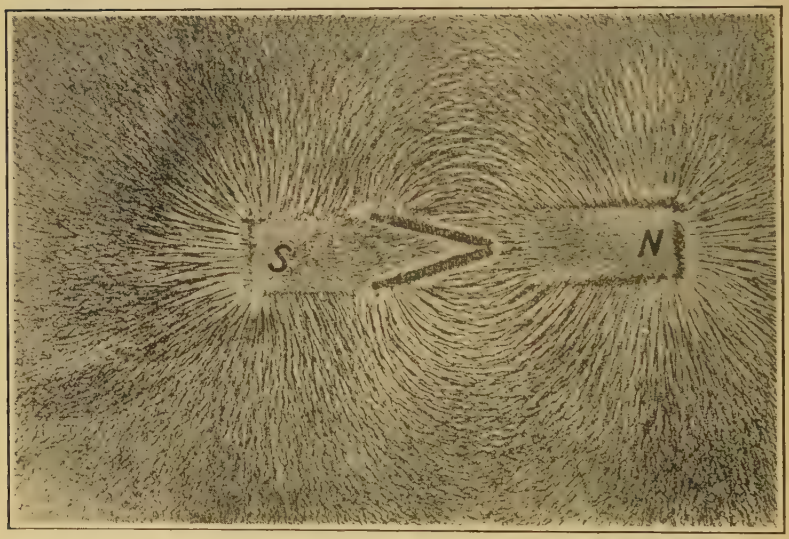


Fig. 337*b*.

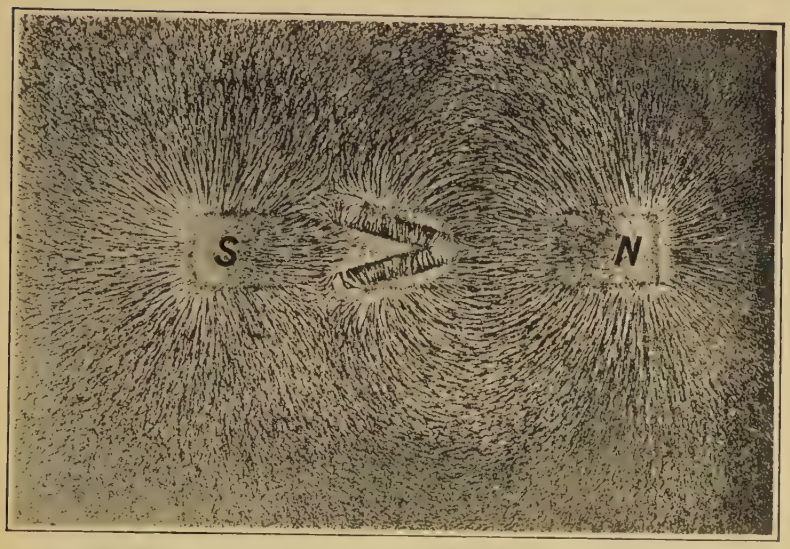


FIG. 337*c*.

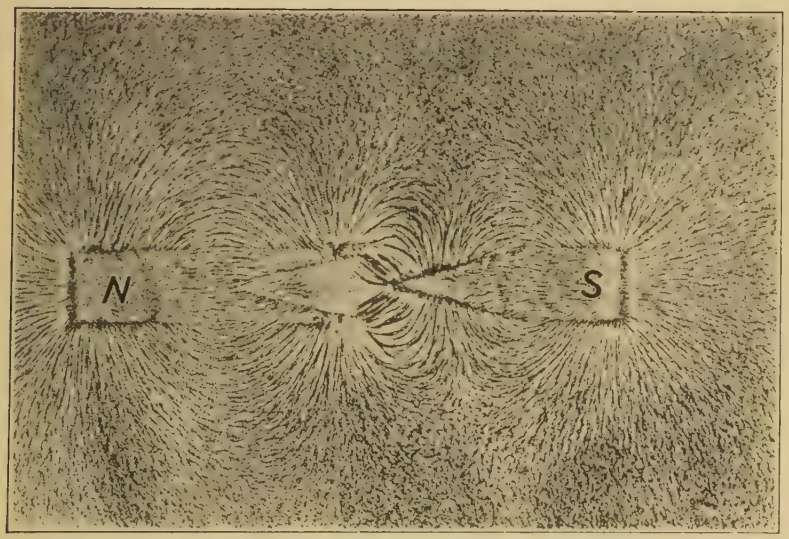


FIG. 337*d*.

FIGS. 337*a*, *b*, *c*, and *d*.—Effect on magnetic field of introducing a V-shaped air gap.

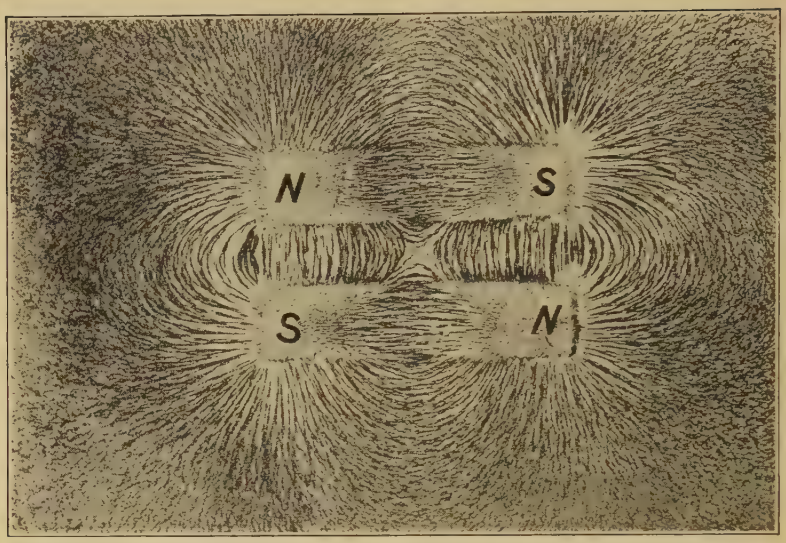


FIG. 338.—Effect on magnetic field of placing two magnets in parallel attractive relationship.

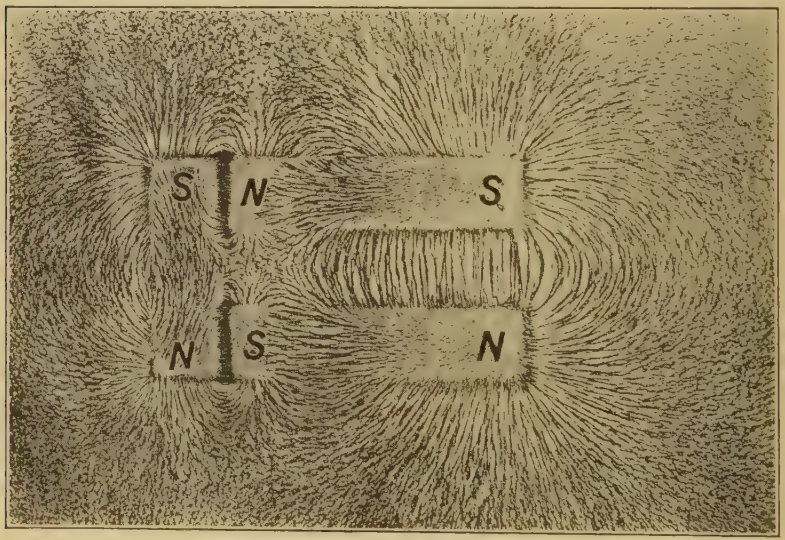


FIG. 339.—Simulative horseshoe magnet.

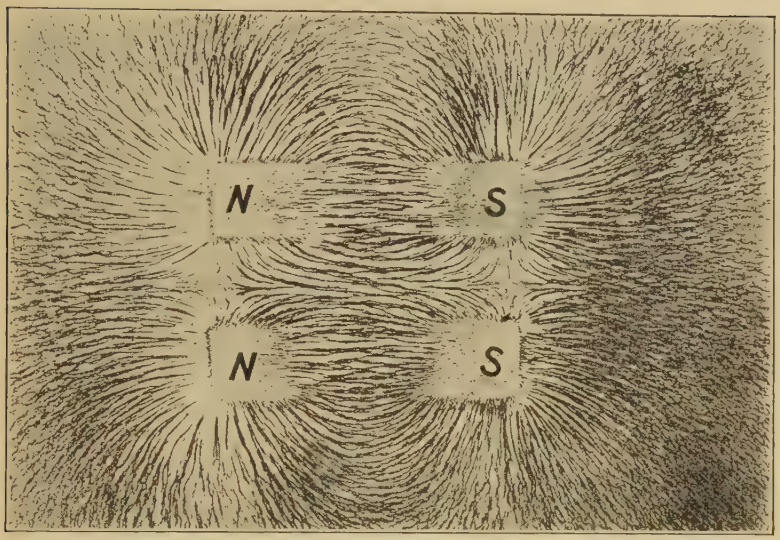


FIG. 340.—Effect of placing two magnets in parallel repulsive relationship.

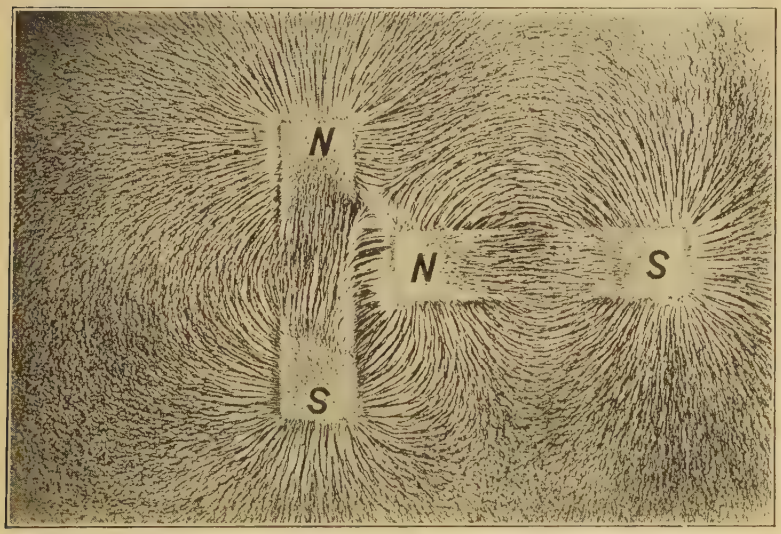


FIG. 341.—Effect of placing the end of one magnet near the neutral-polarity plane of another magnet.

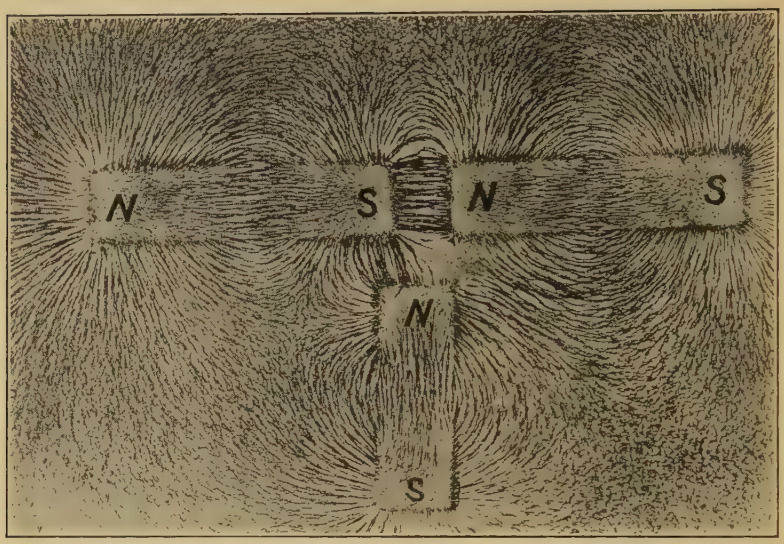


FIG. 342.—Simulating Fig. 341 with an air gap introduced.

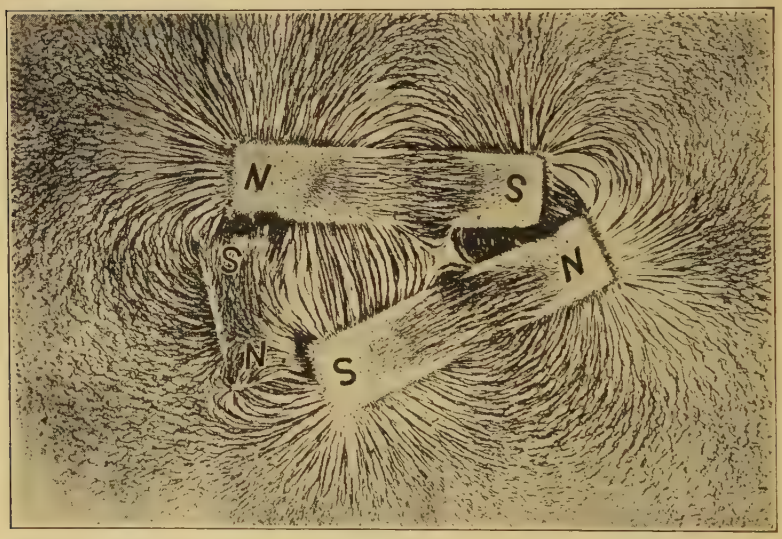


FIG. 343.—Triangular series magnetic circuit with various air gaps.

meances of the flux paths as they really exist. The above figures are given similar numbers when they belong to a common group, the letters indicating the various changes in position of the parts.

A filing figure in the plane of a complete plunger electromagnet is shown in Fig. 344. This was made by winding a thin layer of magnet wire on the coacting parts (which were flat) and placing the paper over the magnet. The fact that some of the ferromagnetic flux leaked to points over the surface of the steel caused details to be brought out very strongly.

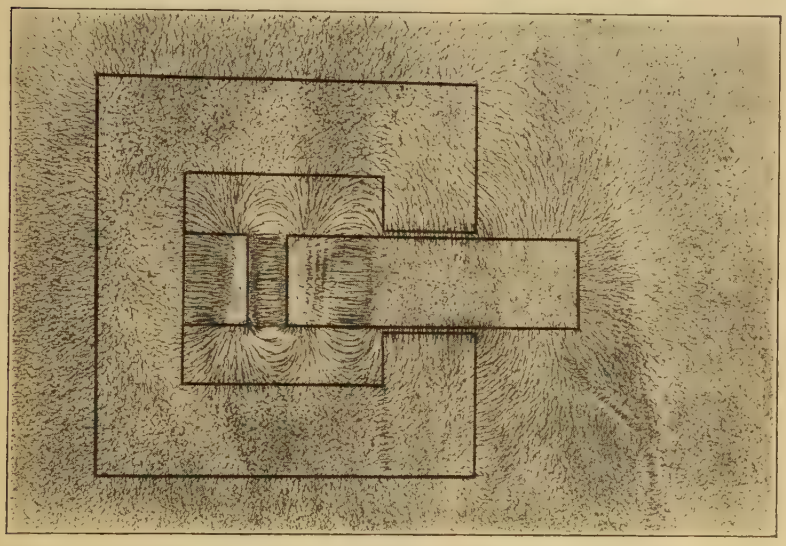


FIG. 344.—Magnetic fields of a plunger electromagnet.

Magnetic flux is constantly tending to shorten its length and to increase its cross-section, that is, it tends to attain the maximum linkage with the currents (either ordinary or molecular) producing it. Hence, two compass needles placed end to end tend mutually to pull themselves together. When placed side by side with like poles adjacent, they tend mutually to push each other away. That is why steel filings in a magnetic field tend to form chains in the directions of the flux paths but tend to repel one another sidewise so that the lines of filings are clearly defined; but these lines of filings are not lines of magnetic flux. Magnetic flux is continuous and does not exist in the forms of lines.

In the magnetic circuits of bar magnets there is no limit to the distribution of magnetic flux in space, but all of the magnetic flux passes through the neutral-polarity plane of the magnet where it can be measured by inductive methods, as by means of a fluxmeter.

In general, the ferromagnetic leakage is a maximum when the air-gap flux is a minimum, and is a minimum when the air-gap flux is a maximum.

Leakage from a magnet is similar to current leakage from an electric circuit having a bare electric conductor immersed in a liquid conducting medium of relatively low conductivity.

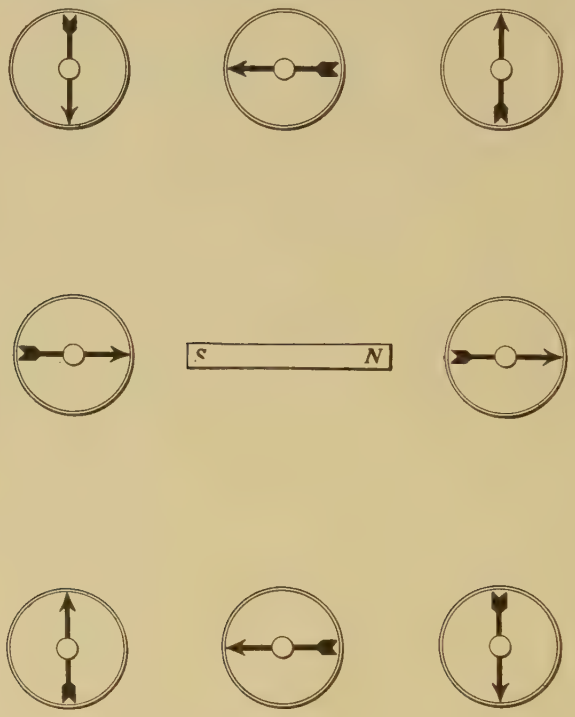


FIG. 345.—Magnetic compasses surrounding a permanent magnet.

218. Conception of Magnetic Flux.—Figure 345 shows compass needles placed about a bar permanent magnet. This figure was made by placing a single compass needle in the various positions about the magnet. Each compass needle is a magnet with N and S polarity. When a number of compass needles are simultaneously employed, they lie in such positions with

respect to one another and the bar permanent magnet that each N-pole points to an S-pole, as in Fig. 346, thus tending to form a ferromagnetic circuit.

The mechanical forces of mutual attraction and repulsion between each compass needle and the magnet act in straight lines. In Fig. 347, the N end of the magnet attracts the S end, but repels the N end, of the compass needle. This would cause the N end of the needle to point straight to the S end of the magnet were it not for the fact that the S end of the magnet repels the S end, but attracts the N end, of the needle. The net result is that the needle is balanced in the various positions shown. In Fig. 347b, the N and S ends of the magnet equally attract and repel the respective ends of the compass needle. In Fig. 347c the reverse of the condition in Fig. 347a exists. The theoretical magnitudes of the mechanical forces are directly proportional to

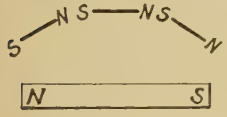


Fig. 346.

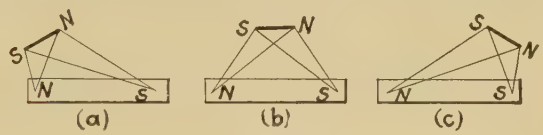


Fig. 347.

Fig. 346.—Compass needles in the field of a magnet.
Fig. 347.—Compass needles balanced by magnet.

the product of the strengths of the coacting poles, but are inversely proportional to the square of the distance between them. When filings or compass needles are exactly opposite the ends of magnets they point straight to those ends, the compass needles reversing their positions if forcibly placed with the wrong pole towards the magnet. This is the fundamental principle of all rotating magnetic mechanisms.

When filings are considered as being so small as to approach molecular dimensions, they may be represented as points on the paper. Then the compass needles in Fig. 347 would be so small that a line drawn from the N end of the magnet to the S end of the filing would coincide with a line drawn from the same end of the magnet to the N end of the filing. The same would be the case of lines drawn from the S end of the magnet to the N and S ends of the filing. This is illustrated in Fig. 348, which may be compared with Fig. 347.

Combining a, b, and c in Fig. 348, Fig. 349 is obtained. Hence, when any number of radial lines representing the directions in

which mechanical forces can act between magnets and magnetized particles are drawn from points near the ends of the magnet representing the poles of the magnet, wherever the straight lines drawn through the respective poles intersect a point representing a magnetized particle may be placed, and when curves are drawn

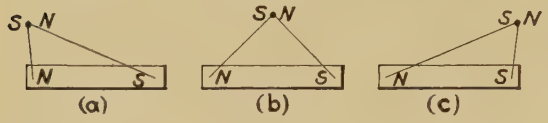


FIG. 348.

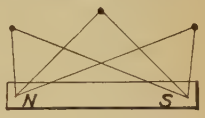


FIG. 349.

FIG. 348.—Compass needles represented as points.

FIG. 349.—Showing how radial mechanical forces act from the poles of a magnet on magnetized particles in the magnetic field.

through these points, as in Fig. 350, the direction of the curve will represent the direction of the magnetic flux in the magnetic field surrounding the magnet at that point. Consequently, when the steel filings are sprinkled on a sheet of paper placed in the magnetic field of a magnet, each filing becomes a magnet and the radial forces of mutual attraction and repulsion between the

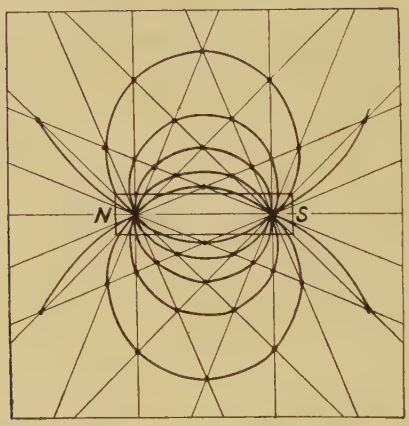


FIG. 350.—Construction of the magnetic-flux paths in the magnetic field surrounding a magnet.

filings and the magnet cause them to turn into the positions shown. Moreover, each molecule in the steel must similarly be affected, since the magnetizing field permeates the structures of all substances.

If the mechanical forces acted only from points near the ends of the magnets, the shapes of the magnetic-flux paths would be

exactly as in Fig. 350. However, these forces do not act from theoretical points but from surfaces extending over each half of the magnet, varying from a maximum at points near the ends to 0 at the neutral-polarity plane.

219. Effects of Air Gaps in Series with Magnetic Materials. The sum of the reluctances of any number of air gaps in series in a ferromagnetic circuit is the total air-gap reluctance of that circuit. The reciprocal of the air-gap reluctance is the air-gap permeance. Both are constant when the air-gap dimensions are constant.

Since the permeance of the leakage magnetic-flux paths is added to the permeance of the air gaps to produce the total air permeance, the reciprocal of the total air permeance is the total

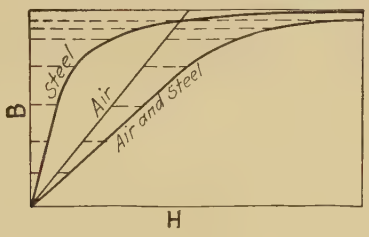


FIG. 351.—Saturation curve of steel and air in series.

air reluctance. Hence, magnetic leakage is included as a part of the air reluctance in estimating the total air reluctance of a magnetic circuit, and this is generally charged to the air gap or gaps.

To find the saturation $(B-H)$ curve of a magnetic material in series with air gaps, draw the normal saturation curve of the magnetic material, as steel, without any air gaps in the magnetic circuit and also draw a straight line expressing $\frac{B}{H} = 1$ for the air gap or gaps, as in Fig. 351. Then add together the magnetizing forces for both the magnetic material and the air gap or gaps. The result is shown in the lower curve, which is the saturation curve for a magnetic circuit consisting partly of magnetic material and partly of non-magnetic materials, as a powdered-iron core of the type described in Art. 213.

CHAPTER XX

ALTERNATING-CURRENT PHENOMENA

Methods of producing alternating currents and voltages are discussed; the phase relations between the voltage and the current with only resistance, only self-inductance, and only capacitance, respectively, are treated, and then follows an introduction to the general method of treatment of the subject.

220. Alternating Current.—By means of the connections in Fig. 352, a condenser may first be charged and then discharged in

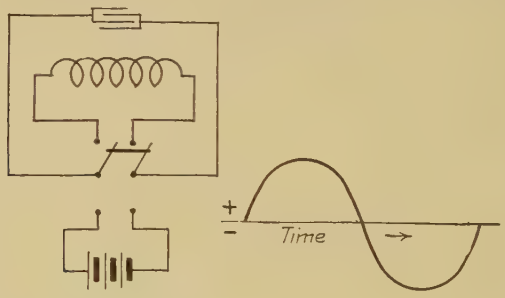


Fig. 352.—Method of producing a harmonic alternating current.

series with a reactor. When the charged condenser is first connected with the reactor, the self-inductance of the reactor prevents the current from instantly attaining a maximum strength, but permits it to increase at a time-rate proportional to the ratio of the voltage of the condenser to the self-inductance of the reactor. As the electric energy stored in the dielectric of the condenser is discharged therefrom and converted into magnetic energy in the magnetic circuit of the reactor, the voltage of the condenser decreases. When the condenser is completely discharged, the self-inductance of the reactor keeps the electricity in motion, with the result that the condenser begins to charge in the opposite direction. The storage of electric energy in the dielectric of the condenser through the displacement of the electricity elastically bound therein causes a counter-

voltage to exist across the dielectric which tends to oppose the displacement of electricity in the dielectric and, therefore, the flow of electricity in the entire circuit. But each increase in the dielectric voltage is opposed by an equal voltage induced in the reactor due to the decrease in the magnetic flux linked with the ampere-turns as the current strength decreases. Naturally, the dielectric is strained the most when the electricity is brought to a state of rest by the increasing voltage of the dielectric.

The time-rate of change in a quantity is greatest when it changes from zero to any finite quantity, and *vice versa*. Consequently, the self-induced voltage of the reactor is a maximum when the current strength is equal to zero, at which instant the condenser is completely charged and the dielectric voltage is a maximum. Hence, at all instants the respective voltages are numerically equal, but opposed, to each other, except at the instant when both are equal to zero. At that instant the current strength is a maximum.

With no electric resistance or other cause of energy loss in the circuit, the electric energy stored in the dielectric of the condenser would be alternately and periodically converted into magnetic energy in the magnetic circuit of the reactor, and thence back into electric energy in the dielectric of the condenser, for an indefinite period of time; but some electric energy would be stored in the dielectric of the condenser, and some magnetic energy would be stored in the magnetic circuit of the reactor, at all instants except when the current strength was equal to zero and all the energy would be stored in the dielectric of the condenser, or at the instant when the current strength was a maximum and all of the energy would be stored in the magnetic circuit of the reactor.

The balance wheel and hair spring of a watch form an excellent analogy, considering the inertia of the balance wheel as the self-inductance, the hair spring as the dielectric, and the velocity of the balance wheel as the current strength. The energy of the system is alternately and periodically transferred from the hair spring (where it is stored in the form of potential energy) to the balance wheel (where it is stored as kinetic energy). The to-and-fro motion of the balance wheel is analogous to the to-and-fro motion of the electricity. When the balance wheel is moving at its maximum angular velocity, the spring is neither under tension nor compression.

In all electric circuits containing self-inductance and capacitance, the electricity tends to move to and fro at a natural frequency. When an alternating voltage is impressed upon such a circuit at any other frequency than the natural frequency, the electricity tends to oscillate at the natural frequency but is compelled to oscillate at the frequency of the impressed voltage. However, when an alternating voltage is impressed upon such a circuit at the natural frequency of that circuit, the resulting current strength at any instant is equal to the ratio of the impressed voltage to the resistance in ohms at that instant, since the self-induced voltage and the dielectric voltage exactly neutralize each other at all instants. This condition is called resonance.

221. Oscillating Circuit.—The square root of the product of the self-inductance in henries and the capacitance in farads is called the oscillation constant. The condition for resonance is that the product of the oscillation constant and the frequency in cycles per second is equal to the number 0.159 16. Hence, the oscillation constant is found by dividing the number 0.159 16 by the frequency in cycles per second, while the natural frequency is found by dividing the number 0.159 16 by the oscillation constant.

The product of the self-inductance in henries and the capacitance in farads is, of course, equal to the square of the oscillation constant and, therefore, is equal to the quotient obtained by dividing the number 0.025 33 by the square of the frequency in cycles per second. Hence, the number of henry-farads is found by dividing the number 0.025 33 by the square of the frequency in cycles per second, while the natural frequency is found by dividing the number 0.025 33 by the number of henry-farads and then extracting the square root of the quotient thus found.

For instance, the product of the self-inductance and the capacitance of a circuit having a natural frequency of 60 cycles per second is equal to $\frac{0.025\ 33}{3{,}600} = 0.000\ 007\ 036$ henry-farad = 7.036 henry-mf. = 7,036 millihenry-mf. Conversely, the natural frequency of a circuit containing 0.000 007 036 henry-farad is equal to $\sqrt{\frac{0.025\ 33}{0.000\ 007\ 036}} = \sqrt{3{,}600} = 60$ cycles per second.

222. Production of the Impressed Alternating Voltage.—This is accomplished by an alternating-current generator, wherein magnetic flux alternately and periodically links and unlinks with

electric conductors. One method of accomplishing the above is to rotate a loop of wire at a constant angular velocity in a magnetic field of constant strength, as in Fig. 353. This method is referred to as generator action. The production of voltage by varying the current and flux is termed transformer action. One complete revolution constitutes one cycle. Hence, there are 360 deg. per cycle. The angle whose arc is equal to the radius is the radian, or 57.296 deg. Hence, there are $\frac{360}{57.296} = 6.2832$ radians per cycle. The product of the number of radians per cycle and the frequency in cycles per second is the angular velocity. Consequently, the angular velocity is equal to 6.2832 times the number of cycles per second. When the frequency is 60 cycles per second, the angular velocity is equal to $6.2832 \times 60 = 377$.

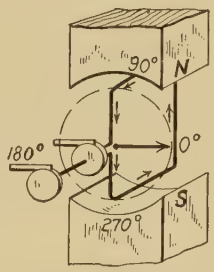


FIG. 353.—Production of alternating current by rotation.

Referring to Fig. 353, at 0 deg. the linkage of magnetic flux with the single turn of conductor is a maximum. When the loop or turn has rotated through 90 deg. the linkage of magnetic flux with the loop has decreased to zero. From 90 to 180 deg., the linkage increases in the reverse direction, becoming a maximum at 180 deg. The linkage then decreases in this reverse direction from 180 deg. until at 270 deg. the linkage is again equal to zero, becoming a maximum in the original direction at 360 deg.

But the voltage induced in the looped conductor is, at any instant, proportional to the time-rate of change in the linkage of magnetic flux with the electricity in the looped conductor, and this time-rate is greatest when the linkage is changing from 0 to a finite quantity, and *vice versa*. When the linkage is a maximum, the time-rate of change in the linkage is equal to 0 and the voltage is equal to 0. When the linkage is equal to 0, as shown in Fig. 353, the induced voltage is a maximum. Consequently, there are four complete changes in the linkage during each cycle, and, since the induced voltage is equal to the time-rate of change in the number of weber-turns in series, it follows that the average voltage during any cycle is equal to four times the product of the maximum number of weber-turns in series and the frequency in cycles per second.

The above statement may be better understood when presented in a slightly different form. Since there are four changes in the linkage of weber-turns per cycle, only one quarter cycle need be considered. But the time required for one quarter cycle is but one-quarter of the time required for one cycle. Therefore, the time-rate of change during each quarter cycle is equal to four times the time-rate for the time based on the number of cycles per second. The statement regarding the number of weber-turns in series refers to the number of turns in series linked with the number of webers.

At first the instantaneous values of the voltage and the resulting current strength will be considered.

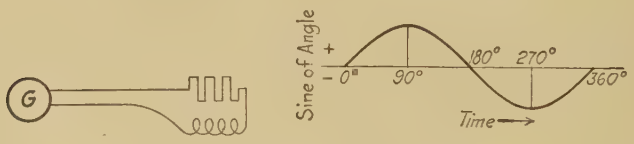


Fig. 354.—Single-phase circuit.

223. Phase.—If there is but one alternating voltage generated in or impressed upon a simple two-wire circuit, the system is single-phase. When two or more coils are placed at various angles with respect to one another and are rotated in a magnetic field, the system is polyphase. When two coils placed 90 deg. apart generate two separate voltages displaced by 90 electrical degrees, as in a four-wire system, the system is either two-phase or

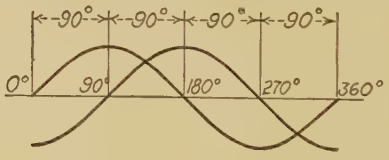


Fig. 355.—Two-phase currents.

quarter phase. When three coils placed 120 deg. apart generate three voltages displaced by 120 electrical degrees in a three-wire circuit, the system is three-phase.

Figure 354 shows a single-phase circuit with a generator at one end, and a load at the other end, of the line. Polyphase systems are discussed further on. However, two-phase and three-phase voltage or current waves are respectively shown in Figs. 355 and 356. Polyphase systems may be reduced to terms of

single-phase systems, the calculations made on a single-phase basis, and the single-phase results then converted into those for polyphase systems.

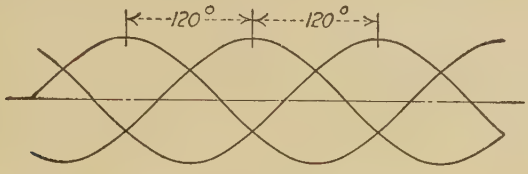


FIG. 356.—Three-phase waves.

224. Phase Difference.—This is determined by the difference in the angles where two phenomena are equal to 0. The one first arriving at 0, when both approach zero from the same direction, leads while the other lags.

225. Single-phase Relations.—In what follows, the maximum magnitude of a quantity is expressed by a capital letter with suitable subscripts, and the instantaneous value is expressed by the equivalent small letter with subscripts. Thus $E_{i\,max}$ and e_i are the maximum and instantaneous magnitudes, respectively, of the impressed voltage.

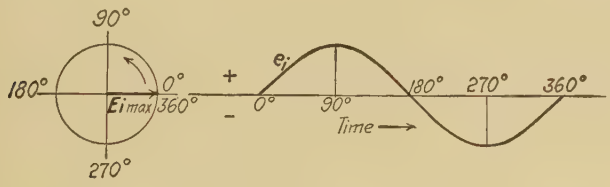


FIG. 357.—Sinusoidal voltage wave.

The maximum magnitudes of voltage and current strength are equal to 1.5708 times the average magnitudes. Hence, the maximum induced voltage is equal to 1.5708 times the average voltage, or to 6.2832 times the product of the maximum number of weber-turns in series and the frequency in cycles per second.

In Fig. 357, the length of the vector $E_{i\,max}$ represents the maximum magnitude of the generated or impressed voltage. When this vector is rotated in the conventional counterclockwise direction, the product of the length of the vector and the sine of the angle through which it has rotated up to any instant at a constant angular velocity gives the magnitude of the impressed voltage e_i at that instant. It is as though the terminal of the vector simply moves upward and downward on the paper, like the

swing of a pendulum, as the paper is moved to the left as time elapses. In fact, the sine curve or sinusoid at the right is the record which a swinging pendulum would leave in the absence of friction. If the circuit is not closed, there will be no resulting current.

In a closed alternating-current circuit containing only constant resistance,[1] as in Fig. 358, the sinusoidal impressed voltage will cause a current to flow in phase with it, the strength of the current at any instant being equal to the ratio of the instantaneous

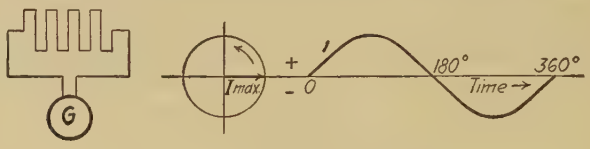


Fig. 358.—Sinusoidal current wave.

voltage to the resistance at that instant. The active voltage under such circumstances is equal to the impressed voltage at all instants. This is represented in Fig. 358, wherein I_{max} and i are the respective maximum and instantaneous current strengths. The active voltage is always in phase with the current strength. When dealing with voltages, the horizontal axis is always taken as the axis of current.

In an alternating-current circuit containing only constant self-inductance, as in Fig. 359, the instantaneous magnitude of the

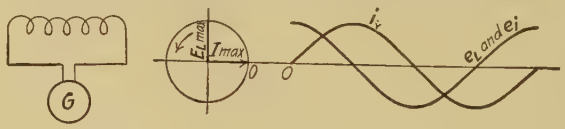


Fig. 359.—Single-phase relations with only self-inductance present.

e.m.f. of self-induction is equal to the product of the self-inductance in henries and the time-rate of change in the current strength in amperes per second at any instant, and also is equal to the impressed voltage. The cosine curve e_L represents the rate of change in the slope of the sine curve i for equal angles and times, and the values of the cosine curve are 90 deg. ahead (leading) of the values of the sine curve. Hence, when the instantaneous values of the current strength are represented by a sine curve, the instantaneous magnitudes of the e.m.f. of self-induction are represented by a cosine curve, the current lags 90 deg. behind the

[1] Cyclic changes occur in lamp loads.

e.m.f. of self-induction, and they are said to be in time quadrature. These phase relations are shown in Fig. 359, wherein $E_{L\,max}$ and e_L are, respectively, the maximum and instantaneous magnitudes of the e.m.f. of self-induction.

The time-rate of change in the current strength is a maximum when the current strength is equal to 0, and it is equal to 0 when the current strength is a maximum. Upward slopes have positive rates of change, while downward slopes have negative rates of change.

The magnetic flux is always in phase with the current when the self-inductance is constant, and both then always lag behind the e.m.f. of self-induction by 90 deg. unless the self-inductance is changing, as when the permeability of a steel core within a coil changes with corresponding changes in the magnetizing force.

In an alternating-current circuit containing only constant self-inductance, the e.m.f. of self-induction is in phase with the

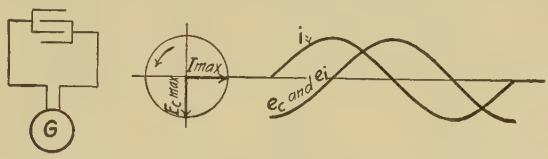


FIG. 360.—Single-phase relations with only capacitance present.

impressed voltage. Under all other circumstances, the two voltages are not in phase. Since electric resistance always is present, the e.m.f. of self-induction never is exactly in phase with the impressed voltage.

Referring to Fig. 359, the instantaneous value of the current strength is equal to the product of the maximum current strength I_{max} and the sine of the angle; the instantaneous value of the e.m.f. of self-induction is equal to the product of the maximum voltage $E_{L\,max}$ and the cosine of the angle, or to the product of the self-inductance in henries and the time-rate of change in the current strength in amperes per second; and the time-rate of change in the current strength at any instant is equal to the ratio of the e.m.f. of self-induction to the constant self-inductance at that instant.

In an alternating-current circuit containing only capacitance, as in Fig. 360, the current always leads the e.m.f. of capacitance by 90 deg., that is, they are in time quadrature. Hence, the e.m.f. of capacitance and the e.m.f. of self-induction always oppose each

other in circuits containing both self-inductance and capacitance. In Fig. 360, $E_{c\,max}$ and e_c are the respective maximum and instantaneous magnitudes of the e.m.f. of capacitance. In an alternating-current containing only constant capacitance, the e.m.f. of capacitance is equal to, and in phase with, the impressed voltage However, this ideal condition never is realized because there is always some resistance present.

In an alternating-current circuit containing both self-inductance and capacitance, as in Fig. 361, the e.m.f. of self-induction opposes the e.m.f. of capacitance, and the current tends to become very great as the two e.m.fs. tend to balance each other, unless there is sufficient resistance present to limit the current strength.

When the two e.m.fs. are numerically equal to one another, series resonance, or voltage resonance, occurs and the electricity

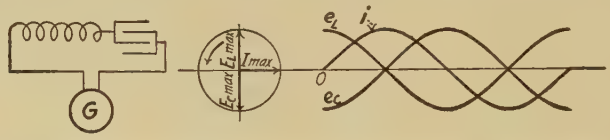


Fig. 361.—Single-phase relations with self-inductance and capacitance in series.

oscillates at the natural frequency of the circuit. This is due to the fact that the self-inductance and the capacitance are so proportioned that the natural frequency of the circuit is equal to the frequency of the impressed voltage and, consequently, there are no reactions on the impressed voltage due to the tendency of the electricity to remain in motion when moving, or to remain in a state of rest when not moving, due to self-inductance, or to its tendency to rush instantly at maximum current strength in the presence of capacitance and the absence of self-inductance. The only reaction is due to the presence of some resistance, and the active voltage is equal to and in phase with the impressed voltage at all instants. Therefore, the current is in phase with the impressed voltage.

The foregoing may be summed up as follows:

1. When an alternating-current circuit contains only constant resistance, the active voltage is equal to the impressed voltage, and is in phase therewith, the current also being in phase with the impressed voltage.

2. When an alternating-current circuit contains only constant self-inductance, the e.m.f. of self-induction is equal to the

impressed voltage, and is in phase therewith, the current lagging 90 deg. behind the voltage.

3. When an alternating-current circuit contains only constant capacitance, the e.m.f. of capacitance is equal to the impressed voltage, and is in phase therewith, the current leading the voltage by 90 deg.

4. In an alternating-current circuit containing both self-inductance and capacitance (*a*) the e.m.f. of self-induction and the e.m.f. of capacitance oppose each other, that is, their phase difference is 180 deg., (*b*) the e.m.f. of self-induction leads the current by 90 deg., and (*c*) the e.m.f. of capacitance lags behind the current by 90 deg.

5. When the e.m.f. of self-induction and the e.m.f. of capacitance are numerically equal (*a*) the current is in phase with the impressed voltage, (*b*) the active voltage is in phase with the impressed voltage and is numerically equal thereto, (*c*) the e.m.f. of self-induction leads the impressed voltage by 90 deg. (*d*) the e.m.f. of capacitance lags 90 deg. behind the impressed voltage, and (*e*) the current strength at any instant is equal to the ratio of the impressed voltage to the resistance at any instant.

6. The current and flux are always in phase with each other when the self-inductance is constant, as when there is no magnetic material in the magnetic circuit.

7. The active voltage is always in phase with the current.

8. The current always lags 90 deg. behind the e.m.f. of self-induction.

9. The current always leads the e.m.f. of capacitance by 90 deg.

10. The current is in phase with the impressed voltage only when (*a*) there is only resistance in the circuit, and (*b*) when voltage resonance occurs.

All waves are assumed to be sinusoidal.

226. Energy Conversions and Volt-amperes.—When an alternating voltage is impressed only upon a resistor, all of the electric energy is converted into heat, regardless of the instantaneous magnitudes and directions of the impressed voltage and the current, that is, the energy conversion is cumulative and there are no energy reactions or reconversions. When an alternating voltage is impressed upon a reactor, electric energy is converted into magnetic energy during the first quarter cycle. During the next quarter cycle, the magnetic energy is converted into electric energy. During the following alternation or half cycle, the above

is repeated, the only difference being that of direction of the power. When an alternating voltage is impressed upon a condenser, electric energy is alternately and periodically stored in and discharged from the dielectric of the condenser.

These energy conversions and transferences in reactors and condensers are real, and their time-rates during any quarter cycle can be expressed as true power in watts. However, the net energy converted, stored, or otherwise transferred in either a reactor or a condenser during any half cycle, and, therefore, during any cycle, is equal to zero. In alternating-current circuits containing reactors and condensers, either in series or in parallel, the reactor always receives its store of energy during the quarter cycle in which the condenser is discharging its energy, and *vice versa*. Consequently, the condenser discharges at least some of its energy into the reactor, and *vice versa*. When the energy stored in the reactor during any quarter cycle is equal to the energy discharged from the condenser during the same quarter cycle, and *vice versa*, no electric energy is abstracted from the main source of electric energy after the initial store is supplied to the reactor or to the condenser, except to convert electric energy into heat or work.

While all of the above time-rates of conversion, transference, and storage of energy can be expressed in watts, the net conversions and storages of energy in perfect reactors and condensers are equal to zero (except during the last quarter cycle of any series of cycles) and the product of the impressed voltage and the current strength is called the number of volt-amperes. In alternating-current circuits containing resistance, self-inductance, and capacitance, the number of volt-amperes which expresses the time-rate of conversion of electric energy into work and heat is the true power component of the total number of volt-amperes.

227. Algebraic and Geometric Addition and Subtraction.— The algebraic sum of 4 and 3 is 7, and their algebraic difference is 1. The algebraic sum of 4 and -3 is 1, and their algebraic difference is 7. But the geometric sum of 4 and 3 is equal to the square root of the sum of their squares and, therefore, is equal to $\sqrt{16 + 9} = \sqrt{25} = 5$, and their geometric difference is equal to $\sqrt{7} = 2.646$. Likewise, the geometric sum of 4 and -3 is equal to $\sqrt{16 + (-3)^2} = \sqrt{16 + 9} = 5$, and their geometric difference is equal to $\sqrt{16 - 9} = 2.646$, the square of a negative quantity always being positive.

The geometric sum of two positive quantities is always less than their algebraic sum, but their geometric difference is always greater than their algebraic difference.

The geometric sum of a positive and a negative quantity is the same as though they were both positive, because the square of a negative quantity is positive. Likewise, the geometric difference of two quantities is the same whether or not the quantities are positive or negative.

Instantaneous values of voltage and current strength may be algebraically added and subtracted, but maximum, average, and effective (r.m.s.) values must be geometrically added or subtracted.

228. Components of the Impressed Voltage.—The active voltage is the component of the impressed voltage which is in

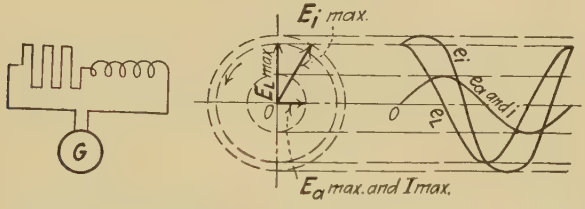


FIG. 362.—Phase relations with resistance and self-inductance in series.

phase with the current. The e.m.f. of self-induction is the leading reactive or quadrature component of the impressed voltage. The e.m.f. of capacitance is the lagging reactive or quadrature component of the impressed voltage.

The active voltage is equal to the product of the resistance and the current strength; the e.m.f. of self-induction is equal to the product of the self-inductance and the time-rate of change in the current strength; and the e.m.f. of capacitance is equal to the product of the elastance and the quantity of electricity displaced in the dielectric.

Figure 362 shows the phase relations in a circuit containing resistance and self-inductance in series. Let the maximum value of the e.m.f. of self-induction be 100 volts and let that of the active voltage be 50 volts. The instantaneous values of the impressed voltage are found by algebraically adding together the instantaneous values of the active voltage and the e.m.f. of self-induction, but the maximum value of the impressed voltage may be found by geometrically adding together the maximum values

of the active voltage and the e.m.f. of self-induction. Thus, the maximum value of the impressed voltage is equal to

$$\sqrt{10{,}000 + 2{,}500} = \sqrt{12{,}500} = 111.8 \text{ volts.}$$

The phase relations of a circuit containing resistance and capacitance in series are shown in Fig. 363. Let the respective maximum values of the active voltage and the e.m.f. of capacitance be 50 and 100. Then the maximum impressed voltage will be 111.8, as in the case of resistance and self-inductance in series, but the phase relations with respect to the current will be entirely different, as noted.

If, in the circuit in Fig. 364, the maximum active voltage is 50 volts, the maximum e.m.f. of self-induction is 150 volts, and the maximum e.m.f. of capacitance is 50 volts, the net maximum

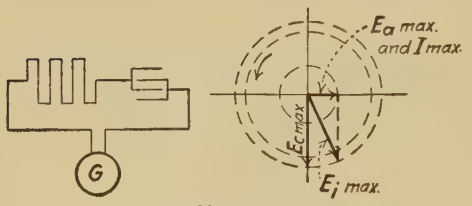


Fig. 363.

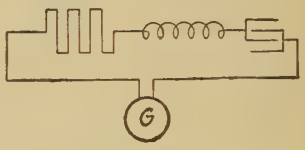


Fig. 364.

Fig. 363.—Phase relations with resistance and capacitance in series.
Fig. 364.—Conditions for the example.

reactive or quadrature voltage will be equal to 150 volts (leading) − 50 volts (lagging) = 100 volts (leading, 90 deg. in advance of the current and the active voltage) and the maximum impressed voltage will be the same as in the case of Fig. 362. Aside from the fact that the condenser is present, all conditions and phase relations will be the same as in Fig. 362.

The lengths of the vectors in Fig. 363 show the relative magnitudes of the various voltages or current strengths, and the angles between them are the phase angles. When such a group of vectors is rotated, all of the instantaneous values of the various phenomena may be found by taking the sines, cosines, or tangents of the various angles between the vectors and the horizontal axis, the angles being measured in the conventional counterclockwise direction from 0 deg. Figure 262, for instance, is laid out on this principle.

229. Maximum, Average, and Effective Values.—The average or arithmetical mean values of voltage and current strength dur-

ing each half cycle is equal to $\frac{2}{3.1416} = 0.637$ times the maximum value.

Except for the duration of one half cycle, the electric energy which is alternately and periodically converted into magnetic energy, and *vice versa*, when the self-inductance is constant (in the aggregate) is not converted at all. The same general statement holds for the electric energy which is alternately and periodically stored in and discharged from a dielectric when the capacitance is constant. However, the electric energy converted into heat is converted at a time-rate which, at any instant, is equal to the product of the resistance and the square of the current strength at that instant.

When the current-time relations can be represented by a sine curve, as here assumed, the electrothermal or heating power in watts is equal to 0 when the current strength is 0. If the current strength is 10 amp. and the resistance is, say, 5 ohms, then at the instant the current strength becomes 10 amp. the power is 500 watts (0.5 kilowatt, or 0.5 kw.). Consequently, if the instantaneous values of the current strength throughout the half cycle are squared and then multiplied by the resistance, the electrothermal power will be equal to $\frac{500 + 0}{2} = 250$ watts. This is the average time-rate at which the conductor will be heated. But it would be heated at the same time-rate if a direct current flowed in its structure. What would be the equivalent strength of the direct current required?

The resistance is 5 ohms. Hence, the square of the current strength would be equal to $\frac{250 \text{ watts}}{5 \text{ ohms}} = 50$, and the current strength would be equal to the square root of 50, or to 7.07 amp. The ratio of the maximum current strength to the effective or root-mean-square (r.m.s.) value of the current strength is, therefore, equal to $\frac{10}{7.07} = 1.41$, and the r.m.s. value of the current strength is equal to 0.707 times the maximum current strength.

The r.m.s. active voltage is equal to the product of the r.m.s. current strength and the resistance. Similarly, all sinusoidal voltages may be expressed in terms of their r.m.s. values, which are equal to 0.707 times their maximum sinusoidal values.

The foregoing may be taken directly from sine or cosine curves or tables. The maximum value of any sine or cosine is 1 and the minimum value is 0. Therefore, the square root of the average sum of their squares, as measured over a half cycle, is equal to $\sqrt{\frac{0+1}{2}} = \sqrt{0.5} = 0.707$. Hence, 0.707 is the square root of ½ and 1.41 is the square root of 2.

Multiplying the maximum value of the voltage or current strength by 0.707 gives the r.m.s. or effective value.

Alternating-current ammeters and voltmeters give direct readings in terms of the r.m.s. values. These readings must be multiplied by 1.41 when it is desired to know the maximum values. The maximum value of the voltage always should be taken in calculating the dielectric strength of an insulating material from test data.

The use of the r.m.s. or effective values of the various electric and magnetic phenomena greatly simplifies alternating-current calculations because the instantaneous values do not generally have to be considered.

Table XX shows the relations between the various values, the numbers expressing the ratios of the values at the top to those in the first column.

TABLE XX.—RELATIONS OF THE VARIOUS VALUES

	Maximum	Average	Effective
Maximum	1.0	0.737	0.707
Average	1.57	1.0	1.11
Effective	1.41	0.901	1.0

The symbols used for the r.m.s. values of the various voltages and the constant current strength are given in Table XXI.

TABLE XXI.—SYMBOLS FOR THE EFFECTIVE VALUES

E_i = impressed voltage.
E_a = active voltage.
E_L = e.m.f. of self-induction.
$-E_L$ = self-induced voltage.
E_c = e.m.f. of capacitance.
$-E_c$ = dielectric voltage.
I = total current strength in amperes.

230. The Self-induced Voltage and the Maximum Number of Weber-turns.—The average self-induced voltage (or its balancing e.m.f. of self-induction) during any cycle is equal to four times the product of the maximum number of weber-turns in series and the frequency in cycles per second, since there are four complete changes in the linkage during each cycle. The r.m.s. self-induced voltage is equal to 1.11 times the average self-induced voltage. Therefore, the r.m.s. self-induced voltage is equal to 4.44 times the product of the maximum number of weber-turns in series and the frequency in cycles per second. The r.m.s. current strength in amperes is equal to 0.707 times the maximum current strength in amperes. Therefore, the ratio of the r.m.s. self-induced voltage to the r.m.s. current strength in amperes is equal to $\frac{4.44}{0.707} = 6.2832$ times the product of (1) the frequency in cycles per second and (2) the ratio of the maximum number of weber-turns to the maximum current strength in amperes. But (2) is the self-inductance in henries. Consequently, the ratio of the r.m.s. e.m.f. of self-induction (when the sign is changed) in volts to the r.m.s. current strength in amperes is equal to 6.2832 times the product of the frequency in cycles per second and the self-inductance in henries.

But 6.2832 times the frequency in cycles per second is the angular velocity in radians per second. Therefore, the ratio of the r.m.s. e.m.f. of self-induction in volts to the r.m.s. current strength in amperes is equal to the product of the angular velocity in radians per second and the self-inductance in henries, and the e.m.f. of self-induction in r.m.s. volts is equal to the continued product of the angular velocity in radians per second, the self-inductance in henries, and the r.m.s. current strength in amperes.

When there is only self-inductance in an alternating-current circuit, the r.m.s. e.m.f. of self-induction is equal to and in phase with the r.m.s. impressed voltage. When the frequency is 60 cycles per second, the angular velocity is 377 radians per second.

Since the r.m.s. induced voltage is equal to 4.44 times the product of the maximum number of weber-turns and the frequency in cycles per second, the number of r.m.s. induced volts per maximum weber-turn can never exceed $4.44 \times 60 = 266.4$ in a 60-cycle circuit. This statement refers to all kinds of sinusoidal induced voltages and the balancing e.m.fs., as the e.m.f. of self-induction. To find the maximum number of r.m.s. induced volts per maximum weber-turn in any case, multiply

the frequency by 4.44. To find the number of maximum weber-turns per induced r.m.s. volt, divide $\frac{1}{4.44} = 0.225$ by the frequency in cycles per second. Thus, the number of maximum weber-turns per r.m.s. induced volt in a 60-cycle circuit is equal to $\frac{0.225}{60} = 0.003\ 75$. The maximum number of weber-turns in a 110-volt, 60-cycle circuit, even when only self-inductance is present, is equal to $0.003\ 75 \times 110 = 0.412$, and it can never exceed this number. For instance, if a coil contains 1,000 turns, the maximum number of webers that could be linked with all of the turns under the above conditions would be equal to $\frac{0.412}{1,000} = 0.000\ 412$, or 41,200 maxwells, which is just about enough magnetic flux to saturate 2 sq. cm. of cross-section of steel.

231. Skin Effect and Alternating-current Resistance.—When an alternating current which is linked with magnetic flux flows in the structure of a conductor under the action of an alternating voltage, the electromagnetic inertia of the electricity causes it to be forced towards the surface of the conductor. The effect is to decrease the effective cross-section of the conductor, and thus increase the effective resistance. The phenomenon is called the skin effect.

The skin effect in steel wires is much more pronounced than in copper wires. For ordinary commercial frequencies and sizes of copper wires, the alternating-current resistance may be assumed to be about the same as the direct-current resistance, since slight changes in temperature will cause greater changes in resistance than does the skin effect. When the product of the frequency in cycles per second and the number of circular mills of conductor is equal to 10,000,000, the alternating-current and direct-current resistances are considered equal to one another. When the above product of frequency and cross-section is equal to 20,000,000, the ratio of the alternating-current resistance to the direct-current resistance is 1.01.

The resistance in ohms is equal to the ratio of the r.m.s. active voltage to the r.m.s. current strength in amperes; the r.m.s. active voltage is equal to the product of the resistance in ohms and the r.m.s. current strength in amperes, and the r.m.s. current strength in amperes is equal to the ratio of the r.m.s. voltage to the resistance in ohms.

CHAPTER XXI

SERIES AND PARALLEL ALTERNATING-CURRENT CIRCUITS

By using the effective values of voltage and current strength, alternating-current phenomena may be readily treated by simple trigonometrical methods or by the use of vectors. Only the simplest methods are herein employed. The subjects of power-factor correction by means of static condensers and polyphase systems are briefly discussed.

232. Phase Angle θ.—The angle between the impressed or total voltage and the total current strength is the phase angle. In an alternating-current circuit containing only self-inductance, the phase angle is 90 deg. lagging, meaning that the current lags 90 deg. behind the impressed voltage. In an alternating-current circuit containing only capacitance, the phase angle is 90 deg. leading, meaning that the current leads the impressed voltage by 90 deg. In an alternating-current circuit containing only resistance, or resistance, self-inductance, and capacitance with the e.m.fs. of self-induction and capacitance balancing each other, the phase angle is equal to 0, the current and the impressed voltage are in phase, and all of the volt-amperes are true watts, or in-phase volt-amperes, because all the electric energy is being converted into work and heat.

The e.m.f. of self-induction (in leading quadrature with the current) is equal to the product of the impressed voltage and the sine of the phase angle θ.[1] The active voltage (in phase with the current) is equal to the product of the impressed voltage and the cosine of the phase angle θ. Hence, the ratio of the e.m.f. of self-induction to the active voltage is equal to the ratio of the sine to the cosine of the phase angle θ, that is, to the tangent of the phase angle θ. Consequently, the ratio of the e.m.f. of self-induction to the active voltage is the tangent of the phase angle θ by which the current lags behind the impressed voltage, while the ratio of the e.m.f. of capacitance to the active voltage is the tangent of the phase angle θ by which the current leads the impressed voltage.

[1] Greek letter theta.

The tangent of the phase angle θ is equal to the ratio of any quadrature component to any in-phase component. The actual angle θ is found by referring to a table of tangents.

In a certain alternating-current circuit, the resistance R is 10 ohms, the impressed voltage E_i is 110 volts, and the current strength I is 5 amp. The active voltage E_a is equal to 5 amp. $\times$ 10 ohms = 50 volts. The quadrature voltage is equal to $\sqrt{110^2 - 50^2} = \sqrt{12{,}100 - 2{,}500} = \sqrt{9{,}600} = 98$ volts. The tangent of the phase angle θ is equal to $\frac{98 \text{ volts}}{50 \text{ volts}} = 1.96$. Referring to a table of tangents, the phase angle θ is found to be equal to 62 deg., 58 min. If the quadrature voltage is due to the

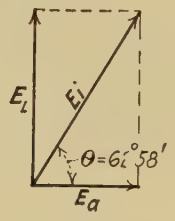


Fig. 365.

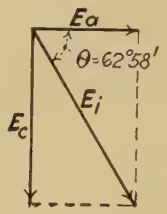


Fig. 366.

Fig. 365.—Quadrature voltage due to self-inductance.
Fig. 366.—Quadrature voltage due to capacitance.

presence of self-inductance in the circuit, the current will lag 62 deg. 58 min., behind the impressed voltage. If the quadrature voltage is due to the presence of capacitance in the circuit, the current will lead the impressed voltage by 62 deg., 58 min. Figures 365 and 366 show the relations.

233. The Use of Symbols.—Unless otherwise stated, all values of voltage, current, and power are r.m.s. or effective, the symbols in Table XXII representing the various quantities.

Table XXII.—General Symbols

E_i = impressed voltage.	E_iI = total volt-amperes (v.-a.).
E_a = active voltage.	Sin θ = sine of angle θ.
E_L = e.m.f. of self-induction.	Cos θ = cosine of angle θ.
E_c = e.m.f. of capacitance.	= power factor.
I = total current strength.	Tan θ = tangent of angle θ.
θ = phase angle.	ω = angular velocity in radians per second.
P = power in watts.	qv.-a. = quadrature volt-amperes.

234. Power Factor (Cos θ).—The power factor of an alternating-current circuit, or any component thereof, is the ratio of the true power P in watts (as measured with a wattmeter) to the

total number of volt-amperes E_iI (as separately measured with a voltmeter and an ammeter).

The true power component P of the total number of volt-amperes E_iI is equal to the product of the active voltage E_a and the total current strength I.

The quadrature component qv.-a. of the total number of volt-amperes E_iI is equal to the product of the quadrature voltage (E_L or E_c) and the total current strength I.

The active voltage E_a is equal to the product of the impressed voltage E_i and cos θ, or $E_a = E_i \cos \theta$. Consequently, the power P in watts is equal to $E_iI \times \cos \theta$, or $P = E_iI \cos \theta$.

The quadrature (or wattless) component qv.-a. of the total number of volt-amperes E_iI is equal to $E_iI \times \sin \theta$, or qv.-a. = $E_iI \sin \theta$. qv.-a. may be equal to either E_LI or to E_cI, depending upon the predominance of E_L or E_c.

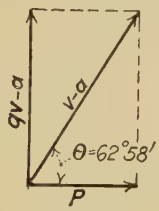


FIG. 367.

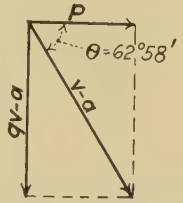


FIG. 368.

FIG. 367.—Power with current lagging due to self-inductance.
FIG. 368.—Power with current leading due to capacitance.

In an alternating-current circuit containing only resistance R, the power factor cos θ is unity and, therefore, a maximum. In a circuit containing only self-inductance L, the power factor cos θ is equal to 0 and I lags 90 deg. behind E_i. When only capacitance C is present, cos $\theta = 0$ and I leads E_i by 90 deg. In a circuit containing R, L, and C, the nearer E_L balances E_c the nearer does cos θ approach 1, or 100 per cent.

In the example in Art. 232, θ = 62 deg., 58 min. Referring to a table of cosines, cos θ = 0.455, or 45.5 per cent, and this is the power factor. In the same example, $E_i = 110$ and $I = 5$. Hence, $E_iI = 110 \times 5 = 550$ v.-a., and the true power is $P = 550 \times 0.455 = 450.3$ watts. The sine of 62 deg., 58 min. is 0.891, and may be called the quadrature volt-ampere factor or the qv.-a. factor. Hence, the number of quadrature volt-amperes is qv.-a. = $550 \times 0.891 = 494.5$. The phase relations

are shown in Figs. 367 and 368, which are the same as Figs. 365 and 366 multiplied by the current strength I.

235. The Reactances.—These are the apparent resistances which limit the current strength in alternating-current circuits without converting electric energy into heat. They are of two kinds, namely, inductive reactance X_L and condensive reactance X_c. The former may simply be called reactance and the latter condensance. The unit of each kind of reactance is the ohm.

236. Reactance X_L.—The reactance X_L in ohms is equal to 6.2832 times the product of the frequency f in cycles per second and the self-inductance L in henries, or to the product of the angular velocity ω (Greek letter omega) in radians per second and the self-inductance L in henries, or $X_L = \omega L$. In 60-cycle circuits, $X_L = 377L$. Hence, in a 60-cycle circuit containing 0.1 henry, $X_L = 377 \times 0.1 = 37.7$ ohms.

The e.m.f. of self-induction is equal to the product of the reactance X_L and the current strength I, or $E_L = X_L I$. Hence, if $X_L = 37.7$ ohms and $I = 5$ amp., $E_L = 37.7 \times 5 = 188.5$ volts.

The current strength I is equal to the ratio of the e.m.f. of self-induction E_L to the reactance X_L in ohms, or $I = \frac{E_L}{X_L}$ amp.

The reactance X_L in ohms is equal to the ratio of the e.m.f. of self-induction E_L to the current strength I, or $X_L = \frac{E_L}{I}$ ohms.

237. Further Symbols.—These are given in Table XXIII, and include some of the symbols used in the foregoing paragraphs.

TABLE XXIII.—SYMBOLS FOR VARIOUS PHENOMENA

π = 3.1416 (Greek letter pi).	Z = impedance in ohms.
2π = 6.2832.	Y = admittance in mhos.
f = frequency in cycles per second.	I = total amperage.
$\omega = 2\pi f$ = angular velocity in radians per second.	I_a = in-phase amperage.
R = resistance in ohms.	I_L = quadrature lagging amperage.
L = self-inductance in henries.	I_c = quadrature leading amperage.
C = capacitance in farads.	G = effective conductance in mhos.
C_m = capacitance in microfarads.	B = susceptance in mhos due to self-inductance.
X_L = reactance in ohms.	$-B$ = susceptance in mhos due to capacitance.
X_c = condensance in ohms.	

238. Condensance X_c.—The condensance X_c is equal to the reciprocal of the product of the angular velocity and the capacitance, or $X_c = \frac{1}{\omega c} = \frac{1}{2\pi fC} = \frac{0.159\ 16}{fC} = \frac{159{,}160}{fC_m}$ ohms. X_c is also equal to the ratio of the elastance S to the angular velocity, or $X_c = \frac{S}{\omega}$ ohms. In 60-cycle circuits, $X_c = \frac{0.002\ 65}{C} = \frac{2{,}650}{C_m} =$ 265 ohms when the capacitance is 10 mf.

The general relations are: $E_c = X_c I$, $I = \frac{E_c}{X_c}$ and $X_c = \frac{E_c}{I}$.

239. Impedance Z.—The product of the impedance Z in ohms and the total current strength I gives the impressed voltage E_i. The general relations are: $E_i = ZI$, $I = \frac{E_i}{Z}$, and $Z = \frac{E_i}{I}$.

When only R is present, $Z = R$; when only X_L is present, $Z = X_L$; and when only X_c is present, $Z = X_c$.

When only R and X_L are in series, $Z = \sqrt{R^2 + X_L^2}$. Hence, if $R = 10$ ohms and $X_L = 37.7$ ohms, $Z = \sqrt{100 + 1{,}421} = \sqrt{1{,}521} = 39$ ohms.

When only R and X_c are in series, $Z = \sqrt{R^2 + X_c^2}$. Hence, if $R = 10$ ohms and $X_c = 26.5$ ohms, $Z = \sqrt{100 + 704} = \sqrt{804} = 28.3$ ohms.

When there are only reactance X_L and condensance X_c in series, $Z = X_L - X_c$. If the reactance X_L is 37.7 ohms and the condensance X_c is 26.5 ohms, $Z = 37.7 - 26.5 = 11.2$ ohms, showing that the reactance X_L predominates, and I will lag behind E_i.

With R, L, and C in series, that is, when R, X_L, and X_c are in series, proceed as follows:

1. Find $X_L - X_c$.
2. Add $(X_L - X_c)^2$ to R^2.
3. Extract the square root.

Then $Z = \sqrt{R^2 + (X_L - X_c)^2}$.

If $X_L = X_c$, Z will be equal to R. This is the condition for series resonance. When X_L is greater than X_c, I will lag behind E_i. When X_c is greater than X_L, I will lead E_i.

If $R = 10$, $X_L = 37.7$, and $X_c = 265$, Z is found as follows:

1. $37.7 - 265 = -227.3$.
2. $10^2 + (-277.3)^2 = 100 + 51{,}665 = 51{,}765$.
3. $Z = \sqrt{51{,}765} = 227.5$ ohms.

This example shows how little the resistance affects the impedance when the condensance (or the reactance) is so relatively great.

The impedance Z is found by dividing the impressed voltage E_i by the total current strength I. The reactance X_L or the condensance X_c may be calculated from the impedance when the resistance R is known. Let $Z = 39$ ohms and $R = 10$ ohms. Then $X_L = \sqrt{39^2 - 10^2} = \sqrt{1{,}521 - 100} = \sqrt{1{,}421} = 37.7$ ohms. Whether X_L, X_c, $X_L - X_c$, or $X_c - X_L$ is equal to 37.7 ohms depends upon the amount of self-inductance and capacitance in series in the circuit.

In a circuit containing only resistance R and condensance X_c in series, X_c is equal to the geometric difference between Z and R. If $Z = 28.3$ ohms and $R = 10$ ohms, $X_c = \sqrt{804 - 100} = \sqrt{704} = 26.5$ ohms.

Dividing E_i, E_a, E_L, and E_c by I in Figs. 369 and 370 gives the corresponding values of Z, R, X_L and X_c in Figs. 371 and 372

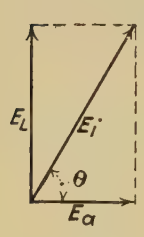


FIG. 369.

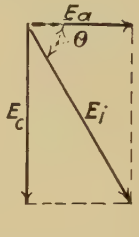


FIG. 370.

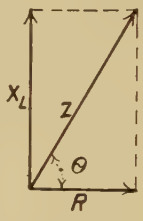


FIG. 371.

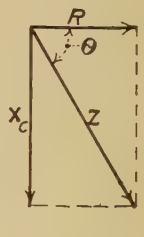


FIG. 372.

FIG. 369.—Components of the impressed voltage with current lagging.
FIG. 370.—Components of the impressed voltage with current leading.
FIG. 371.—Components of the impedance with current lagging.
FIG. 372.—Components of the impedance with current leading.

240. Components of the Total Current Strength I.—Just as the products of I and Z, R, X_L, or X_c give the respective values of E_i, E_a, E_L, or E_c, so may the total current strength I be resolved into the active or in-phase current strength I_a and the reactive or quadrature current strengths I_L and I_c, as in Figs. 373 and 374, which are inverted with respect to the preceding voltage relations. The axis of the active current strength I_a is the axis of the impressed voltage E_i in Figs. 373 and 374. Hence, in Fig. 373 the total current I lags behind E_i while in Fig. 374 I leads E_i.

241. Admittance Y is the Reciprocal of the Impedance Z.—The unit is the mho. $I = E_i Y$, $E_i = \frac{I}{Y}$, and $Y = \frac{I}{E_i}$.

The admittance Y is equal to the geometric sum of the effective conductance G and the susceptance B, that is, $Y = \sqrt{G^2 + B^2}$.

Figures 375 and 376 show the phase relations, and may be compared with Figs. 373 and 374.

$$I_a = E_i G, I_L = E_i B_L, \text{ and } I_c = E_i B_c.$$

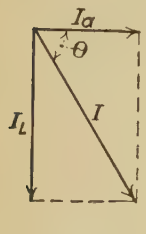


Fig. 373.

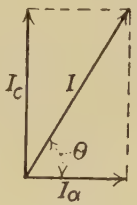


Fig. 374.

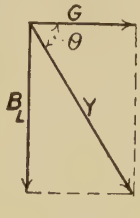


Fig. 375.

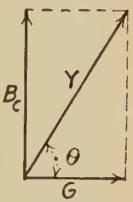


Fig. 376.

Fig. 373.—Components of the total lagging current.
Fig. 374.—Components of the total leading current.
Fig. 375.—Components of the admittance with current lagging.
Fig. 376.—Components of the admittance with current leading.

242. Effective Conductance G is the Active Component of the Admittance Y.—The unit is the mho. $G = \frac{R}{Z^2} = \sqrt{Y^2 - B^2}$.

The effective conductances G of perfect reactors and condensers are equal to 0, because no electric energy is converted into heat in either of them.

When there is no reactance X_L or condensance X_c in a circuit, $Z^2 = R^2$, and $G = \frac{1}{R}$, which is the direct-current conductance.

In a 60-cycle circuit wherein $R = 10$ ohms and $L = 0.05$ henry, $X_L = 377 \times 0.05 = 18.85$ ohms, and $Z^2 = 10^2 + 18.85^2 = 100 + 355 = 455$. Hence, $G = \frac{10}{455} = 0.202$ mho. If $E_i = 110$ volts, $I_a = 110 \times 0.022 = 2.42$ amp., and the true power is $P = 110 \times 2.42 = 266$ watts.

Similarly, in a 60-cycle circuit containing $R = 10$ ohms and $C_m = 141$ mf. in series, $X_c = \frac{2{,}650}{141} = 18.85$ ohms. Then G will be found to be 0.022 mho, $I_a = 2.42$ amp., and $P = 266$ watts, as in the former example.

In both of the above examples, the total current strength is $I = \frac{E_i}{Z}$, $Z = \sqrt{455} = 21.3$ ohms, and $I = \frac{110}{21.3} = 5.16$ amp. But $G = 0.022$ mho, and $I_a = 2.42$ amp. in both cases. Hence, I_L or $I_c = \sqrt{5.16^2 - 2.42^2} = \sqrt{26.6 - 5.85} = \sqrt{20.75} = 4.56$ amp., and the number of quadrature volt-amperes is qv.-a. $= 110 \times 4.56 = 500$.

243. Susceptance B is the Reactive or Quadrature Component of the Admittance Y.—The unit is the mho. B_L is the inductive susceptance due to self-inductance and $-B$ or B_c is the condensive susceptance due to the capacitance. Calling the former B_L and the latter B_c, $I_L = E_i B_L$ and $I_c = E_i B_c$. The general susceptance $B = B_L = B_c = B_L - B_c = B_c - B_L$, depending upon whether self-inductance or capacitance is present and which of the susceptances predominates. $B = \sqrt{Y^2 - G^2}$.

When only R is present, $B = 0$. When only L is present, $B_L = \frac{1}{X_L}$. When only C is present, $B_c = \frac{1}{X_c}$. When L and C are in series, $B = \frac{1}{(X_L - X_c)}$. When both R and X are in series, $B = \frac{X}{Z^2}$.

In a 60-cycle circuit containing only such self-inductance that $X_L = 18.85$ ohms, $B_L = \frac{1}{18.85} = 0.0531$ mho. If $E_i = 110$ volts, $I_L = 110 \times 0.0531 = 5.85$ amp., and qv.-a.$_L = 110 \times 5.85 = 642$ quadrature or reactive volt-amp., lagging.

In a 60-cycle circuit containing only such capacitance that $X_c = 24.2$ ohms, $B_c = \frac{1}{24.2} = 0.0414$ mho. If $E_i = 110$ volts, $I_c = 110 \times 0.0414 = 4.55$ amp., and qv.-a.$_c = 110 \times 4.55 = 500$ quadrature or reactive volt-amp., leading.

In a 60-cycle circuit containing only $R = 10$ and $L = 0.05$, $X_L = 377 \times 0.05 = 18.85$ ohms, and $Z^2 = 100 + 335 = 455$. Then $B_L = \frac{18.85}{455} = 0.0414$ mho. If $E_i = 110$, $I_L = 110 \times 0.0414 = 4.56$ amp., and qv.-a.$_L = 110 \times 4.56 = 500$.

In a 60-cycle circuit containing only $R = 10$ and $C_m = 14$ $I_c = \frac{2{,}650}{141} = 18.85$ ohms, and $Z^2 = 100 + 355 = 455$. Then $B_c = \frac{18.85}{455} = 0.0414$ mho. If $E_i = 110$, $I_c = 110 \times 0.0414 = 4.56$ amp., and qv.-a.$_c = 110 \times 4.56 = 500$.

In both of the above examples, the impedance $Z = \sqrt{455} = 21.3$ ohms, and the total current strength $I = \frac{110}{21.3} = 5.16$ amp. The active current strength

$I_a = \sqrt{5.16^2 - 4.56^2} = \sqrt{26.6 - 20.75} = \sqrt{5.85} = 2.42$ amp. and the total number of watts is equal to $P = 110 \times 2.42 = 266$.

The examples may be compared with those in the preceding article.

244. Parallel Connections.—In finding the admittance Y of a circuit like that in Fig. 377, or any portion thereof, algebraically add together the effective conductance G of each branch of the circuit. Then algebraically add together the susceptances B of each branch, remembering that all condensive susceptances $-B$ or B_c are negative. Next take the geometric sum of the total effective conductance G and the total susceptance B, that is, take the square root of the sum of their squares. The result is the admittance Y of the entire circuit, and its reciprocal is the impedance Z.

Then $I_a = E_i G$, and I_L or $I_c = E_i B$, whether or not the entire circuit or a single branch is considered. The quadrature current in any branch is equal to the product $E_i B$ of that branch.

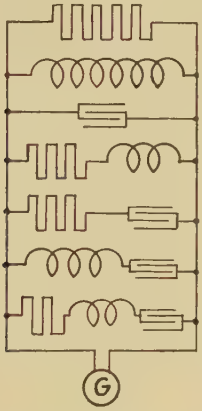


FIG. 377.

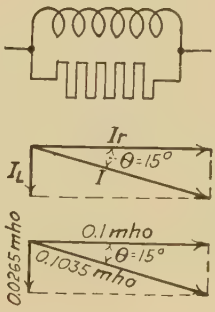


FIG. 378.

FIG. 377.—Resistance, reactance, condensance, and impedance in parallel.
FIG. 378.—Resistance and reactance.

$\text{Tan } \theta = \dfrac{B}{G}$ is the tangent of the phase angle θ in any case.

245. Resistance and Reactance in Parallel.—Referring to Fig. 378, let $L = 0.1$ and $R = 10$. Furthermore, let $E_i = 110$ and $f = 60$. The conductance G of the resistor is 0.1 mho. The reactance of the reactor is $X_L = 377 \times 0.1 = 37.7$ ohms, and its susceptance is $B_L = \dfrac{1}{37.7} = 0.0265$ mho. The admittance of the combination is $Y = \sqrt{G^2 + B_L^2} = \sqrt{0.1^2 + 0.0265^2} = \sqrt{0.01 + 0.000\,704} = \sqrt{0.010\,704} = 0.1035$ mho, and the total

current strength $I = 110 \times 0.1035 = 11.4$ amp. The active component of the total current strength of 11.4 amp. is $I_a = E_i G = 110 \times 0.1 = 11$ amp., and the quadrature component of the total current strength of 11.4 amp. $I_L = E_i B_L = 110 \times 0.0265 = 2.92$ amp. Hence, the current strength in the resistor is 11 amp. and the current strength in the reactor is 2.92 amp., and $I = \sqrt{I_a^2 + I_L^2} = \sqrt{11^2 + 2.92^2} = \sqrt{121 + 8.53} = \sqrt{129.53} = 11.4$ amp., as the total current strength first calculated. The impressed voltage E_i is in phase with $I_a = 11$ amp.

The tangent of the angle by which the total current I lags behind the impressed voltage E_i is $\tan \theta = \frac{G}{B_L} = \frac{I_a}{I_L} = \frac{0.1}{0.0265} = \frac{11}{2.92} = 3.77$. Referring to a table of tangents, the phase angle θ is found to be very close to 15 deg. The relations are graphically shown in Fig. 378.

246. Resistance and Condensance in Parallel.—Referring to Fig. 379, let $R = 10$, $C_m = 70.4$ ($C = 0.000\ 070\ 4$), $E_i = 110$, and $f = 60$. The conductance of the resistor is $G = \frac{1}{10} = 0.1$ mho, the condensance of the condenser is

$$X_c = \frac{1}{(377 \times 0.000\ 070\ 4)} = \frac{1}{0.0265} = 37.7 \text{ ohms},$$

and the susceptance B_c is 0.0265 mho. Therefore, the admittance Y of the combination is the same as that in Fig. 378, that is, $Y = 0.1035$ mho.

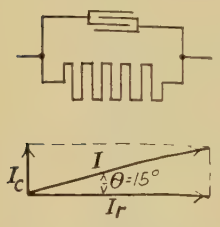


Fig. 379.—Resistance and condensance.

The active current strength in the resistor is $I_r = 11$ amp., the quadrature current strength in the condenser is $I_c = 2.92$ amp., and the total current strength is $I = 11.4$ amp., as in the preceding example. However, in this case, I leads E_i by 15 deg., as indicated in Fig. 379.

247. Condensance in Parallel with Impedance.—The resistor in Fig. 380 has a resistance of 10 ohms and the reactor has a self-inductance of 0.005 henry. With the condenser omitted, the circuit conditions would be the same as in Fig. 362. Assuming a frequency of 60 cycles per second, the reactance of the reactor is equal to 377×0.05 henry = 18.85 ohms, and the impedance is 21.3 ohms.

The effective conductance G is equal to the ratio of the resistance to the square of the impedance, or to $\frac{10}{455} = 0.022$ mho. The susceptance B_L is equal to the ratio of the reactance X_L to the square of the impedance, or $B_L = \frac{18.85}{455} = 0.0414$ mho.

Let the impressed voltage be 110 volts. Then the total current strength is equal to $\frac{110}{21.3} = 5.16$ amp. The active component of the total current strength I is $I_a = 110 \times 0.022 = 2.42$ amp., and the quadrature component $I_L = 110 \times 0.0414 = 4.55$ amp.

The above statements may be verified by referring to the preceding articles wherein the same values for the various phenomena have been used.

Now connect the 70.2-mf. condenser in parallel with the above impedance, as in Fig. 380. The condensance X_c of the condenser is 37.7 ohms and its susceptance B_c is 0.0265 mho. Its effective conductance is $G = 0$. The condensance X_c and susceptance B_c are negative with respect to the reactance X_L and the susceptance B_L of the reactor. Algebraically adding the effective conductance G of the resistor and reactor in series to that of the condenser (which is equal to 0) gives the total effective conductance $G = 0.022$. Algebraically adding the susceptance B_L of the reactor in series with the resistor to that (B_c) of the condenser gives $B = 0.0414 - 0.0265 = 0.0149$ mho as the total susceptance.

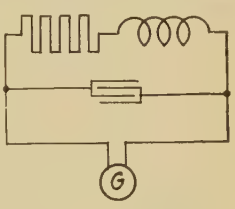


FIG. 380.—Impedance and condensance.

Geometrically adding the total effective conductance G to the total susceptance B gives

$$Y = \sqrt{0.022^2 + 0.0149^2} = \sqrt{0.000\,482 + 0.000\,222} =$$

$\sqrt{0.000\,704} = 0.0265$ mho as the total admittance. The total current strength is now $I = 110 \times 0.0265 = 2.92$ amp. The active current strength in the branch containing the reactor and resistor in series is $I_a = 110 \times 0.022 = 2.42$ amp. Thus, the total current strength of 5.16 amp. has been reduced to 2.92 amp. without changing the active current strength (in phase with the impressed voltage) of 2.42 amp.

The quadrature component of the total current strength I is now equal to $\sqrt{2.92^2 - 2.42^2} = \sqrt{8.51 - 5.85} = \sqrt{2.66} = 1.63$ amp. Hence, the quadrature component of the total current strength has been reduced from 4.55 to 1.63 amp., without having changed the useful or active component of the total current strength, by connecting a 70.2-mf. condenser in parallel with the resistor and reactor in series. In consequence, the total number of volt-amperes has been reduced from $100 \times 5.16 = 567$ to $110 \times 2.92 = 321$, or to $\frac{321}{567} = 0.566$, or 56.6 per cent, of the former value.

Now let the condensance X_c of the condenser be made equal to the reciprocal of the susceptance B_L of the reactor in series with the resistor, that is, let it be equal to $\frac{1}{0.0414} = 24.2$ ohms. Then the capacitance will be equal to $\frac{1}{377 \times 24.2} = \frac{1}{9{,}120} = 0.000\,110$ farad, or 110 mf. The susceptance of this new condenser being 0.0414 mho, when it is connected in parallel with the impedance the total susceptance will be equal to $0.0414 - 0.0414 = 0$. The effective conductance is $G = 0.022$ mho. Consequently, the admittance is $Y = 0.022$ mho because the susceptance has disappeared and the new total current strength is $I = 110$ volts $\times$ 0.022 mho $= 2.42$ amp., which is also the in-phase or active current strength I_a, and the quadrature current strength is equal to zero. This is the condition for 100 per cent power factor.

It must be remembered that the above are effective values. Actually, energy is being transferred from the reactor to the condenser, and *vice versa*, but the net effect is as though there were no such transferences of energy.

248. Further Methods of Finding the Phase Angle θ.—The e.m.f. of self-induction E_L is equal to the product of the total current strength I and the reactance X_L, or $E_L = IX_L$. The active voltage E_a is equal to the product of the total current I and the resistance R, or $E_a = IR$. Hence, $\tan\theta = \frac{E_L}{E_a} = \frac{X_L}{R}$. But $X_L = 6.2832fL$. Therefore, $\tan\theta = 6.2832\, f\left(\frac{L}{R}\right) = \omega t_c$, wherein ω is the angular velocity in radians per second (377 for 60 cycles per second) and t_c if the time constant $\left(\frac{L}{R}\right)$.

In a 60-cycle circuit containing $L = 0.1$ henry and $R = 5$ ohms in series, $t_c = \frac{0.1}{5} = 0.02$ sec., $\tan \theta = 377 \times 0.2 = 7.54$, and $\theta = 82$ deg., 29 min. (current lagging).

In a circuit containing capacitance C and resistance R in series, $\tan \theta = \frac{X_c}{R} = \frac{0.159\,16}{ft_c}$, wherein $t_c = RC$. When $f = 60$, $\tan \theta = \frac{0.002\,65}{t_c}$.

In a 60-cycle circuit containing 100 mf. in series with 100 ohms, $t_c = 0.0001$ farad $\times$ 100 ohms $= 0.01$ sec., $\tan \theta = \frac{0.002\,65}{0.01} = 0.265$, and $\theta = 14$ deg., 51 min. (current leading).

The tangent of the phase angle θ between the impressed voltage and the total current strength is equal to any of the following ratios:

$$\text{Current lagging: } \frac{E_L}{E_a}, \frac{I_L}{I_a}, \frac{\text{qv.-a.}_L}{P}, \frac{X_L}{R}, \frac{B_c}{G}.$$

$$\text{Current leading: } \frac{E_c}{E_a}, \frac{I_c}{I_a}, \frac{\text{qv.-a.}_c}{P}, \frac{X_c}{R}, \frac{B_L}{G}.$$

249. Phase Compensation.—The phase angle θ may be adjusted to any desired value between the limits $+90$ deg. and -90 deg. by compensating the reactances of the circuit. Usually, reactance X_L predominates, so condensers are used for phase compensation, the object of the condenser being to absorb the quadrature volt-amperes qv.-a.$_L$ (lagging) due to reactance X_L and thus obtain the desired power factor $\cos \theta$, which is equal to any of the following ratios: $\frac{E_a}{E_i}, \frac{I_a}{I}, \frac{P}{\text{v.-a.}}, \frac{R}{Z}, \frac{G}{Y}$, or the ratio of the power in watts as measured with a wattmeter to the total number of volt-amperes as separately measured with a voltmeter and an ammeter.

Phase compensation may be accomplished by connecting the condenser either in series or in parallel with the device in which the phase angle θ is to be changed, but the series arrangement produces relatively great voltages across the terminals of both the condensers and the reactors which tend to break down the dielectric or insulation, and there are other objections. Hence, condensers are generally connected in parallel with the devices.

Assume that the measured power is 1,000 watts (1 kw.) and that the total number of volt-amperes is found to be 1,667 (1.667 kv.-a.). Then the power factor $\cos \theta$ is equal to $\frac{1}{1.667} =$ 0.6, or 60 per cent. Now assume that a power factor of 0.9 or 90 per cent is desired without changing the number of watts (1 kw. = 1,000 watts) but by reducing the total number of kv.-a. to $\frac{1}{0.9} = 1.111$. The function of the condenser is to absorb a portion of the lagging kv.a., or qkv.-a.$_L$, and the capacitance of the condenser is made such that the condenser will absorb the difference between the qkv.-a.$_L$ corresponding to the lower power factor and that for the higher power factor.

In the present example, qkv.-a.$_L = \sqrt{1.167^2 - 1^2} = \sqrt{2.78 - 1} = \sqrt{1.78} = 1.33$. If it was desired to increase the power factor $\cos \theta$ to unity, the condenser should have a capacitance sufficient to absorb the entire qkv.-a.$_L = 1.33$. But it is more economical to increase the power factor to about 0.9 as a maximum on account of the increasing cost of the condenser as the power factor approaches $\cos \theta = 1$, as compared with the reduction in the total qkv.-a.$_L$. Therefore, the problem is to ascertain what will be the qkv.-a.$_L$ when the power factor is, say, 0.9, or 90 per cent.

The power P is to remain constant at 1 kw. When $\cos \theta$ becomes equal to 0.9, due to the installation of the condenser, the total kv.-a. will be equal to 1.111, as found. Hence, the number of kilovolt-amperes in lagging quadrature with the total kv.-a. will be qkv.-a.$_L = \sqrt{1.111^2 - 1^2} = \sqrt{1.235 - 1} = \sqrt{0.235} =$ 0.485. Therefore, the qkv.-a. to be absorbed by the condenser is equal to the difference between the original or high qkv.-a.$_L$ and the low qkv.-a.$_L$ at the new or higher power factor, or to $1.33 - 0.485 = 0.848$ qkv.-a. With this information, and the known impressed voltage and frequency, the capacitance of the condenser is easily found.

The power factor is $\cos \theta$. Sin θ may be called the qkv.-a. factor. When $\cos \theta = 0.9$, $\theta = 25$ deg., 51 min., and $\sin \theta =$ 0.436. For the phase angle $\theta = 25$ deg., 51 min., $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{0.436}{0.9} = 0.485$, and this is the ratio of the lagging qkv.-a. to the power P.

$$\cos\theta = \text{power factor} = \frac{P}{\text{kv.-a.}}.$$

$$\sin\theta = \text{qkv.-a.factor} = \frac{\text{qkv.-a}}{\text{kv.-a.}}.$$

$$\tan\theta = \frac{\text{qkv.-a.}}{P}.$$

$$\text{qkv.-a.} = \text{kv.-a.} \sin\theta = P \tan\theta.$$

These values are tabulated in Table XXIV, with $\cos\theta$ as a basis.

The constant power P is 1 kw. in the above example. When $\cos\theta = 0.6$, $\tan\theta = 1.33$ and qkv.-a. $= 1 \times 1.33 = 1.33$. When $\cos\theta = 0.9$, $\tan\theta = 0.485$ and qkv.-a. $= 1 \times 0.485 = 0.485$. The difference in qkv.-a. is, therefore, equal to $1.33 - 0.485 = 0.848$, as previously determined, and this is the qkv.-a. to be absorbed by the condenser. While the foregoing is based on kilovolt-amperes and kilowatts, volt-amperes and watts may be substituted therefore.

250. The Capacitance of a Condenser Required to Absorb a Given Qkv.-a.—Assuming that a condenser is to be connected across the terminals of an electromagnetic device, as a large electromagnet or an induction motor, connected directly across the line, to improve the power factor of the device, the condenser will receive the full-line voltage.

The current strength in a condenser is $I_c = \omega C E_c$. Consequently, the number of volt-amperes (v.-a.) is equal to $E_c I_c = \omega C E_c^2$, and this is the number of quadrature volt-amperes required when the e.m.f. of capacitance E_c is numerically equal to the line voltage E_i. Hence, the number of farads of capacitance per volt-ampere is equal to the reciprocal of the product of the angular velocity in radians per second and the square of the terminal voltage.

When the terminal voltage of the condenser is 110 volts and the frequency is 60 cycles per second, the capacitance in farad per volt-ampere is equal to

$$\frac{1}{(377 + 121{,}000)} = \frac{1}{4{,}561{,}700} = 0.000\ 000\ 219\ 2$$

farad per volt- ampere = 0.2192 mf. per v.-a. = 219.2 mf. per kv.-a., or per qkv.-a.

Referring to the preceding example, the qkv.-a to be absorbed by the condenser was found to be 0.848. Since the impressed voltage is 110 volts and the frequency is 60 cycles per second, the

TABLE XXIV.—TRIGONOMETRIC RELATIONS

Power factor cos θ	qkv. - a. factor sin θ	Ratio tan θ	Product (slide rule)	Phase angle θ		Angle θ in decimals
				Deg.	Min.	
0.60	0.800	1.33	0.480	53	8	53.13
0.61	0.792	1.30	0.483	52	25	52.42
0.62	0.785	1.27	0.486	51	41	51.68
0.63	0.777	1.23	0.490	50	57	50.95
0.64	0.768	1.20	0.492	50	12	50.20
0.65	0.760	1.17	0.494	49	27	49.45
0.66	0.751	1.14	0.496	48	42	48.70
0.67	0.742	1.11	0.498	47	56	47.93
0.68	0.733	1.08	0.499	47	9	47.15
0.69	0.724	1.05	0.500	46	22	46.37
0.70	0.714	1.02	0.500	45	34	45.57
0.71	0.704	0.992	0.500	44	46	44.76
0.72	0.694	0.964	0.499	43	57	43.95
0.73	0.683	0.936	0.498	43	7	43.16
0.74	0.673	0.909	0.497	42	16	42.27
0.75	0.662	0.882	0.496	41	25	41.42
0.76	0.650	0.855	0.495	40	32	40.53
0.77	0.638	0.829	0.492	39	39	39.65
0.78	0.626	0.803	0.488	38	45	38.75
0.79	0.613	0.776	0.484	37	49	37.81
0.80	0.600	0.750	0.480	36	52	36.87
0.81	0.586	0.724	0.475	35	54	35.90
0.82	0.572	0.698	0.469	34	55	34.92
0.83	0.558	0.672	0.463	33	54	33.90
0.84	0.543	0.646	0.456	32	52	32.87
0.85	0.527	0.620	0.448	31	47	31.78
0.86	0.510	0.593	0.438	30	41	30.68
0.87	0.493	0.567	0.428	29	33	29.55
0.88	0.475	0.540	0.418	28	21	28.35
0.89	0.456	0.512	0.406	27	8	27.13
0.90	0.436	0.485	0.393	25	51	25.85
0.91	0.415	0.456	0.378	24	30	24.50
0.92	0.392	0.426	0.360	23	4	23.07
0.93	0.368	0.395	0.342	21	34	21.57
0.94	0.341	0.363	0.321	19	57	19.95
0.95	0.312	0.329	0.296	18	12	18.20

capacitance of the condenser should be equal to 0.848×219.2 mf. per qkv.-a. = 186 mf.

251. Resonance.—The condition for either series or voltage resonance or for parallel or current resonance is that the reciprocal of 6.2832 times the square root of the product of the self-inductance in henries and the capacitance in farads shall be equal to the frequency of the impressed voltage and current. The voltage and current are then in phase and the phase angle $\theta = 0$.

Voltage resonance causes the e.m.f. of self-induction and the e.m.f. of capacitance greatly to exceed the impressed voltage.

Current resonance causes the strength of the current flowing in the local oscillating circuit, that is, in the reactor and the condenser connected in parallel, to be greater than the current strength in the rest of the circuit.

In the examples given below, with a line voltage of 110 volts, the voltage across the reactor and the condenser in series is 829 volts in each case, while with the reactor and the condenser connected in parallel the line current strength is 0.022 amp. and the current strength in the reactor and the condenser is 11 amp.

The above phenomena are comparable to those obtained with a swinging mass, each added impulse causing a greater swing and a consequent greater reactive or arresting force, until the motion is limited only by friction.

The oscillation constant is equal to $\frac{0.159\ 16}{f}$. Hence, when $f = 60$, the oscillation constant is equal to 0.025 33 and its square is 0.000 007 036.

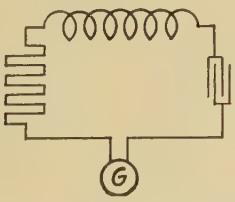


FIG. 381.

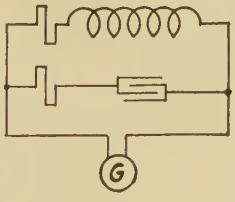


FIG. 382.

FIG. 381.—Series arrangement.
FIG. 382.—Parallel arrangement

In the circuit in Fig. 381, let $R = 5$ ohms and $E_i = 110$ volts. Then, when series resonance occurs, the current strength will be 22 amp. For the condition of resonance, $LC = 0.000\ 007\ 036$. Let $L = 0.1$ henry, and $C = 0.000\ 070\ 36$ farad = 70.36 mf.

The reactance $X_L = 377 \times 0.1$ henry $= 37.7$ ohms, and the condensance $X_c = \frac{0.002\,65}{0.000\,070\,36} = 37.7$ ohms. Hence, the voltage across the terminals of the reactor and the condenser will be equal to 37.7 ohms $\times$ 22 amp. $= 829$ volts in each case.

There is always some resistance in series with reactance and condensance in commercial circuits. In the parallel arrangement in Fig. 382, let the reactance and the condensance each be 10 ohms, and let the resistance of each resistor be 0.01 ohm.

The square of the impedance of each branch is equal to $0.01^2 + 10^2 = 0.0001 + 100 = 100.0001$. The effective conductance of each branch is equal to $\frac{R}{Z^2} = G = \frac{0.01}{100.0001} = 0.0001$ mho, and the susceptance of each branch is $B = \frac{X}{Z^2} = \frac{10}{100.0001} = 0.1$ mho (either B_L or B_c).

The total susceptance B is equal to the difference between the inductive susceptance B_L and the condensive susceptance B_c and, since the susceptances are equal to each other, the total susceptance is equal to 0.

The total effective conductance is equal to $0.0001 + 0.0001 = 0.0002$, which also is the admittance since there is no susceptance.

If the impressed voltage is 110 volts, the in-phase or active current strength is equal to 0.0002 mho $\times$ 110 volts $= 0.022$ amp. and this is the total line current.

The current strength in the reactor and in the condenser is equal to 0.1 mho $\times$ 110 volts $= 11$ amp.

The stronger current is comparable to the accumulated motion of a large swinging body due to a succession of alternate pushes and pulls in time-phase with it.

252. Polyphase Systems.—A polyphase system is said to be symmetrical when the impressed voltages are equal and are displaced in phase by equal angles. In a balanced polyphase system, the sum of the instantaneous power in all of the phases is constant regardless of the fact that the power in each phase is pulsating. If the total power is pulsating, the system is unbalanced.

Polyphase systems are usually of the three-phase class, although the two-phase and quarter-phase systems also are used in distribution.

253. Phase Voltages and Currents.—The phase voltage is the voltage as measured across the terminals of any coil. The phase current is the current in any coil.

254. Two-phase System.—The coils in the circuit in Fig. 383 are displaced by 90 deg. The voltage between the phases is equal to $\sqrt{2} = 1.414$ times the voltage between each phase and the common return wire. The current in the common return wire is equal to 1.414 times the current in each phase.

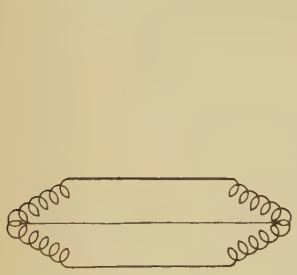


FIG. 383.

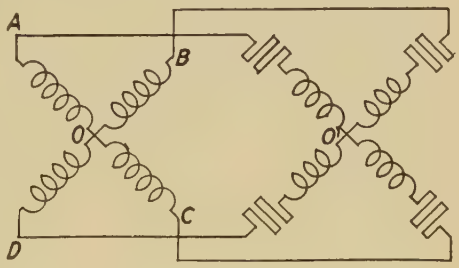


FIG. 384.

FIG. 383.—Two-phase system.
FIG. 384.—Quarter-phase system.

255. Quarter-phase Systems.—The circuit in Fig. 384 is the equivalent of a two-phase circuit without any connection between the coils (except that they may be connected at O and O') and the voltages are displaced by 90 electrical degrees. If O and O' are connected by a neutral wire, three different voltages may be obtained. If E_1 is the voltage $OA = OB = OC = OD$, then voltages $AB = BC = CD = DA = 1.414 \times E_1$. Voltages $AC = BD = 2 \times E_1$.

256. Balanced Three-phase Delta (Δ) or Mesh System.—Referring to Fig. 385, wherein voltameters and ammeters are shown connected across and in the line and phases, respectively, E_1 = line voltage, E_p = phase voltage, E_n = voltage to neutral, I_1 = line current, and I_p = phase current.

In this system, the line and phase voltages are equal.

The phase current is equal to $\frac{1}{\sqrt{3}} = 0.577$ times the line current.

The line current is equal to $\sqrt{3} = 1.732$ times the phase current.

The voltage to neutral (imaginary in this case) is equal to 0.577 times the line voltage.

The total number of volt-amperes is equal to 3 times the product of the phase voltage and the phase current, or to 1.732 times the product of the line voltage and the line current.

257. Balanced Three-phase Star or Y System.—Figure 386 shows the various voltages and currents corresponding to those in Fig. 385. In this system:

The line and phase currents are equal.

The phase voltage is equal to 0.577 times the line voltage.

The line voltage is equal to 1.732 times the phase voltage.

The voltage to neutral is equal to the phase voltage.

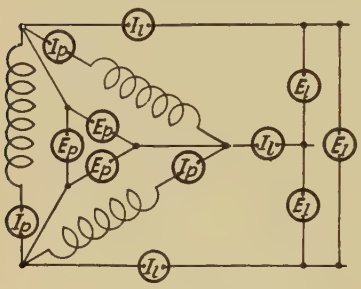


FIG. 385.

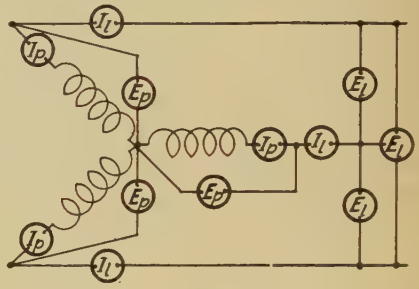


FIG. 386.

FIG. 385.—Three-phase delta (Δ) or mesh system.
FIG. 386.—Three-phase star or Y system.

The total number of volt-amperes is equal to 3 times the product of the phase voltage and the phase current, or to 1.732 times the product of the line voltage and the line current.

258. Characteristics Common to Both Δ- and Y-connected Systems.—For equal line voltages, the line currents are equal and, therefore, the total volt-amperes are equal. These are the only common characteristics.

259. Equivalent Y-connection.—Assuming sinusoidal voltage and current waves, a three-phase Δ-connected electromagnetic device may be considered the equivalent of a Y-connected electromagnetic device.

The equivalent Y-connection voltage per phase is equal to 0.577 times the corresponding Δ-connection voltage per phase.

The equivalent Y-connection resistance per phase is equal to $\frac{1}{3}$ (or 0.333 33) times the corresponding Δ-connection resistance per phase.

The equivalent Y-connection reactance (inductive or condensive) per phase is equal to 0.333 33 times the corresponding Δ-connection reactance per phase.

260. Reduction to Equivalent Single Phase.—Each of the three phases of a Y-connected three-phase system, or its equivalent, may be treated as a single-phase circuit under the condition shown in Fig. 387, wherein the dotted line represents a neutral (imaginary) having zero impedance. Each voltage is then the voltage to neutral and the currents are line currents. In the *Y*-connection, the voltage to neutral is the phase voltage (or the number of volts per phase). Consequently, the usual single-phase calculations may be made for any one phase, and the

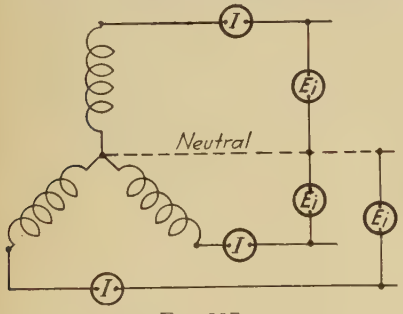


Fig. 387.

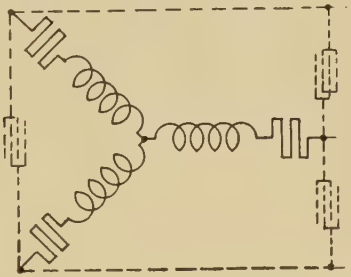


Fig. 388.

Fig. 387.—Y system with neutral.
Fig. 388.—Y-connection.

voltages, impedances, and so forth then multiplied by the proper factor to obtain the line or the total values. The Y-connection values thus deduced may, in turn, be reduced to Δ-connection values.

In a balanced Y-connected three-phase system, as in Fig. 388, the resistance is 5 ohms and the reactance is 8.66 ohms, for each phase, making the impedance 10 ohms per phase. If the current strength is 10 amp., the number of volts per phase is 100 and the line voltage is equal to 1.442×100 volts = 144.2 volts. The total number of volt-amperes is equal to 3 times the product of the line voltage and the line current. Hence, the total number of volt-amperes is 3,000, or 3 kv.-a. The power factor is equal to the ratio of the resistance to the impedance (as per phase) or to $\frac{5 \text{ ohms}}{10 \text{ ohms}} = 0.5$, or 50 per cent. Hence, the power is equal to $3{,}000 \times 0.5 = 1{,}500$ watts, or 1.5 kw., and the number of

quadrature volt-amperes is qv.-a. $= 3{,}000 \times \left(\frac{8.66}{10}\right) = 2{,}598$, or qkv.-a. $= 2.598$. In order to absorb the 2.598 qkv.-a., three condensers, each having a condensance of $8.66 \times 1.442 = 12.5$ ohms, should be connected across the respective phases of the circuit, or a three-phase condenser having a condensance of 12.5 ohms in each phase should be connected across the circuit.

261. Equivalent Δ-connection.—Having converted the single-phase values to Y-connection values, the Δ-connection values are found as follows:

The equivalent Δ-connection voltage per phase is equal to 1.442 times the Y-connection voltage per phase.

The equivalent Δ-connection resistance per phase is equal to 3 times the corresponding Y-connection resistance per phase.

The equivalent Δ-connection reactance per phase is equal to 3 times the corresponding Y-connection reactance per phase.

CHAPTER XXII

ALTERNATING-CURRENT SOLENOIDS

The general force-distance characteristics of a given constant-current electromagnet are about the same whether the current is alternating or direct. Solid cores or plungers heat at high time-rates, thus requiring a greater electric energy input per cycle than corresponding laminated cores. In the calculation of alternating-current solenoids the structural flux must be separated from the electromagnetic flux, as in the cases of direct-current solenoids, the condition for the maximum pull per unit cross-section of plunger between a constant-voltage, alternating-current solenoid and its plunger, for a given relative position, being that the magnitude of the structural flux shall be numerically equal to the magnetizing force.

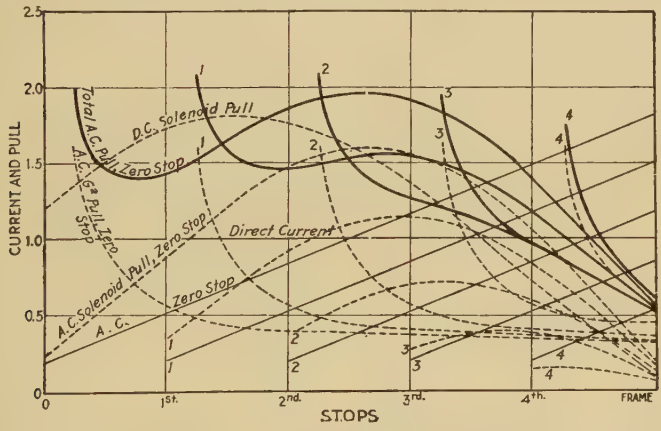


FIG. 389.—General principle of alternating-current electromagnet characteristics.

262. Alternating-current Solenoid Force Characteristics.—In what follows, the plunger is assumed to move towards the left-hand end of the solenoid. In a constant-current solenoid-and-plunger, the general mechanical force characteristics are the same as those for a direct-current solenoid-and-plunger. However, in a constant-voltage, alternating-current solenoid-and-plunger, the current strength is a maximum when the plunger is entirely outside of the core hole of the solenoid, and is a minimum when the middle of the plunger is at the middle of the solenoid.

Figure 389[1] shows the relations in the latter case, wherein the total and air-gap (G^2) pulls are to be temporarily ignored. Assuming that the current strength decreases in direct proportion to the position of the end of the plunger within the core hole of the solenoid, the constant-voltage, alternating-current solenoid-pull curve would have the same form shown in Fig. 389 as the "Alternating-current Solenoid Pull, Zero Stop." However, it should be borne in mind that, while the general shape of a direct-current solenoid-and-plunger-pull curve may always be the same with constant direct voltage, the position of maximum pull due to constant alternating voltage impressed across the same solenoid may vary considerably, depending upon the impedance of the solenoid.

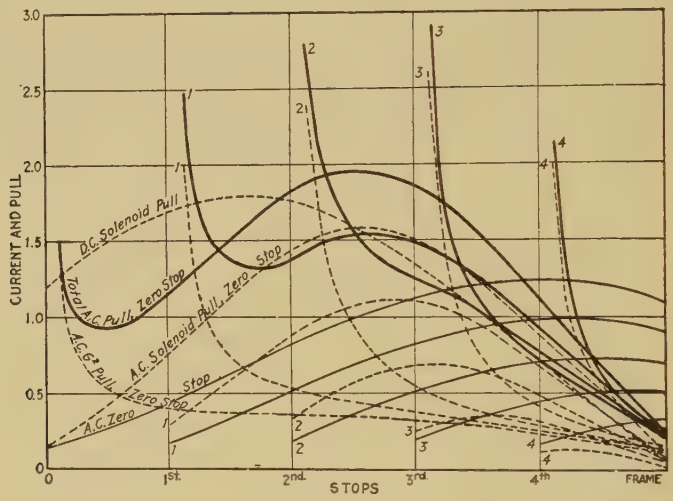


Fig. 390.—Corrected characteristics of alternating-current electromagnet.

263. Plunger Electromagnets.—The principle above outlined also applies to the constant-voltage, alternating-current iron-clad solenoid or plunger electromagnet of either the single-coil or the two-coil type. The air-gap pulls are likewise affected by the decrease in the current strength with corresponding decreases in air-gap lengths. However, the total pull is equal to the sum of the solenoid pull and the air-gap (G^2)pull. The symbol G^2 as here used has no relation to conductance. Expressed in terms of dynes of force, $G^2 = \frac{b^2}{8\pi} S = 0.0398\ b^2S$, wherein b is the

[1] Underhill, C. R., "Characteristics of Alternating-current Electromagnets," *Elec. World*, vol. 63, p. 982, 1914.

structural flux density and S is the cross-section of the air gap in square centimeters (see page 178).

But the presence of resistance in the electric circuit, and the reactance due to the presence of a stop in the ferromagnetic circuit (which has the general effect of the insertion of a plunger into the core hole of the solenoid) as well as the presence of the

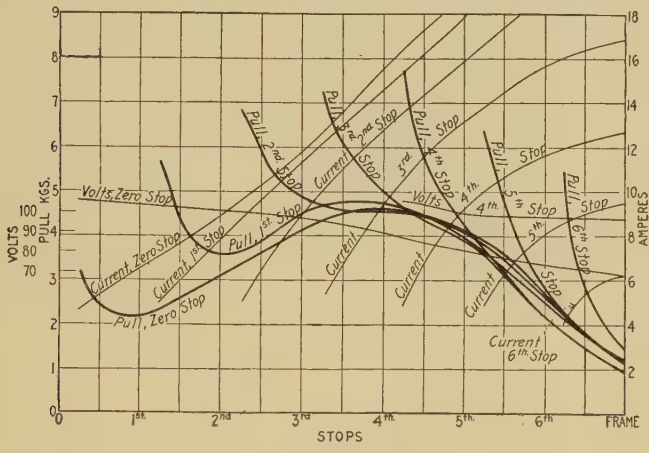


FIG. 391.—Pull and current characteristics of the test electromagnet.

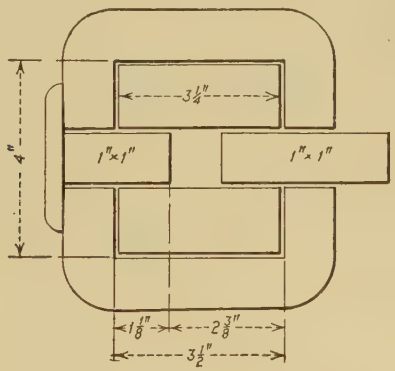


FIG. 392.—Test electromagnet.

magnetic frame, tend to make the current and pull characteristics about as illustrated in Fig. 390.

Figure 391[1] shows the test characteristics of the plunger electromagnet in Fig. 392, which had all of its magnetic parts laminated (although the laminations were rather thick). This test

[1] UNDERHILL, C. R., "Alternating-current Electromagnets," *Elec. World*, vol. 63, p. 260, 1914.

was made by the author on a 60-cycle lighting circuit at a point situated some distance from a small transformer, so the voltage drop was quite great. The current strengths, total power, power factor, terminal volts, and iron losses for various positions are given in Figs. 393 and 394.

In these, as well as in other similar tests, a spring scale was employed for measuring the pull. A non-magnetic spacer was placed between the plunger and the stop to represent an air gap.

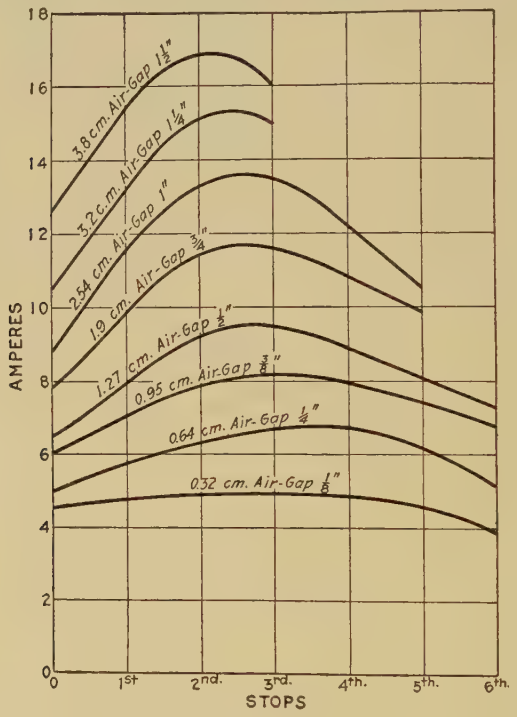


FIG. 393.—Current characteristics of the test electromagnet.

When the spring was extended in measuring the pull, the pointer showed very little evidence of vibration and, since the mass of the spring and its connected moving parts was relatively small, the pull thus measured was taken as the effective pull.

The total number of flux-turns in a perfect constant-voltage, alternating-current electromagnet is constant, but there never was a perfect alternating-current electromagnet. The presence of resistance in the electric circuit requires that a portion of the constant impressed voltage be exerted in converting electric

energy into heat. Then the e.m.f. of self-induction is decreased, and the number of flux-turns is thereby reduced. Even when the resistance may be negligible, and the number of flux-turns is constant, magnetic hysteresis and eddy or other secondary currents increase the number of ampere-turns, which also varies with the amounts of magnetic reluctance and leakage present.

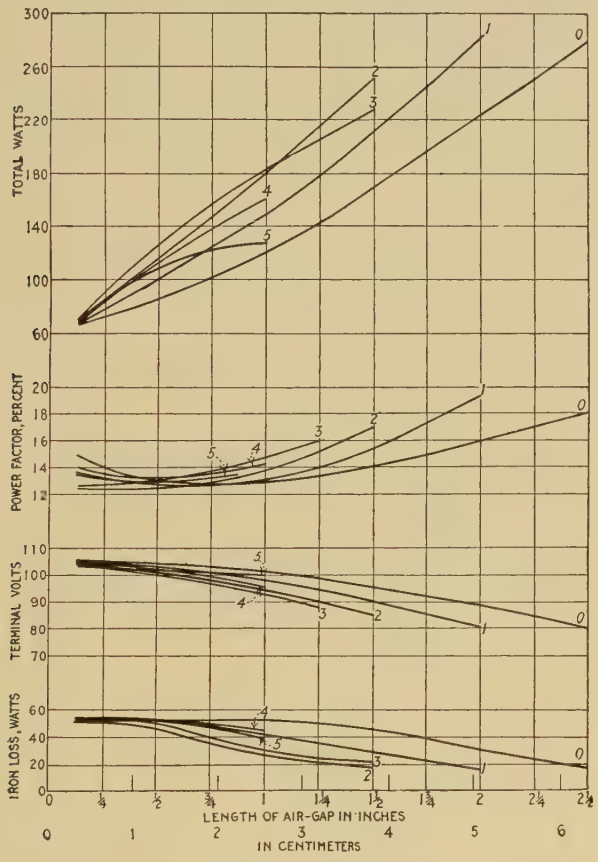


Fig. 394.—General characteristics of the test electromagnet.

The ratio of the ferromagnetic flux that passes through an air gap of a plunger electromagnet to that which leaks back from it is approximately as represented in Fig. 395. Generally speaking, the leakage pull is a maximum when the length of the air gap is a minimum, but when the length of the air gap is equal to zero, as when there is no air gap at all, so that the plunger and stop are

integral, there can be no leakage that can produce an effective net solenoid pull at that place.

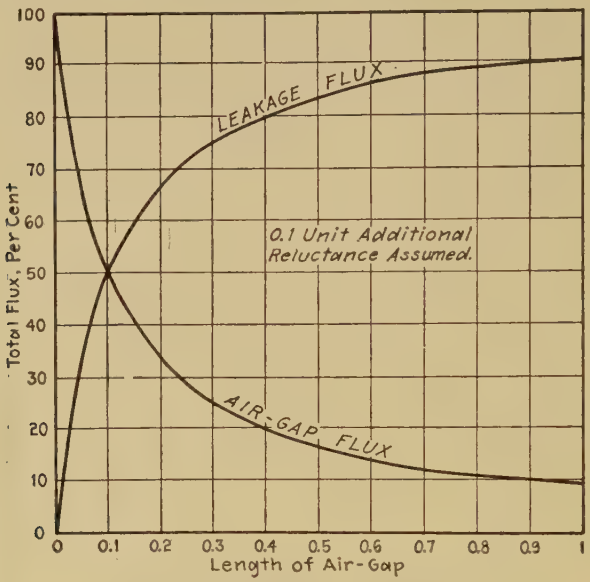


FIG. 395.—Relative values of flux.

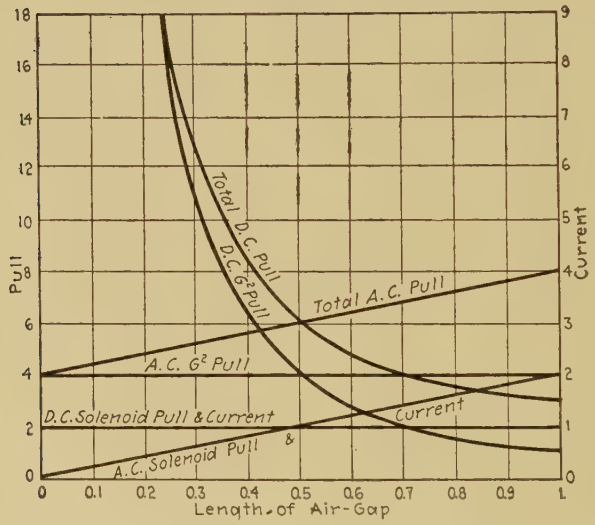


FIG. 396.—Characteristics when the alternating current falls to zero.

If the number of flux-turns for the air gap were constant, all of the reluctance were in the air gap, there were no energy losses,

and no magnetic leakage, the current strength would decrease in direct proportion to the decrease in the length of the air gap, as illustrated in Fig. 396, wherein the constant-current characteristics also are shown. However, the presence of the usual reluctance and the energy losses prevents the current strength from being reduced to zero when the air gap closes. Consequently, the results are as shown in Fig. 397.

The test in Fig. 398 was made with the electromagnet in Fig. 392 under line conditions similar to those described in connection with Fig. 391. After obtaining the constant-voltage, alternating-

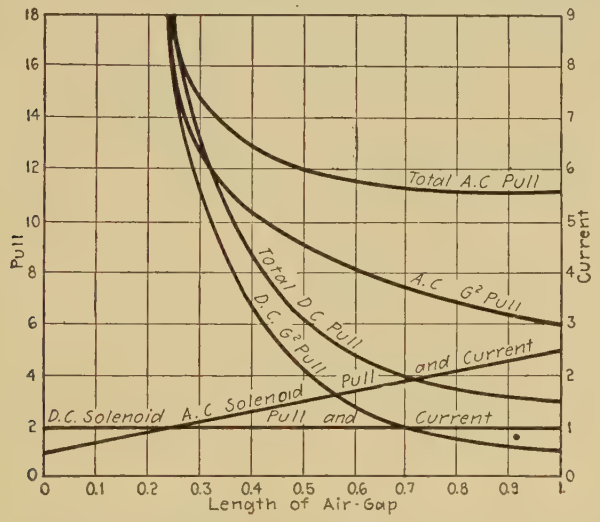


FIG. 397.—Characteristics when the alternating current does not fall to zero.

current characteristics, the electromagnet was again tested with direct current, the latter being adjusted so that the pull in each case was the same as with the r.m.s. current strength at constant r.m.s. voltage. The stop was then removed and the solenoid pull found for all points, through the former air gaps, with direct current. A test also was made with direct current, and with the stop in place, in order to find the total direct-current pull. The "Direct-current G^2 Pull with 7 Amp." was obtained by subtracting the solenoid pull from the total pull. The points show the calculated air-gap (G^2) pulls. More r.m.s. amperes were required with alternating than with direct current because of the iron losses, which can only be supplied through the ampere-turns or

the m.m.f. The illustration shows that about 20 per cent more r.m.s. amperes were required when alternating current was applied than when direct current was used.

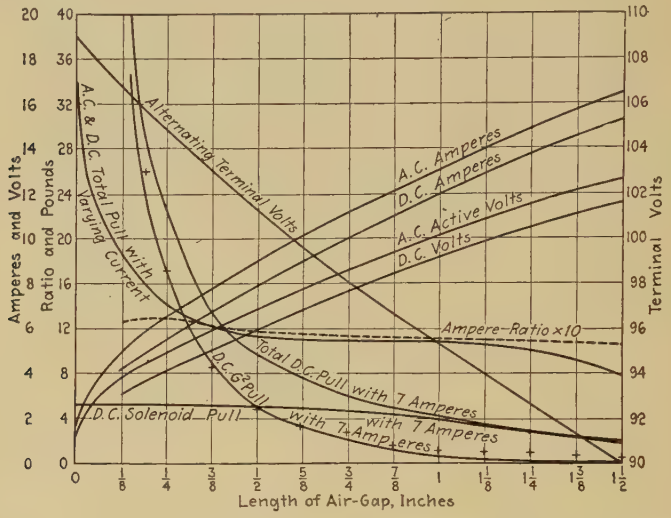


FIG. 398.—Test characteristics with both alternating and direct current.

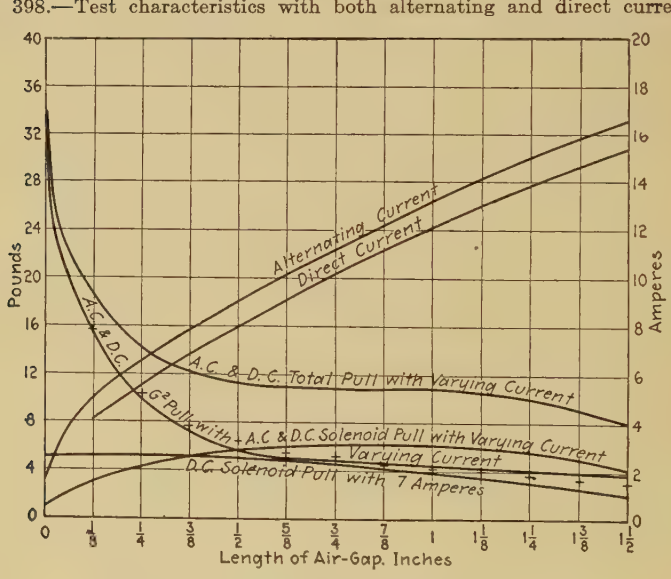

Pounds
Amperes
Length of Air-Gap, Inches
Alternating Current
Direct Current
A.C. & D.C.
A.C. & D.C. Total Pull with Varying Current
G² Pull with A.C. & D.C. Solenoid Pull with Varying Current
Varying Current
D.C. Solenoid Pull with 7 Amperes

FIG. 399.—Analysis of the test characteristics.

The test characteristics in Fig. 398 are further analyzed in Fig. 399. The air-gap (G^2) pull is proportional to the square of

the ratio of the current strength to the length of the air gap. By allowing an additional reluctance of the ferric parts corresponding to one-sixteenth unit of length of air gap, and by dividing by 30 to bring these values

$$\left(\frac{\text{amperes}}{\text{air-gap length}}\right)^2$$

to the same scale as the G^2 pull values in both Figs. 398 and 399, and using direct-current values, the calculated G^2 pulls coincide favorably with the actual G^2 pulls, as indicated by the calculated points.

By inserting a stop about one-third coil length inside of the coil of a plunger electromagnet, the effective pull with constant r.m.s. terminal voltage will be quite uniform over the usual range of travel required of plunger electromagnets of this type. By giving the stop various lengths, as many different shapes of pull curve may be obtained. It is to be noted that the usual air-gap pull is all that may be obtained when the plunger or armature does not pass within the core hole of the coil or quite close to the end of the coil. The characteristics of two-coil plunger electromagnets are similar to those of the single-coil type.

Charles C. Raitt made a rather long series of tests at the University of Cincinnati to check the author's results as published in the Jan. 31, 1914 issue of the *Electrical World* and described above. He found that

> The results of the test show, first, that, in an alternating-current plunger magnet the pull obtained from a given value of current is independent of frequency, but is not independent of wave form. Second, they show the difference between direct-current amperes and alternating-current amperes required to produce a given pull at a given air gap is approximately constant for all values of voltage and frequency.

264. Volt-amperes and Mechanical Force.—A little reflection will show that there should be a definite relation between the number of terminal volt-amperes for both alternating and direct currents in the same electromagnet. In a constant-voltage, alternating-current electromagnet, the number of terminal volt-amperes is equal to the product of (1) the geometric sum of the e.m.f. of self-induction and the active voltage and (2) the total current strength. The number of active volt-amperes, or the power in watts, in a constant direct-voltage electromagnet, under

static conditions, is equal to the product of the active voltage and the total current strength. Hence, the ratio of the alternating terminal voltage to the direct terminal voltage, for a common current strength in both cases, is equal to the ratio of (3) the geometric sum of the e.m.f. of self-induction and the active voltage to (4) the active voltage, or to the square root of 1 plus (5) the squares of the ratio of the e.m.f. of self-induction to the active voltage. (5) Is so great with respect to 1 that the e.m.f. of self-induction may be considered as being equal to the total impressed voltage. Therefore, for a given current strength, the ratio of the impressed alternating voltage to the impressed direct voltage is equal to the ratio of the e.m.f. of self-induction to the active voltage.

The e.m.f. of self-induction is equal to 6.2832 times the continued product of (1) the frequency in cycles per second, (2) the self-inductance in henries, and (3) the amperage. The active voltage is equal to the product of the resistance and the amperage. Consequently, the ratio of the alternating impressed voltage to the direct impressed voltage is equal to the product of the angular velocity in radians per second and the time constant in seconds $= \omega t_c = \tan \theta$ (page 358). The phase angle θ is substantially 90 deg. (current lagging), particularly when the air gap is short and the current strength is small.

The effect of hysteresis, secondary currents, or the movement of the armature or plunger under the influence of the mechanical force is to reduce the self-inductance, for constant flux-turns, and to increase the current strength. The time-rate of change in the current strength is increased as the self-inductance is decreased, so that the e.m.f. of self-induction is approximately constant, although the active voltage, or the in-phase component of the impressed voltage, varies directly with the current strength. In such cases, the phase angle may be in the neighborhood of 80 deg. under static conditions. Referring to Fig. 394, the power factor $\cos \theta$ is about 0.13, on the average, for air gaps up to 1 in. in length. Under those conditions, the angle of lag is about 82 deg., 30 min.

When $f = 60$, $\omega = 376.9$ (usually taken as 377). Referring to Fig. 398, when the length of the air gap is slightly greater than $\frac{3}{4}$ in., the number of alternating terminal volts is about 98, the number of direct terminal volts is about 8, and the ratio of the former to the latter is $\frac{98}{8} = 12.25$. Hence, the time constant is

equal to $\frac{12.25}{377} = 0.0325$ sec. The resistance is equal to $\frac{8 \text{ volts}}{10 \text{ amp.}} =$ 0.8 ohm, and $L = Rt_c = 0.8 \times 0.0325 = 0.026$ henry.

265. Stored Magnetic Energy and Volt-amperes.—The stored magnetic energy in joules is equal to one-half the product of the self-inductance in henries and the square of the amperage. The maximum number of volt-amperes, as when $\theta = 90$ deg., is equal to 6.2832 times the product of the self-inductance in henries and the square of the amperage. Consequently, the number of joules per volt-ampere is equal to $\frac{0.079\ 577}{f}$, and the number of volt-amperes per joule of stored magnetic energy is equal to at least 12.567 times the frequency f in cycles per second. Hence, the maximum amount of magnetic energy that can be stored in a 60-cycle electromagnet per volt-ampere is equal to $\frac{0.079\ 577}{60} =$ 0.001 326 joule, and the number of volt-amperes required to store 1 joule of magnetic energy in the same electromagnet is equal to at least $12.567 \times 60 = 754$ volt-amp. Hence, if the terminal voltage is 110 volts and the current strength is 7.76 amp., 1 joule of magnetic energy, as a maximum, is stored in the magnetic circuit of a 60-cycle electromagnet.

Since 1 joule = 8.8513 in.-lb., the number of inch-pounds is equal to 8.8513 times the number of joules, and joules = 0.112 98 $\times$ inch-pounds. Consequently, the number of inch-pounds per volt-ampere is equal to $\frac{(8.8513 \times 0.079\ 577)}{f} = \frac{0.7044}{f}$, and the number of volt-amperes per inch-pound of stored magnetic energy is equal to $1.42f$. Hence, the maximum amount of magnetic energy that can be stored in a 60-cycle electromagnet per volt-ampere is equal to $\frac{0.7044}{60} = 0.011\ 74$ in.-lb., and the number of volt-amperes required to store 1 in.-lb. of magnetic energy in the magnetic circuit of the same electromagnet must be at least equal to $1.42 \times 60 = 85.2$.

As an example, let it be assumed that the phase angle θ is 90 deg. (lagging), which is the condition for the maximum magnetic energy storage per volt-ampere. Then, if the number of terminal volt-amperes is, say, 1,000 in the initial position of the armature or plunger of a 60-cycle electromagnet, and is 500 in the final position, the maximum amount of work done by the electro-

magnet is equal to $(1{,}000 - 500) \times 0.011\ 74 = 500 \times 0.011\ 74 = 5.87$ in.-lb., regardless of how the self-inductance or the current strength, or both, change during the stroke.

266. Losses in Solid Cores.—A solenoid-and-plunger similar to that described in Art. 137, page 216, was tested with alternating current in order to determine the self-inductance for various positions of the plunger, which was 1.04 in. in diameter and 10 in. long. The outside diameter of the solenoid was 3 in. and its length was 7¼ in. It contained 1,053 turns of wire wound in 12 layers. The resistance was 1.3 ohms, and the frequency was 60 cycles per second. The measured and calculated self-inductance of the solenoid alone was 0.011 henry. The constant-current strength was 3.4 amp. The heat loss in the conductor was equal to $3.4^2 \times 1.3 = 15$ watts.

The test was conducted for the purpose of obtaining data under the worst possible conditions with a fairly good steel, that is, with a solid steel core. Therefore, the self-inductance herein referred to is not the self-inductance that might be obtained with a laminated core under otherwise similar circumstances. The effect of iron loss is to decrease the self-inductance by requiring a greater m.m.f. than as though the iron loss was not present, thus requiring a greater r.m.s. current strength for the same number of flux-turns.

In making the test, the current strength was maintained constant by means of a variable reactor, and the terminal voltage was measured with each change in the position of the core or plunger. The self-inductance was found by the following rule:

1. Divide the terminal voltage by the current strength in amperes and square the ratio thus found.
2. Subtract the square of the resistance from (1).
3. Extract the square root of (2).
4. Divide (3) by the angular velocity (377 in this case).

The active voltage was constant, being equal to $3.4 \times 1.3 = 4.42$ volts. Since the e.m.f. of self-induction is equal to the geometric difference of the terminal voltage and the constant active voltage, the e.m.f. of self-induction was approximately equal to the terminal voltage.

The data obtained from this test are shown in Fig. 400, wherein the terminal voltage and the self-inductance are plotted against the distance that the plunger is inserted into the core hole of the solenoid.

The same solenoid-and-plunger was tested on a lighting circuit wherein the voltage drop was quite large when the plunger was removed from the coil and the maximum current strength resulted. This test was conducted in approximately the same manner as previously described, but no attempt was made to keep the r.m.s. current strength constant. The self-inductance was calculated by the rule previously employed.

The data obtained from this test are shown in Fig. 401. V.-a. is the number of volt-amperes, I is the current strength in amperes, $\frac{1}{2}LI^2$ is the stored magnetic energy in joules, $\frac{NI}{L}$ is the ampere-turn gradient (L being the length of the solenoid), E_i is the terminal voltage, E_L is the e.m.f. of self-induction, and E_a is the active voltage. It is to be noted that the ratio of the stored magnetic energy in joules to the number of terminal volt-amperes is a constant quantity (see page 379). It is to be noted that the total self-inductance is equal to the sum of the self-inductance of the solenoid alone and the self-inductance due to the presence of the plunger, as will be noted in Fig. 401.

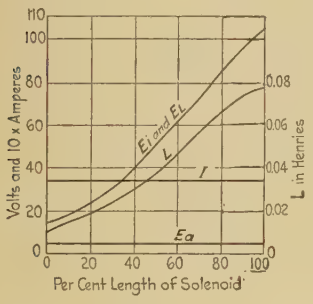


Fig. 400.

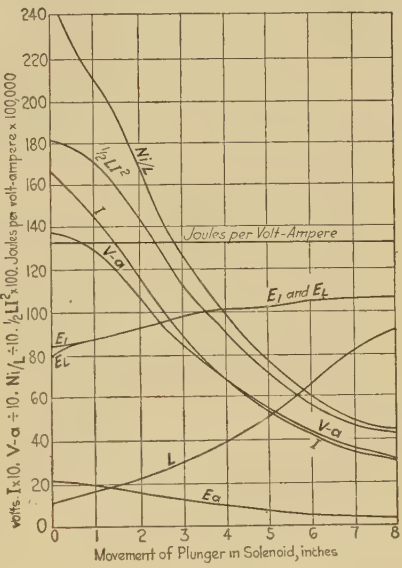


Fig. 401.

Fig. 400.—Test characteristics of alternating-current solenoid with solid core.
Fig. 401.—Further test characteristics of the alternating-current solenoid in Fig. 400.

Referring to Fig. 401, when the end of the plunger is 3.5 in. inside of the solenoid, the length-rate of change in the self-inductance is about 0.01 henry per inch, the current strength is 7.7 amp., and the square of 7.7 is 59.3. By the method described

in Art. 132, page 209, the mechanical force or pull should be equal to about $4.426 \times 59.3 \times 0.01 = 2.63$ lb. The ampere-turn gradient for this position of the plunger is 1,130 amp.-turns per inch, the total number of ampere-turns being 8,200. The dimensions of the solenoid described on page 216 are almost identical with those of the present solenoid, and the test characteristics in Fig. 244 were obtained with a plunger 0.7854 sq. in. in cross-section, but $8\frac{1}{8}$ in., instead of 10 in., long. Therefore, neglecting iron losses in the plunger, the pulls for similar plunger positions, cross-sections of plunger, and ampere-turn gradients should be approximately the same.

Referring to Fig. 244, when the end of the plunger is 3.5 in. inside of the solenoid and the number of ampere-turns is 8,468, the mechanical force is 5.1 lb. If the cross-section of the plunger used in connection with Fig. 244 was 0.85 sq. in. (as in the case of the alternating-current solenoid), the force would have been equal to about $\frac{0.85}{0.7854} \times 5.1 = 5.5$ lb., but with 8,200 amp.-turns it would have been equal to about $\left(\frac{8,200}{8,468}\right) \times 5.5 = 5.3$ lb. Therefore, 5.3 lb. is about the greatest pull that can be obtained with the alternating-current solenoid when the plunger is 3.5 in. inside of the solenoid.

The effect of iron loss is to flatten the self-inductance curve in Fig. 401 because less magnetic flux is produced for a given r.m.s. current strength when eddy currents and hysteresis oppose the magnetizing current. The fact that the pull calculated from the length-rate of change in the self-inductance is but $\frac{2.63}{5.3} = 0.495$, or about 50 per cent, of what the pull would be if there were no iron losses shows that the total iron loss is about 50 per cent in the case of the alternating-current solenoid. There is no iron loss in the direct-current solenoid under static conditions, as in the case above referred to. Therefore, the alternating-current strength is double what it otherwise would be in order to supply the iron loss. Hence, 50 per cent of the total current strength is engaged in balancing the effects due to eddy currents and hysteresis, while the other 50 per cent is useful in producing mechanical force.

267. Separating the Magnetic Fluxes.—When resistance, hysteresis, and eddy currents may be neglected or compensated

for, the e.m.f. of self-induction may be considered equal to the terminal voltage. Either voltage may then be taken as equal to 0.707 times the product of the angular velocity and the maximum number of weber-turns. Hence, the maximum number of weber-turns will be equal to 1.41 times the ratio of the impressed voltage to the angular velocity. When the frequency is 60 cycles per second, the angular velocity is 377 radians per second. Therefore, the maximum number of weber-turns per volt is equal to $\frac{1.41}{377} = 0.003\ 74$ at 60 cycles, $0.003\ 74 \times (60/25) = 0.008\ 97$ at 25 cycles, and so forth. For instance, if a solenoid is to be connected across a 110-volt, 60-cycle lighting circuit, the maximum number of weber-turns will be equal to $0.003\ 74 \times 110 = 0.411$. If the number of turns is 1,000, and the magnetic flux links with all of the turns, the maximum number of webers linking and unlinking with the turns as the current alternates will be equal to 0.000 411, or to 41,100 maxwells.

From the foregoing, the number of flux-turns tends to remain constant when the frequency and the voltage are constant. Let a constant voltage at constant frequency be impressed across the terminals of a solenoid having no appreciable resistance. Then the resulting current strength will be the maximum r.m.s. current strength that can be obtained with that impressed voltage and frequency, and the resulting magnitude of the magnetic flux will be the maximum magnitude that can result whether or not iron, steel, or any other magnetic material is placed in the core hole. Therefore, when steel is placed in the core hole, the total maximum magnitude of the magnetic flux will be the same as before, but the maximum magnitude of the electromagnetic flux will be reduced, the difference being replaced by structural or ferromagnetic flux in the steel. Since the current strength is directly proportional to the magnitude of the electromagnetic flux, and *vice versa*, it is reduced in the same proportion that the electromagnetic flux is reduced.

The product of the densities of the electromagnetic flux and structural flux is a measure of the mechanical force produced between the solenoid and the steel. The total flux density is the induction, and is constant, being equal to the sum of the two magnetic flux densities. The magnitude of the total maximum magnetic flux in webers is always equal to the ratio of the e.m.f. of self-induction to 4.44 times the product of the frequency and the

number of turns linked with the total magnetic flux. To convert from webers to maxwells, multiply by 100,000,000.

The self-inductance of an air-cored solenoid is equal to the ratio of the maximum number of weber-turns to the maximum current strength in amperes, or to the ratio of the respective r.m.s. values. The self-inductance also may be calculated by the method described on page 173. Let wires or strips of magnetic material be placed full length within the core hole of a solenoid having a length approximately equal to the common length of the wires or strips. Every time a wire or strip is placed in the core hole of a solenoid having a constant voltage across its terminals, at constant frequency, the self-inductance is increased thereby because the constant number of flux-turns is maintained with a smaller current strength. As more steel is added, the current strength is correspondingly reduced, but the number of flux-turns remains constant.

The product of the self-inductance in henries and the r.m.s. amperage is equal to 0.707 times the maximum number of weber-turns. As the self-inductance increases, due to the addition of magnetic material, the current strength decreases, but the product of the self-inductance and the current strength is constant when there are no iron or secondary-current losses, and no relative motion results. Hence, when the constant impressed voltage and frequency are known, the self-inductance may be computed from the measured current strength, as described on page 380. Having calculated the self-inductance for the air-cored solenoid (page 173) the r.m.s. current strength is calculated for the known impressed voltage and frequency. For instance, if the self-inductance of the air-cored solenoid is found to be 0.05 henry, the impressed voltage is 110 volts, and the frequency is 60 cycles per second, the r.m.s. current strength is equal to $\left(\frac{110}{377 \times 0.05}\right) = \frac{110}{18.85} = 5.85$ amp. It is known that the maximum number of weber-turns in any half cycle will remain (page 383) constant at 0.411. Therefore, when there is no magnetic material in the core hole of the solenoid, the number 0.411 represents 100 per cent electromagnetic flux and zero structural flux. If, after the magnetic material has been placed in the core hole, the r.m.s. current strength is reduced to, say, 0.5 amp., the maximum current strength is equal to $1.41 \times 0.5 = 0.707$ amp. It is known that the maximum number of weber-turns is constant

and equal to 0.411. Therefore, the self-inductance is now equal to $\frac{0.411}{0.707} = 0.581$ henry.

The method of separating the magnetic fluxes is based on the following premises:

1. The r.m.s. magnitude of the total magnetic flux is constant.

2. The r.m.s. magnitude of the electromagnetic flux is directly proportional to the r.m.s. current strength.

3. The product of the self-inductance in henries and the r.m.s. current strength in amperes is constant and equal to the r.m.s. number of weber-turns, and also is equal to the ratio of the r.m.s. impressed voltage to the angular velocity in radians per second; or self-inductance $L \times$ r.m.s. amperage $I = 0.707 \times$ the number of maximum weber-turns $= \frac{\text{voltage}}{\text{angular velocity}} =$ a constant number $= 0.2915$ for a 110-volt, 60-cycle circuit.

4. The total maximum magnetic flux in webers is equal to the ratio of (*a*) the r.m.s. voltage to (*b*) $0.707 \times 6.2832 = 4.44$ times the product of the frequency in cycles per second and the number of turns linked with the total maximum magnetic flux in webers.

5. The ratio of the electromagnetic flux to the total magnetic flux is equal to the ratio of the current strength with steel present to the current strength without steel present.

6. From (5), the magnitude of the electromagnetic flux is equal to the product of (*a*) the total magnetic flux and (*b*) the ratio of the current strength with steel present to the current strength without steel present.

7. The maximum magnitude of the electromagnetic flux in webers is equal to the product of (*a*) the ratio of the r.m.s. voltage to 4.44 times the product of the frequency and the number of turns linked with the total magnetic flux and (*b*) the ratio of the current strength with steel present to the current strength without steel present.

8. The magnitude of the structural flux is equal to the difference between that of the total magnetic flux and the electromagnetic flux.

Consequently, if 5.85 amp. will produce 0.411 weber-turns with no steel present, 0.5 amp. will produce $0.411 \times \left(\frac{0.5}{5.85}\right)$ $= 0.411 \times 0.0855 = 0.0352$ electromagnetic weber-turns, or $\frac{0.0352}{0.411}$ $= 0.0855$, or 8.55 per cent, of the total number of weber-turns.

and the number of ferromagnetic weber-turns is equal to 0.411 − 0.0352 = 0.3758.

This rule is only approximate because of the changes in average permeability with corresponding changes in magnetizing force, and also because of the decrease in self-inductance due to iron loss, but it is very useful in approximate calculations and as a check on test data.

268. Condition for Maximum Pull.—The condition for the maximum pull that may be obtained with a constant-voltage, alternating-current solenoid-and-plunger is when the magnitude of the structural flux is numerically equal to the magnetizing force. This is easily proved by the following test:

In this test, a 12-in. solenoid containing 2,346 turns was used. The other dimensions were as follows: inside diameter, 1 in.; outside diameter, 3 in.; coil-wall thickness, 1 in.; average radius, 1 in. The plungers were of solid soft steel and were 15 in. long. The self-inductance of the solenoid alone was 0.0364 henry. Only r.m.s. values are here assumed. The current strength with no steel present was 7.2 amp. The frequency was 60 cycles per second. The various cores always were placed inside of the coil with their middles at the middle of the coil. Each time a core or plunger of greater cross-section was thus inserted, the self-inductance and the structural flux increased while the current strength and the electromagnetic flux decreased. The total number of flux turns was substantially a constant quantity. Figure 402 shows the results of the test.

The pull between a solenoid and its plunger is directly proportional to the continued product of (1) the magnetizing force or the electromagnetic flux density, (2) the ferromagnetic flux density, and (3) the cross-section of the plunger. The product of (2) and (3) is simply the ferromagnetic flux. Therefore, the conditions for maximum pull between a solenoid and its plunger for any fixed relative position is that the product of the magnetizing force and the ferromagnetic flux shall be a maximum. In the direct-current solenoid-and-plunger, the magnetizing force is not decreased by increasing the amount of magnetic material inside of the core hole under static conditions. In the constant-voltage alternating-current solenoid-and-plunger, the magnetizing force is decreased in about the same proportion that the amount of magnetic material increases. Therefore, the separation of the magnetic fluxes leads to the important fact that there

is only one cross-section of plunger for any given alternating-current solenoid-and-plunger, or for any given alternating-current iron-clad solenoid or plunger electromagnet, that will produce the maximum pull with a given number of turns, voltage, and frequency.

Referring to Fig. 403, which was developed from Fig. 402, since the cross-section of the core hole is 0.785 sq. in., the magnetizing force is equal to 1.27 times the total magnitude of the electromagnetic flux. Hence, the maximum product of the magnitudes of the electromagnetic flux and the ferromagnetic flux locates 1.27 times the cross-section of the plunger, which will give the maximum pull between the solenoid and the plunger, and

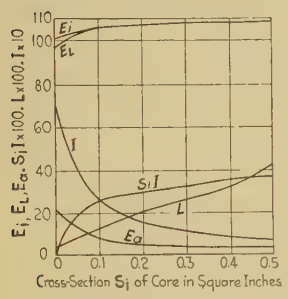


Fig. 402.

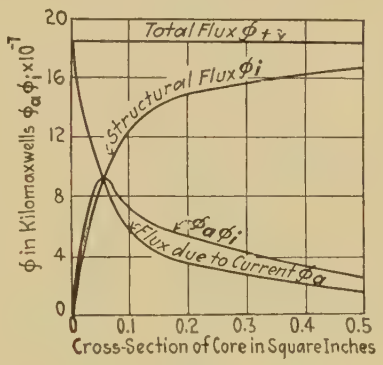


Fig. 403.

Fig. 402.—Conditions for maximum pull.
Fig. 403.—Conditions for maximum pull.

this is found to be approximately 0.05 sq. in. making the proper cross-section of the plunger, with the number of turns, voltage, and frequency employed, equal to $0.785 \times 0.05 = 0.0394$ sq. in., since the maximum pull is obtained when the product of the magnetizing force and the magnitude of the ferromagnetic flux is a maximum. If this plunger was cut across at its middle, as in the Maxwellian hypothesis, the maximum opposed pulls would be obtained.

The foregoing shows that small plungers must be used where there are many turns. Constant-voltage, alternating-current electromagnets in general have the usual sizes of cores and plungers, but the number of turns is relatively small as compared with direct-current electromagnets.

269. The Effect of Winding the Coil in Sections.—The solenoid above described was made up of six separate coils placed end to end and connected in series, as in the case of the solenoid in Fig. 404. In the tests above described, however, the solenoid was used as a unit

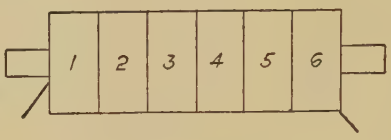


Fig. 404.—Twelve-section solenoid divided into 6 coils.

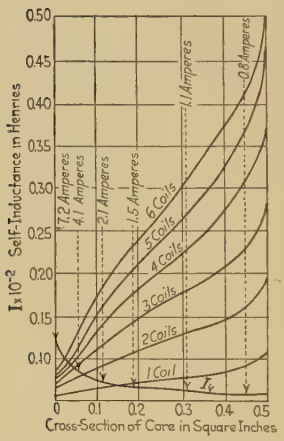


Fig. 405.

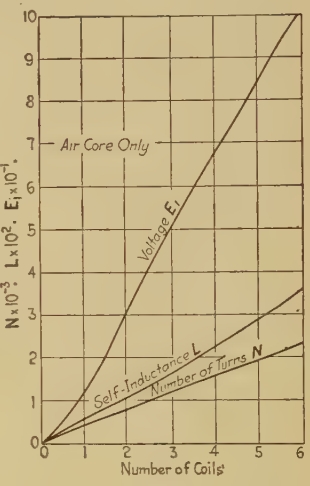


Fig. 406.

Fig. 405.—Alternating-current solenoid with various cross-sections of solid plunger.

Fig. 406.—Air-cored solenoid divided into sections.

The solenoid in Fig. 404 also was tested with a constant-current strength of 7.2 amp. in the entire coil, and the voltage was then measured from one end terminal to each of the intermediate taps or connections between the individual sections, each section being 2 in. long. The results are shown in Figs. 405 and 406.

CHAPTER XXIII

ALTERNATING-CURRENT ELECTROMAGNETS AND MECHANISMS

Specific applications of alternating currents are herein treated, with special reference to the construction of alternating-current electromagnets and electric hammers.

270. Single-phase Electromagnets.—Figure 407 shows a simple arc-lamp type of two-coil single-phase electromagnet, Figure 408 shows the coil and moving elements of a single-phase brake magnet. The undervoltage trip coil[1] in Fig. 409 has two

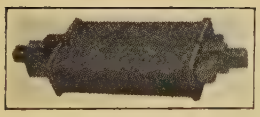


Fig. 407.

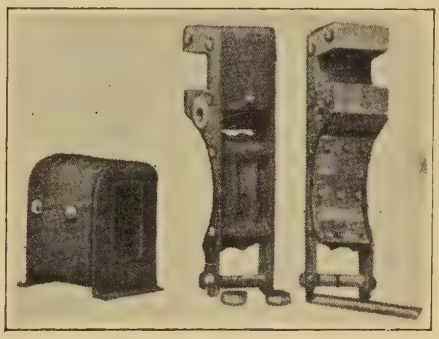


Fig. 408.

Fig. 407.—Arc-lamp type alternating-current plunger electromagnet.
Fig. 408.—Elements of single-phase brake magnet.

opposing U-shaped armatures which normally float in the magnetic field. Both armatures fall, upon a reduction of voltage, armature *A* coming to rest on its stop. When a further reduction in voltage permits the armatures to separate, armature *B* falls and strikes the breaker trip latch a hammer blow.

Figure 412 shows the parts of the brake magnet illustrated in Figs. 410 and 411. The parts of the contactor in Figs. 413 and 414 are illustrated in Fig. 415. In Figs. 412 and 415, each division represents 1 in. These are general examples of design of standard equipment. The spring interposed between the end

[1] "Alternating-current Magnet," *Elec. World*, vol. 76, p. 104, 1920.

of the plunger and the clevis in Figs. 410 and 411 assists in starting the load and also tends to prevent noise.

The above spring feature is shown in U. S. Patent No. 607589, July 19, 1898, issued to J. H. J. Haines, who states therein that

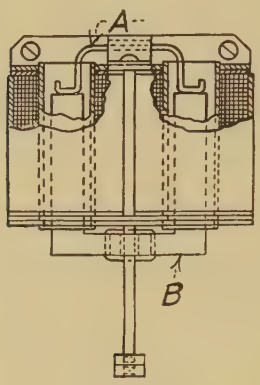


Fig. 409.

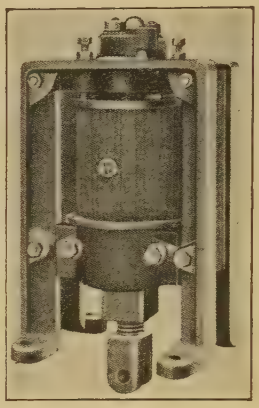


Fig. 410.

Fig. 409.—Trip coil for use with predetermined low-voltage conditions.
Fig. 410.—C-H standard brake magnet with floor-type mounting.

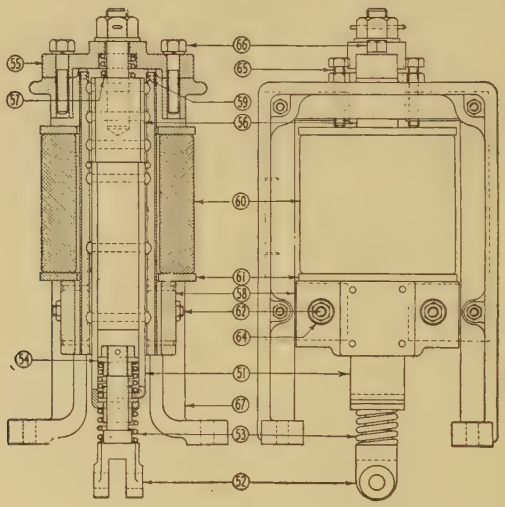


Fig. 411.—Cross-section of the magnet in Fig. 410.

the core consists of a bundle of soft-iron wires housed by an insulating jacket and secured in place by pouring among them some silicate of soda which binds them in a solid mass (Fig. 416).

Single-phase electromagnets are generally noisy unless provided with shading coils. This has led to the application of the term growler to devices for testing the armatures of motors, the

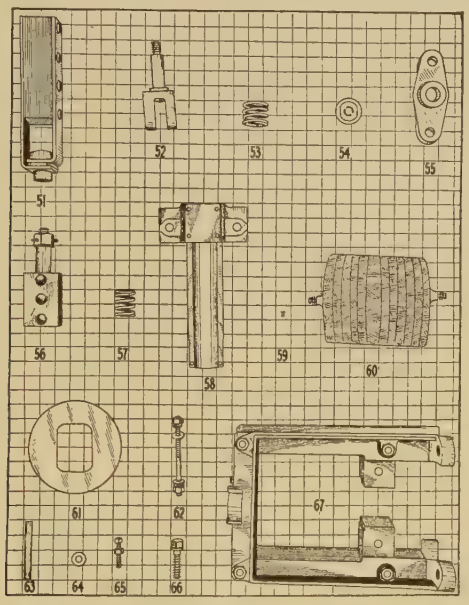


Fig. 412.—Parts of the magnet in Fig. 411.

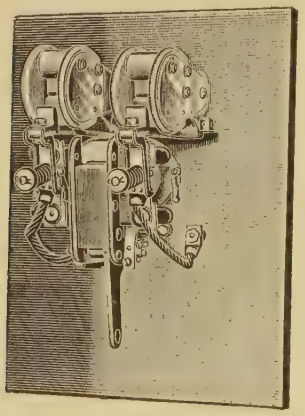


Fig. 413.—Contactor with interlock.

general scheme being illustrated in Fig. 417. Short circuits are detected by rotating the armature in the circular space while a piece of soft iron is held over the top, the strip vibrating strongly

if a short circuit is present. The differential magnetic balance in Fig. 418 is for testing field coils and the like for short circuits.

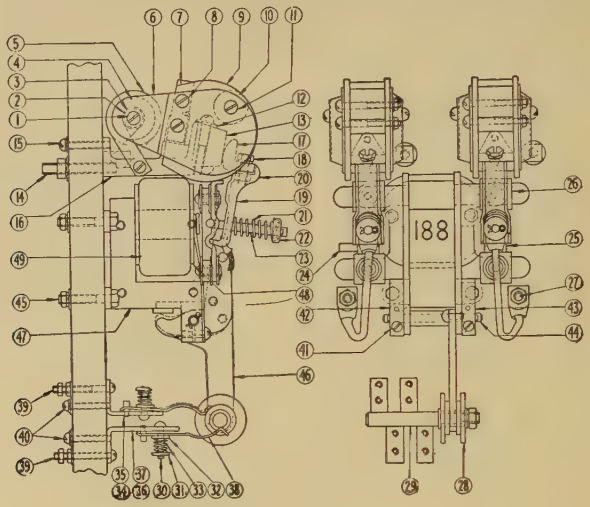


Fig. 414.—Contactor with interlock (Fig. 413).

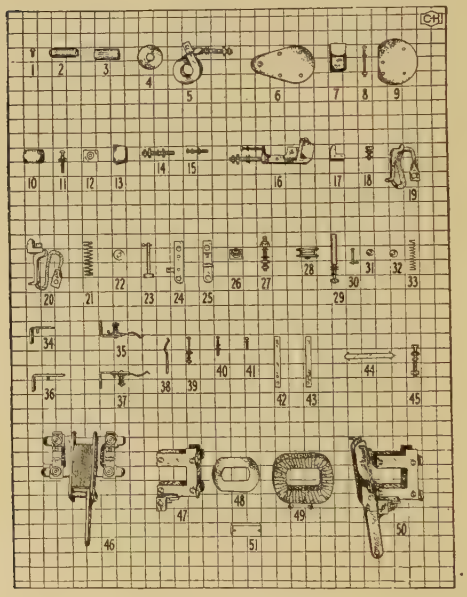


Fig. 415.—Parts of the contactor in Fig. 414.

It has a laminated shunt magnetic circuit with the primary or exciting coil C on the middle leg. The differential secondary

coils A and B are connected to the meter M (or a telephone receiver) to form a local circuit. The laminated keeper or bridge is removable. Without coil D in place no indication at M will be obtained, showing that there are no short-circuited turns in coil D, an indication showing the presence of shorts. An open coil is detected by short-circuiting it and noting whether or not there is an indication. The short-circuited turns are located by their higher temperatures, which increase if the circuit is kept closed for a short time. A single short-circuited turn may be located in this manner and, if the conductor is of large cross-section, it may not be possible to locate it by measuring the resistance.

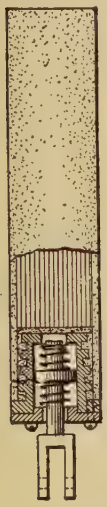


Fig. 416.

271. Splitting the Phase.—By the use of a condenser, the currents in two separate coils may be displaced by 90 electrical degrees and, therefore, the general effect of a two-phase electromagnet may

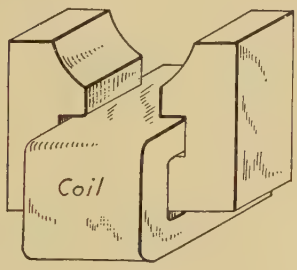


Fig. 417.

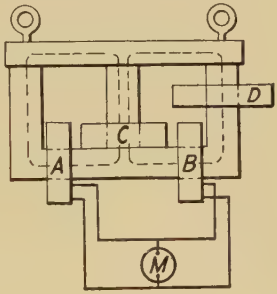


Fig. 418

Fig. 416.—Construction of plunger.
Fig. 417.—Growler.
Fig. 418.—Differential magnetic balance.

be obtained. Frank F. Fowle has described[1] some successful designs of telephone relays for operation on 16.7-cycle circuits with a very small consumption of energy. Two circuit arrangements are possible, as illustrated in Figs. 419 and 420, one of the currents leading 45 deg. and the other lagging 45 deg., with respect to the impressed voltage. The condition for 90 deg. phase difference between the currents in the coils in Fig. 419 is that $\frac{X_{L1}}{R_1} = \frac{-R_2}{(X_{L2}-X_c)}$. If the coils are equal in every respect, the conditions are that $R_1 = R_2 = X_{L1} = X_{L2} = \frac{X_c}{2}$

[1] "An Alternating-current Relay for Low Frequency," *Elec. World*, vol. 44, p. 862, 1904.

and the total impedance is equal to the resistance of one branch. The relation between the self-inductance in henries of one coil and the necessary capacitance in microfarads of the necessary condenser is that their product shall be equal to $\frac{12{,}672}{f^2}$, wherein f is the frequency in cycles per second. For 16.7 cycles, the prod-

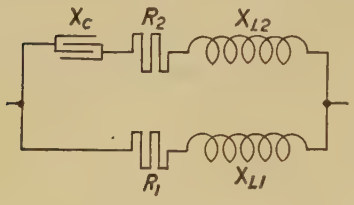


Fig. 419.

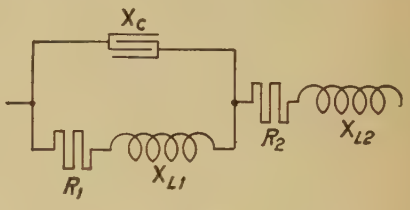


Fig. 420.

Fig. 419.—Split-phase conditions.
Fig. 420.—Split-phase conditions.

uct is 45.5. For 60 cycles, the product is 3.52. For similar conditions in Fig. 420, $X_{L1} = X_{L2}$. If the coils are equal and the currents in the respective coils are equal, the individual resistances, reactances, and capacitances are equal to each other. The total impedance is then equal to twice the resistance of one coil. The product of the self-inductance in henries and the

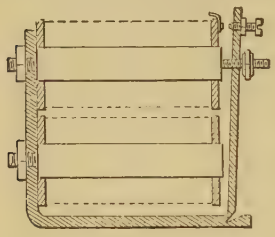


Fig. 421.

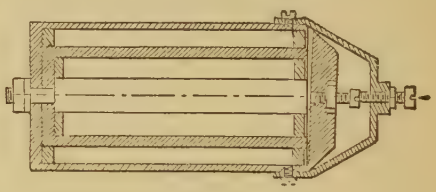


Fig. 422.

Fig. 421.—Split-phase telephone relay.
Fig. 422.—Split-phase telephone relay.

capacitance in microfarads is then equal to $\frac{6{,}386}{f^2}$, and is equal to 90.9 at 16.7 cycles, and 7.04 at 60 cycles. Two forms of relays are shown in Figs. 421 and 422. The former is the cheaper and the latter the more practicable.

In the single-phase arrangement illustrated in Fig. 423, the value of the effective magnetic flux in the outer legs is constant and in time-quadrature with the supply voltage. However, the flux in the central leg is in time phase with the current in the

motor of which the electromagnet may form a part of the brake therefor. If the power factor of the motor were 100 per cent, the flux in the central leg would be in time quadrature with that in the outer legs, with the result that a pull on the common armature would be maintained at all times. Even with the usual power factors, some pull results at all times. It is claimed by C. Krämer, to whom a patent was issued on Feb. 4, 1908, that this arrangement obviates the loss of energy required by the split-phase devices.

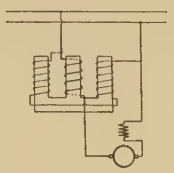


Fig. 423.—Krämer alternating-current magnet.

In one of a series of articles by D. L. Lindquist,[1] it is shown that a two-phase electromagnet may be made to operate as a split-phase electromagnet on a single-phase circuit by the

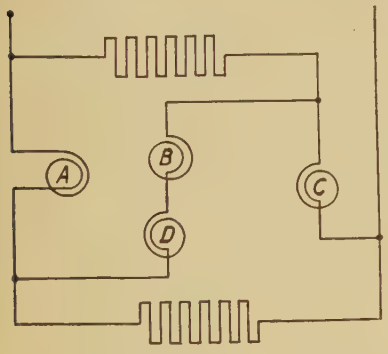


Fig. 424.

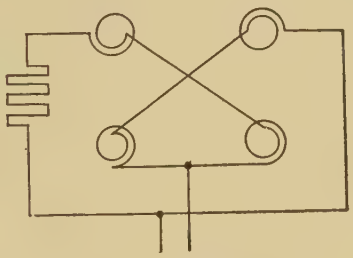


Fig. 425.

Fig. 424.—Two-phase magnet arranged for single-phase operation.
Fig. 425.—Two-phase magnet arranged for single-phase operation.

nsertion of resistors, as in Fig. 424, and by making the air gaps longer in the cases of A and C than in those of B and D. Figure 425 also is due to Mr. Lindquist.

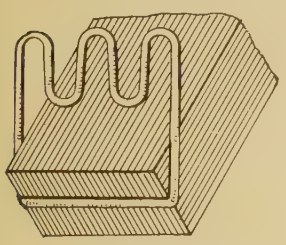


Fig. 426.—One method of applying a shading coil.

272. Shading Coils.—Single-phase electromagnets are made to operate as two-phase electromagnets by simply inserting a looped conductor in the end of a coge, polepiece, armature, or plunger, as in Fig. 426, so that it will enclose only a fractional part of the total flux. In principle, the shading coil is the secondary of a transformer wound on one of two branches of a ferromagnetic

[1] *Elec. World*, vol. 48, pp. 128, 564, 1906.

circuit, the primary winding surrounding both of said branches. When the primary current is equal to zero, the m.m.f. due to the induced current in the shading coil establishes a local magnetic

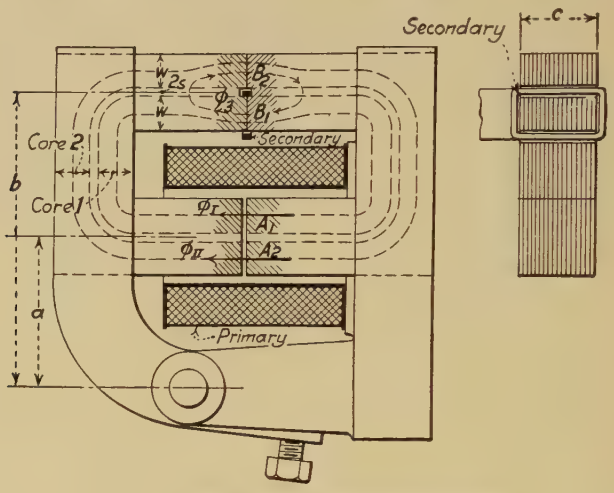


FIG. 427.—Construction of split-phase magnet.

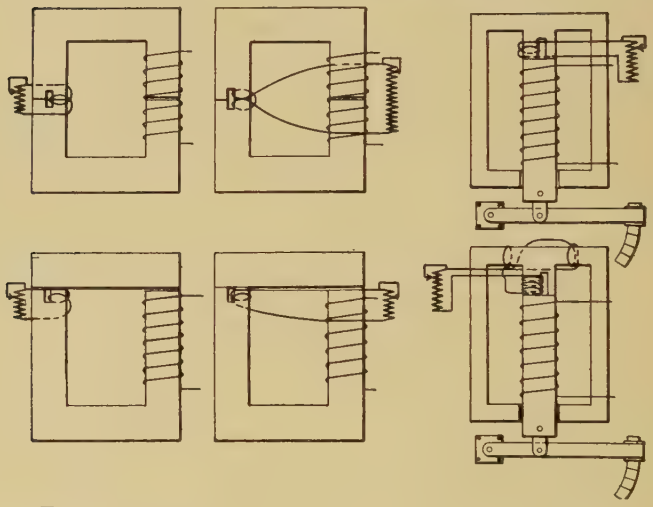


FIG. 428.—Alternating-current magnets with shading coils.

flux in a closed magnetic circuit, thus causing the coacting parts to be held together during this interim. The split-phase effect is thus obtained in a very simple manner. Figure 427 is from an

article by F. Hymans,[1] wherein the two parallel ferromagnetic circuits in the same structure are represented by dotted lines.

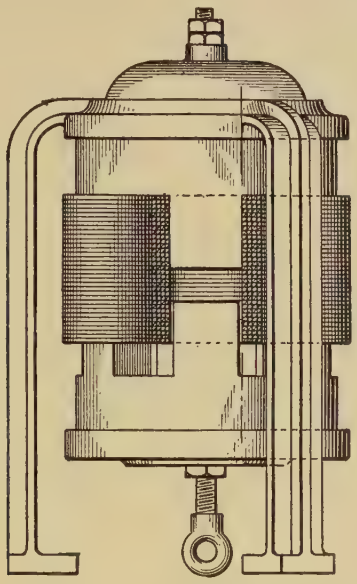


FIG. 429.—Polyphase electromagnet.

FIG. 430.

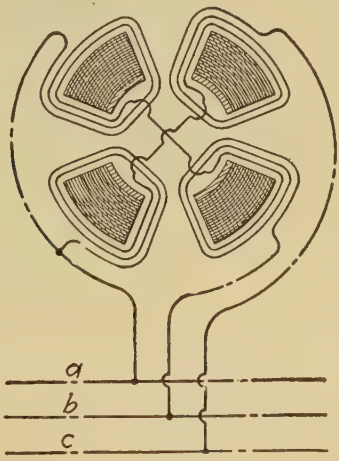


FIG. 431.

FIG. 430.—Y-connection of three-phase magnet.
FIG. 431.—Two-phase magnet connected to three-phase circuit.

Figure 428 shows various applications of shading coils as disclosed in U. S. Patent No. 1255441, issued to D. C. Larson in

[1] "Theory and Operation of Split-phase Magnet," *Elec. World*, vol. 68, pp. 1000, 1192, 1290, 1916.

1918. Shading coils will be noted in many of the alternating-current electromagnets in this volume. It is important that the resistance should be properly proportioned, and that means be provided to dissipate the heat in the cases of large electromagnets, in which cases the shading coils are sometimes made of cast iron or steel.

273. Polyphase Electromagnets.—A number of patents were issued to D. L. Lindquist in 1903 and 1904. Figures 429 and

FIG. 432.—Two-phase magnet with end removed

FIG. 433.—Three-phase magnet with end removed.

430 are taken from U. S. Patent No. 744773. Figure 431, from the same patent, shows a two-phase electromagnet connected across a three-phase circuit. A six-core type also is shown. Figures 432 and 433 were supplied by Mr. Shepard, then of the Warner Elevator Company. These are made from stacked laminations and, after tapering, are wound somewhat similar to the manner in which the stator of an induction motor is wound.

Even polyphase electromagnets are apt to be noisy under heavy loads, and are sometimes provided with shading coils. All forms

of alternating-current electromagnets are sometimes immersed in oil to deaden the noise, especially in connection with elevator brake magnets in hotels and apartments.

274. Mechanical Auxiliaries.—In the article by F. F. Fowle, referred to on page 393, he states:

> It was found by experiment that the simple types of relays designed for direct-current circuits were inoperative, owing to armature vibration and consequent failure to maintain a closed local circuit. This difficulty had been overcome by means of spring contacts and heavy armatures, but relays embodying these features were sluggish in their action and uncertain in releasing their local contacts when adjusted to operate on small energies. These relays were universally of the single magnetic circuit type.

Spring (or floating) contacts have been successfully employed in relays of the telegraph type where more energy is available than in the small and sensitive telephone relays. Three alternating-current relays patents were issued to J. F. D. Hoge on Oct. 17, 1911. In one of these patents a light rod extends beyond the contacts and carries a mass, its distance from the fulcrum being adjustable. The momentum of the mass or weight and the resiliency of the armature and rod cause the contact to be held closed when the current is equal to zero. In another case, a paddle is adapted to move a liquid contained in a suitable dash pot. In the third case he states:

> If a suitable weight mass be mounted on the armature lever between the support for the lever and the contact point, such lever will vibrate in unison with the energizing current, and with a stationary node in contact with the front contact point of the relay, over a considerable frequency range of the energizing current.

Tuned armatures are employed in one form of frequency meter, referred to on page 200, which is adaptable for measuring the frequency of mechanical, as well as electrical, vibration. A number of cases have been mentioned in this volume where masses are employed to prevent sudden motions of the parts under the influence of sudden changes in the mechanical force due to corresponding fluctuations in the current strength, where frictional devices, as dash pots, are employed to make the velocity of the moving parts uniform, and where springs are used to store mechanical energy and then expend it. All of these have their counterparts in tuned electric circuits, self-inductance, resistance,

and capacitance. Dr. W. S. Franklin has shown mechanical models which behave as do the electric phenomena in series and parallel alternating-current circuits. Just as the natural frequency of vibration or oscillation of electricity in an electric circuit is directly proportional to the square root of the ratio of the elastance to the self-inductance, so is the natural frequency of vibration of a spring and mass combination directly proportional to the square root of the ratio of the tension to the mass.

275. Rotating Mechanisms.—A bar electromagnet comprising a core of soft-iron wires about 1 in. in diameter and 7 in. long surrounded by a coil containing about 600 turns of No. 18 magnet wire, with one core end projecting about 1 in. from the coil, is

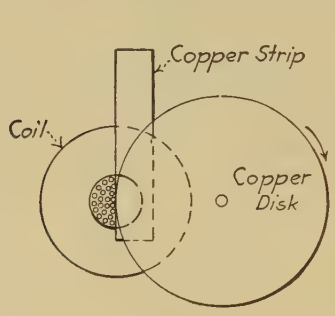


FIG. 434.

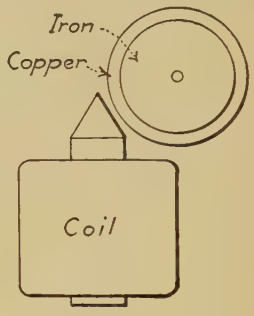


FIG. 435.

FIG. 434.—Rotation due to eddy currents.
FIG. 435.—Rotation due to eddy currents.

suitable for repeating some of the original experiments of Dr. Elihu Thomson. A ring of No. 1 copper wire with a good joint, or else a short-circuited coil of either bare or insulated wire having a total cross-section of copper equal to that of the No. 1 wire, and with an inside diameter such that it may pass freely over the outside of the coil, may be used to demonstrate the repulsion and heating effects previously referred to.

A copper disk, about 0.032 in. thick and 6 in. in diameter, may be supported in any desired manner so that it shall rotate when its edge overlaps the core end, with the shading copper strip in position, as in Fig. 434. Eddy currents are induced in both the disk and shading strip, and these tend to decrease the self-inductance and the permenance of the system. If the eddy currents could neutralize the effects of the eddy currents in the shading strip, the maximum self-inductance would be obtained.

Therefore, the disk rotates to tend to produce said neutralization. By similar means, hollow globes and cups of copper may be made to rotate, as when floated in water. Figure 435 shows another application of the general principle.

The above principle is applied to a device for measuring the total energy delivered to a distributing transformer from its low-

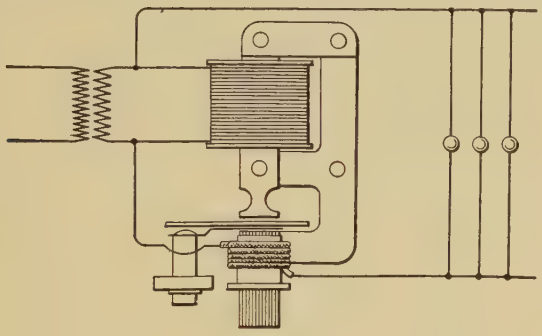


Fig. 436.—Circuits through measuring instrument.

tension secondary side, as described in U. S. Patent No. 1218538, 1917, issued to E. Evans, and illustrated in Fig. 436. The shunt or potential coil is shown at the top and the series or current coil at the bottom. The disk is of aluminum. When the shading strip or vane is so adjusted that the disk operating the recording

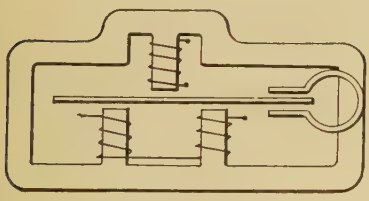


Fig. 437.

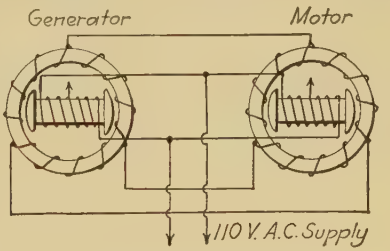


Fig. 438.

Fig. 437.—Principle of the watt-hour meter.
Fig. 438.—Transmitting and indicating units, and connections.

wattmeter records only the no-load power loss with no current in the series coil, the meter registers the aggregate load on the transformer for the low-tension side.

Figure 437 is a schematic diagram of the induction watt-hour meter principle. There are one (upper) potential coil containing many turns of fine wire and two current coils, the latter having

relatively few turns of coarse wire. The disk is of aluminum. A permanent magnet induces eddy currents in the disk as it rotates, thereby acting as a damper or brake. The flux due to the current coils is in phase with the load current. The potential coil has a high self-inductance and, therefore, the current in the potential or voltage coil, and the flux due to it, are approximately in time quadrature with the voltage across the load. Approximately concentric eddy currents are induced in the disk above each of the core ends and, because of the difference in phase, rotation of the disk results.

The principle of the watt-hour meter is applied to electric protective relays of various kinds. For instance, if the direction

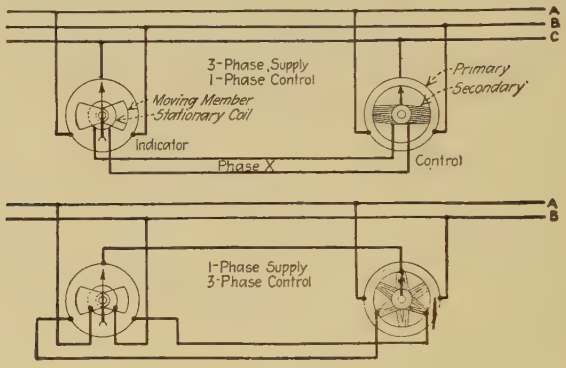


Fig. 439.—Two systems of connections.

of the power reverses, the direction of rotation of the disk reverses. Time delay is obtained through the use of permanent magnets, as above described.

The induction-type watt-hour meter principle has been successfully applied as a phonograph motor.[1]

The Selsyn system of position indication, as applied at the locks of the Panama Canal and the New Orleans Industrial Canal, as well as for signaling between control room and engine room, and so forth, is schematically shown in Fig. 438. This system has been described by E. M. Hewlett in the *General Electric Review*.[2] The generator rotor is operated by turning its shaft which carries a pointer. The motor rotor shaft also carries a pointer which moves in unison with the transmitting pointer.

[1] Hall, C. I., "The Induction Disc Phonograph Motor," *Gen. Elec. Rev.*, vol. 24, p. 218, 1924.

[2] Vol. 24, p. 21, 1921.

H. P. Sparks[1] has described the device in Fig. 439.

The principle of operation of this outfit is the same as that of two synchronous motors, one motor being used as a transformer supplying energy to the second motor from its secondary winding. The second motor, acting as an indicator, receives the energy from the transforming motor into its secondary. At the same time it receives energy into its primary winding from the same source as that from which the primary winding of the transforming motor receives energy. The two primaries of the motors are connected to a polyphase alternating-current system, and, having the same number of poles in the field, they will tend to have magnetic fields operating in synchronism with each other. As the induced current flowing in the secondary winding must become balanced before the motion of the rotor is zero, there will be a tendency of each rotor to move in exact synchronism with the rotor of the opposite motor. According to theory, these two machines will stay in exact synchronism. However, because of friction, there would be some power consumed by the indicating motor when overcoming the initial friction, thus causing an angle of lag. To overcome this error, an indicating instrument is utilized which is similar to the switchboard-type power-factor meter. This instrument has practically a zero friction error, and, as the movement is exceedingly light and well balanced, it is suitable for rugged service. This allows the use of a motor connected to a power-factor meter or synchroscope in initiate angular relation.

A magnet motor is employed in one form of magnet brake, a steel armature being rotated by the alternating field of the motor casing surrounding it.

The Warren Clock is a highly refined small synchronous motor with no winding on the steel rotor.

276. Reactors.—Small reactors may be fixed or variable, and may follow the design of several forms of electromagnets. Reactors are used in generating and distributing systems to limit short-circuit currents, and thus prevent undue mechanical forces in apparatus and the reduction of line voltage. Such reactors are sometimes shunted with resistors to absorb the destructive energy in connection with transient voltages.

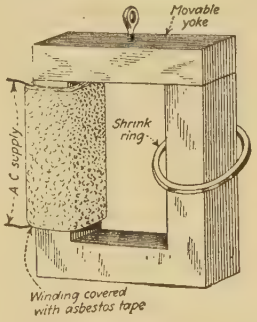


FIG. 440.—Shrink ring expanded by heat caused by secondary current.

One type of reactor resembles a transformer without a secondary. Figure 440 illustrates the use of a reactor with one of its

[1] "Adapting Signal Indicators to Power Plant Use," *Elec. World*, vol. 76, p. 1251, 1920.

coils removed and replaced with a shrink ring to be heated and thus expanded by the electric current induced therein, and illustrates the evil effect of a short-circuited turn in a reactor or other alternating-current coil. This effect was put to good use, as illustrated, and as described by R. H. N. Lockyer in the *Electrical World*.[1] By this method the temperature is easily controlled and the heat is uniformly distributed throughout each ring. However, this is but a single instance of the application of the heating effect due to short-circuited turns.

277. Transformers, Generators, and Synchronous Motors.—The transformer in Fig. 441 has a primary on one leg and a secondary on the other leg. When separated and mounted, as in Fig. 442, with a source of direct current included in the primary circuit, the rotation of the secondary produces an alternating

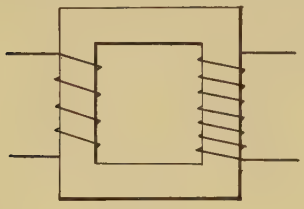


FIG. 441.

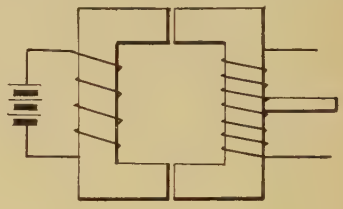


FIG. 442.

FIG. 441.—Core-type transformer.
FIG. 442.—Alternating-current generator or synchronous motor.

voltage therein. If the secondary circuit is closed, a secondary alternating current results. If an alternating voltage is impressed upon the secondary and the latter is set in motion by some means, the secondary will revolve in synchronism with the alternating current.

278. Electric Hammers.—The hammer blow due to the sudden stoppage of an armature, and particularly to that of the movable core or plunger, of an electromagnet long ago suggested its utilization for the purposes of caulking, chipping, heading rivets and cutting and drilling stone and concrete by means of an electromagnetic tool to replace the corresponding pneumatic and other tools. The advantages to be gained by the use of such an electromagnetic tool include the elimination of the air compressor, storage tank, and piping. The flexibility and convenience of such a tool is obvious, as electric power usually can be obtained and connections can be made with flexible cables.

[1] Vol. 77, p. 718, 1921.

The lines along which most inventors worked may be summarized in the following description. One of these hammers, developed about 1905, is illustrated in Fig. 443. It included a common steel plunger hardened at least on the striking end, and two coils arranged end to end, the function of each coil being alternately to pull the plunger in its respective direction. The plunger was given a quick acceleration in the striking direction, but was pulled back more slowly. Included in the outfit was the customary motor-driven switching mechanism for controlling the movements of the plunger. The hammer was 15½ in. long by 2¾ in. in diameter, and it weighed 15 lb. It was designed for operation on 110-volt and 220-volt direct-current circuits, and struck 1,750 blows per minute. The current strength, as indicated on a direct-current ammeter while the hammer was in operation, was 8 amp. on 110-volt circuits and 4.5 amp. on 220-volt circuits. The tool upon which the end of the plunger struck was 4 5/16 in. long by ¾ in. in diameter. The box containing the interrupter was 17 in. long by 12 in. wide, and it was 17 in. high, the weight of the box and its contents being 50 lb.

FIG. 443.—Electric hammer.

Advantages claimed for this hammer included the elimination of the compressor, tank, piping, lubrication of hammer, closely fitted parts (as piston and cylinder), danger of clogging by dirt, freezing, dirty moist air on operator, jar on shoulder of operator, cost when not running, and loss of power when worked against atmospheric conditions, as in tunnel and caisson work. There also were a few other advantages claimed.

While this tool accomplished its caulking, chipping, and heading in a thorough manner—for a few minutes—it soon became so hot that it could not be handled.

The maximum possible efficiency of a direct-current electromagnet is 50 per cent, but the average efficiency of the above device was probably nearer 15 per cent, due largely to the solid plunger and the high resistance of the coils.

In the case of the alternating-current electric hammer, the conditions are quite different from those found in the direct-current device. That devised by Leo Schuler is illustrated in

Fig. 444.[1] In this case, a commutator attached to the shaft of an alternator which supplies the electric energy to the coil closes the electric circuit at the proper instant and then always opens it when the current strength is approximately equal to zero. The timing is thus made so that the current may have reversed sufficiently to demagnetize the steel parts, or it may be otherwise adjusted so that the maximum efficiency may be obtained. Sparking is thereby eliminated and the energy losses are very much less than in the corresponding direct-current device. Mr. Schuler claims an efficiency of 87.5 per cent, and his claim appears

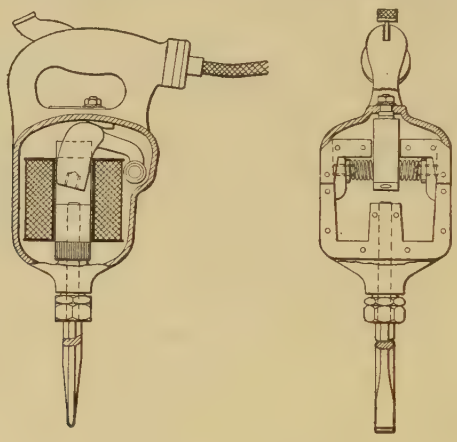


Fig. 444.—Design of electric hammer.

to be justified. The armature is accelerated during one cycle and is returned to its initial position by means of a spring during the following two cycles. Hence, if the frequency is 60 cycles per second, 20 blows per second, or 1,200 blows per minute, will result.

The alternating-current electric hammer in Fig. 445 is a reciprocating device wherein the plunger or "piston" is drawn forward during one half cycle and backward during the following half cycle, the alternating current being rectified by Tungar, Rectigon, or other rectifiers, as used for battery charging. The circuits are shown in Fig. 446, wherein the transformer is for filament lighting only. The single alternating current is thereby split into two separate waves displaced by 180 electrical

[1] "Electromagnetic Hammer," *Elec. World*, vol. 63, p. 1404, 1914 (abstract from *Elek. Z.*, May 14 and 21, 1914); U. S. Patent No. 1115251.

degrees, as in the oscillogram in Fig. 447, each current wave flowing only in its respective solenoid. Thus, the electronic valve is here employed not only to rectify the current but alternately

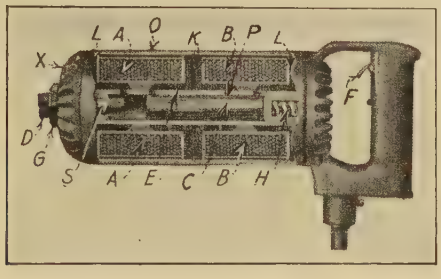


Fig. 445.—Syntron electric hammer.

and periodically to switch it from one solenoid to the other without wear or undue energy loss, although energy is received by both solenoids during a period of about 80 electrical degrees.

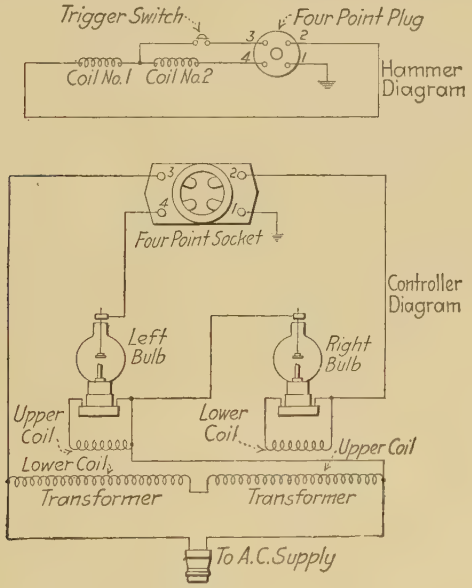


Fig. 446.—Circuits of the Syntron electric hammer.

This is offset by making the laminated section K integral, so that during the period of overlap the m.m.fs. oppose each other in this portion of the magnetic circuit. While this construction requires

about 5 per cent more current, it also causes a leakage flux during the overlap period that demagnetizes the tool steel parts.

Figure 448 is an oscillogram of one impulse of current i with its corresponding voltage characteristic e. The shaded portion s is due to remanent magnetism in the iron. The fundamental corresponds very nearly with e and is about as shown by the dotted curve.

The name Syntron was given this hammer, for the reason that it is a synchronous device controlled by an electronic device, the frequency of the piston being proportional to the square root of the ratio of the mechanical tension produced by the magnetic field to the mass of the piston, and this is made the same as the fre-

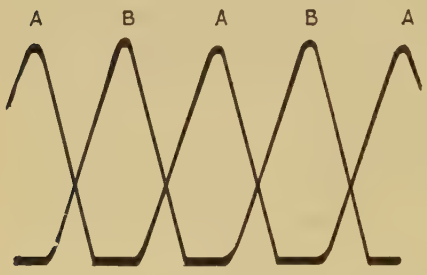


Fig. 447.

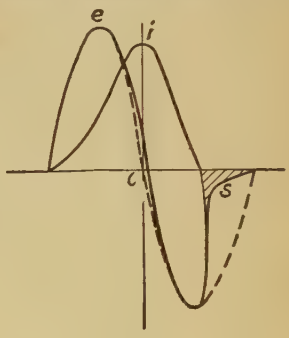


Fig. 448.

Fig. 447.—Oscillogram.
Fig. 448.—Oscillogram.

quency of the alternating current. C. S. Weyandt,[1] who supplied the data on this hammer, states that one mechanical method and one electrochemical method, one of which is satisfactory for direct currents, have been developed wherein the stored magnetic energy is delivered to the electric circuit and utilized for the production of work. However, such hammers are not available at this writing.

Referring to Fig. 445, A and B are the solenoids, K, L, and O comprising the magnetic circuit. K and L are composed of built-up sections of laminated electrical sheet and assembled radially about the cylinder E, which is a non-magnetic non-ferrous alloy. The portion O is made from a good electrical steel with a good finish. The piston C is slotted on its surface as shown, to eliminate eddy currents as much as possible, and is

[1] Vice-president, National Electric Manufacturing Co.

made of a fairly good grade of electrical steel treated on both ends so as to furnish hard impacting surfaces. *H* is an alloy steel bumper with a spring cushion. *S* is the shank of the tool *D*, which extends through hardened steel bushing *G* and is struck by the piston in its travel.

The characteristics of the present line of hammers are given in Table XXV.

At this writing, these hammers are used largely as drilling hammers for concrete, stone, brick, and masonry work, as the high speed obtained on 60 cycles is particularly adapted for this class of work.

TABLE XXV.—ELECTRIC HAMMER DATA

Weight, pounds	Amperes at 110 volts	Watts	Power factor, per cent[1]	Power output ratios[2]
9	1.75	175	90.9	0.50
18	3.80	375	89.7	1.00
25	10.00	600	54.5	2.25[3]

[1] Power factors are of total load, including the filament transfo. mer and tubes.

[2] These tests were made by drilling in stone of a very uniform structure.

[3] Certain advances in design had not been embodied in this hammer at the time of these tests

The 9-lb. tool is also in use as a vibrator on a sanding machine used in the manufacture of porcelain dishware. The principle is being applied to vibrating sieves for use in foundries and in other industries where sifting is required.

In a paper entitled "The Electric Hammer,"[1] P. Trombetta reviews the subject as follows:

The power hammer has been in use for a long time and there are now on the market many types of power-operated hammers, which may be roughly classed according to the nature of the power as follows:

Pneumatic, steam, motor and pulley; cam or crank; and electro-pneumatic drives. The pneumatic drive includes all riveting hammers; the steam drive includes practically all forge hammers, some drop hammers, pile drivers and steam drills; the motor and pulley drive class includes the greater part of drop hammers; the electropneumatic drive includes only small forge hammers. The pneumatic hammer, due to its lightness, holds the field of hand-operated riveting hammers and it is hardly possible that any other means will ever surpass air for driving

[1] *Trans.* A.I.E.E., Apr., 1920, p. 297.

hand-operated riveting hammers; the steam hammer holds its own in very large forging and drop hammers and it is doubtful whether any other kind of hammer can remove it from that place. The field for very large forging or drop hammers is, however, rather limited; they are used only in very large plants in which all sorts of power prevail.

There is an immense field, however, for medium and small forging and drop hammers, which are used to produce the small automobile and other similar parts, as well as name plates, jewelry apparel, etc. It is this field which the electric hammer is supposed to cover. The present methods of driving these hammers are cumbersome, complicated, costly, and very unsafe for the workman.

The electric hammer has been studied and developed by the writer to a point where it seems to show superiority to the present-used hammers, in simplicity, safety, running expenses, cost of installation, cost of upkeep, and in many cases in the original cost.

The development shown herein is of the induction-motor type. Instead of the usual arrangement of concentric armature and field, the slots are punched on long strips of iron in a straight line which makes the field and armature parallel. The armature is not always active under the influence of the field as is the case in the ordinary motor. In other words, in a straight-line motor the armature or runner is continually entering the field of action at one end and leaving it at the opposite end. This constitutes the main difference between the straight-line and the rotary motor. The rotating fields of an induction motor are here replaced by magnetic fields moving in a straight line.

Under the heading "Definitions," the following is stated:

An electric hammer as described in this paper constitutes a hammer in which the reciprocating part is moved solely by the direct action of the electromagnetic forces without interposition of any other mechanism, such as belts, gears, compressed air, etc.

The electrical portion of the reciprocating part, which is analogous to the rotor of a motor, will be called the runner, while the die holder which is attached to the runner is called the ram.

Electric hammers may be divided into four classes: direct-current, semi-synchronous induction, non-synchronous induction, and purely synchronous. A direct-current hammer consists of two symmetrical straight armatures mounted facing each other, between which moves the field carrying with it the brushes; it may also be made with two stationary fields and one moving armature. A semi-synchronous induction hammer is made by setting two polyphase straight-line stators facing each other with a squirrel-cage runner moving between them; the reversal of the current is made by a switch rigidly connected to the generator shaft so that the phases are reversed at zero potential and at predetermined intervals. A two-phase hammer fed with two phases

of different frequency, in which the reversal comes at the nodes of the two frequencies, may also be classed as a semi-synchronous hammer.

A non-synchronous induction hammer is made exactly as a semi-synchronous induction one, with the exception that the reversing of the phases at each end of the stroke is done by the runner itself. A purely synchronous hammer may be defined as a hammer which gives one stroke per cycle. This type of hammer may be of the simple plunger type or of the excited plunger type.

Figure 449 shows a device patented by Van Depoele in 1885. This is some what similar in appearance to the Marcel Deprez electromagnetic hammer described by S. P. Thomson on page 360 of "The Electromagnet," 1892, but with an electrically excited plunger. A patent was issued to R. D. Mershon in Aug.,

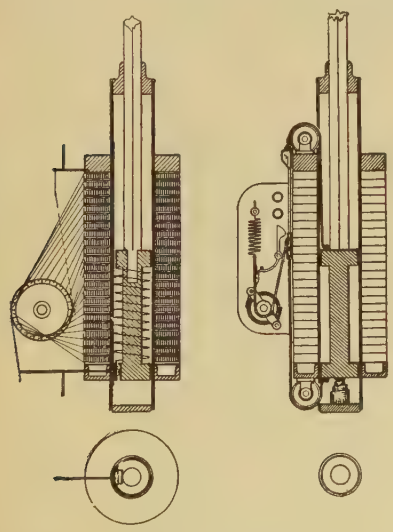


Fig. 449.

Fig. 450.

Fig. 449.—Van Depoele's electric hammer.

Fig. 450.—The first experimental hammer (disassembled view with reciprocator and one stator removed).

1901, covering the first recorded polyphase rectlinear motor. A patent for a two-phase machine wherein each stator was fed with separate sources of alternating current of different frequencies

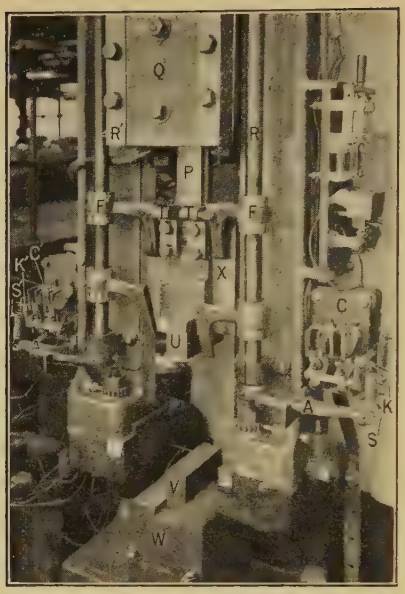


Fig. 451.—Equivalent of a 200-lb. board hammer. Front view.

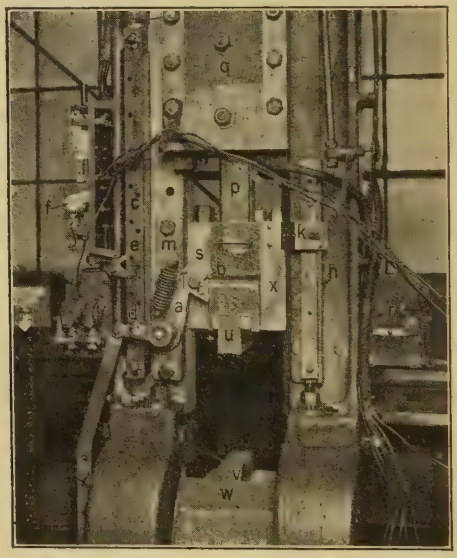


Fig. 452.—Equivalent of a 200-lb. board hammer. Rear view.

was issued to P. Centener in 1905. This machine automatically reversed at the resulting beats without the use of any switches, but it had a constant period and required special generators to operate it.

While the construction of a laminated runner is rather costly, it would be elastic and would absord the shocks. The boards of board hammers frequently break, as do also the piston rods of steam hammers due to crystallization.

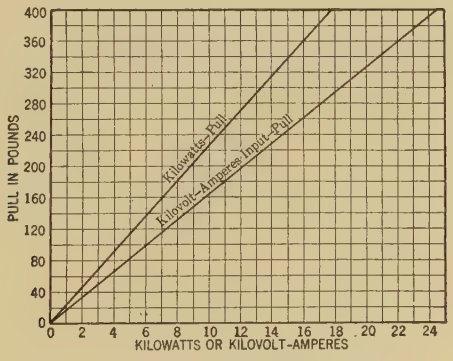


FIG. 453.—Stand-still test characteristics.

Most of the above is generally abstracted from the paper in question. Mr. Trombetta's first electric hammer, which he states proved very satisfactory, is shown in Fig. 450. The equivalent of a 200-lb. board hammer was constructed, as illustrated in Figs. 451 and 452. The stand-still test characteristics are shown in Fig. 453. Oscillograms and various data also are shown in the paper.

CHAPTER XXIV

ELECTROMAGNETS IN GENERAL

The subject is herein generalized. Rules, hints, and shortcuts are given. The efficiency of the electromagnet is considered and some specific conditions are discussed.

279. The Practicability of Using Electromagnets.—The electromagnet without a wound armature has its field of usefulness apart from that of the electric motor. It cannot compete with the motor in its own field any more than the motor can compete with the electromagnet in its field. However, electromagnets are constantly replacing parts of machines that are driven, as units, by electric motors.

Before arranging to use an electromagnet for any given purpose, it should be ascertained whether the branches of the power or lighting circuit in which the electromagnet is to be connected are capable of conducting the necessary current without violating underwriter's rules, and whether or not the voltage drop due to the additional current necessary for the operation of the electromagnet will cause lamps to dim or flicker objectionably, or in any other manner affect other apparatus connected across the circuit.

When a condenser is used to correct the power factor of an electromagnet or motor, every piece of apparatus connected across the circuit must be taken into consideration. For instance, if a condenser is used to correct the power factor of a given electromagnet or motor, and there is another electromagnet or motor connected across the same line, the condenser will tend to correct the power factor of both motors or electromagnets, and thus will not perform the function required of it unless the capacitance is made sufficiently great to provide for all of the reactance connected across the line.

When a battery is used as the source of electric energy supply, it should be remembered that the efficiency of the direct-current electromagnet is generally very low and that all of the electric energy supplied to the electromagnet is converted into heat when the armature is not in motion.

When a heavy-duty electromagnet is operated from a source of energy of low voltage, the conductors between the source of energy and the electromagnet should have ample cross-sections to conduct the current without too much voltage drop. In the cases of lines of fixed resistance, the maximum number of volt-amperes is received by the coil of a direct-current electromagnet under static conditions when the coil resistance is equal to the external resistance. Likewise, the maximum number of volt-amperes is received by the coil of an alternating-current electromagnet when the coil impedance is equal to the external impedance. The external resistance and impedance may be respectively represented by a constant-resistance resistor in the former case, and by a constant-impedance resistor and reactor combination in the latter case.

It has been noted (page 386) that the maximum pull due to a constant-voltage, alternating-current solenoid-and-plunger occurs when the magnetizing force is equal to the ferromagnetic flux.

The maximum force $\times$ length of working air gap in an electromagnet occurs when the reluctance of the working air gap is equal to the reluctance of the rest of the magnetic circuit, when the current strength is constant.

The above phenomena are based on the simple rule that, when the sum of two such variable quantities is constant, their product is a maximum when they are equal to each other.

The author recalls several instances where he was asked to explain the surprising phenomenon that, when the magnet wires in the coils of electromagnets energized from storage batteries were made greater in cross-section so as to decrease the resistance per turn and, therefore, increase the number of ampere-turns to increase the pull, the pull increased to a maximum and then began to decrease, although the current strength constantly became greater as magnet wires of greater cross-sections comprised the coils; yet this is an old and well-known phenomenon.

An electromagnet designed to perform a given amount of work quickly will "draw" more current than one designed to operate more slowly. The quicker the action the coarser must be the magnet wire and the lower will be the resistance and the self-inductance, both of which vary directly with the square of the number of turns in a given space. Low-resistance, direct-current electromagnets require heavier wiring than do slow-acting electro-

magnets, on account of the greater current strength required for their operation.

Where large quantities of electromagnets are in use, as in telephone exchanges or for operating the valves on radiators in large hotels and office buildings, the very number of electromagnets requires that the first cost shall be as low as possible, consistent with their energy consumption. One thousand electromagnets drawing 0.1 amp. each on a 110-volt, direct-current circuit represent a time-rate of energy expenditure of 11,000 watts, or 11 kw., in case they should all happen to be in circuit simultaneously. Where only a few electromagnets are to be used intermittently, the first cost is of the greater importance, provided the arcing at the switch contacts does not necessitate too expensive a switch.

It should be ascertained if there is sufficient space, of the proper cross-section to contain an electromagnet which shall have the necessary cross-section of core and the proper cooling surface, and of length consistent with the type and stroke of the electromagnet. Furthermore, the temperature of the place where the electromagnet is to be used, and the time-rate at which air, oil, or other fluid or substance can carry off the heat produced in the electromagnet, should be ascertained in order to determine whether or not the insulating material of the electromagnet will be impaired.

Electromagnets which are exposed to acid fumes or even to moisture must be so protected that their coils, particularly, shall not be damaged. Even moisture in coils often causes short-circuited turns, and acid or salts will cause open circuits by eating through the wires.

Electromagnets of the general solenoid type may be mounted in horizontal, vertical, or intermediate positions. When in the horizontal position, the friction between the plunger and the tube or other bearing or guide is greatest, but is otherwise unaffected by gravity. Any given solenoid may be placed in such an intermediate position with respect to the horizontal plane that the tangent of the angle between the axis of the solenoid and the horizontal plane shall be equal to the coefficient of friction, in which case the weight of the plunger does not need to be considered except in starting. In cases where the gravitational pull on the vertical plunger is objectionable, the solenoid may be placed in the horizontal or the intermediate position and the

vertical motion obtained through the use of levers or a bell crank. The pulsating pulls of single-phase, alternating-current solenoids tend to reduce friction.

In cases where the work to be accomplished in a given time requires too great a current strength, either for the coil or for the line, the design is impracticable and some form of electric motor should be used to operate the device through some mechanical system, or else the energy may be stored in a spring through a series of impulses and then electromagnetically released.

The use of large alternating-current electromagnets becomes objectionable when their laminated structures become mechanically weak as the size increases. The current strength at the beginning of the stroke of a large alternating-current electromagnet often amounts to a short circuit, or is too great for the circuits or switches therein. The above applies to attempted designs of abnormally large electromagnets.

However, many of the precautions do not apply in the great majority of cases, although it is well to bear them in mind. Furthermore, new electromagnetic devices are constantly being developed.

280. Selection of the Type.—This subject is so general that only the principal requisites are herein mentioned:

1. Whether the current is direct or alternating.
2. The frequency if alternating.
3. The voltage, the amperage, or both.
4. The length of the stroke.
5. The mechanical force at the initial and final positions, and at some of the intermediate points.
6. Whether the mechanical force in the final position is important, or whether the only requirement is to start the load and to keep it moving.
7. How long the current will be on and off, and the order of recurrence.
8. The maximum cross-section permissible for the entire electromagnet.
9. The maximum length permissible for the entire electromagnet.
10. The switching facilities necessary or available.
11. If an alternating-current shunt magnet, the maximum permissible current strength at the beginning of the stroke.

12. If an alternating-current series magnet, the maximum permissible voltage at the end of the stroke.

13. The position of the armature or plunger, whether horizontal, vertical, or intermediate, and the angle.

14. The maximum temperature of the place where the magnet is to be operated.

281. Determining the Mechanical Force and the Length of Stroke Required.—The principal consideration is to have the electromagnet strong enough and to have it move its load within the required time interval. The initial and final pulls may be determined by pulling on the part to be moved with a spring scale. Average static pulls may be determined in a similar manner. In many cases, only the initial pull $\times$ distance is all that is required, although the accelerating pull near the end of the stroke often is desirable.

282. Limits of Mechanical Forces in Electromagnets.—Below the knee of the normal saturation curve, the pull varies approximately as the square of the current strength, but after the steel is saturated the pull varies directly with the current strength, the additional magnetic energy being stored in the electromagnetic circuit and not in the steel. Hence, after the steel is saturated, the only pull is due to the solenoid effect, the latter having no limit when the current strength has no limit.

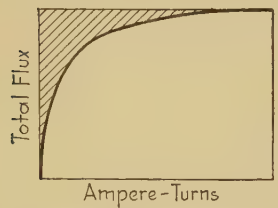


FIG. 454.—Energy in magnetic circuit with armature in place.

283. The Storage of Magnetic Energy. Figure 454 shows that the magnetic energy stored during saturation is equal to the shaded area. This is due to the fact that the stored magnetic energy is equal to one-half the number of the ampere-weber-turns, or to one-half the product of the number of ampere-turns and the number of webers of magnetic flux.

284. Notes on Design.—It is necessary first to make a preliminary design of the magnetic circuit, allowing space for the insulated coil and its support. The actual coil is then calculated, any necessary changes in the magnetic circuit for the accommodation of the coil and its support, as finally designed, being then made.

The magnetic circuit is designed to have the maximum economical flux density, about 13,000 gausses for cast steel and wrought

iron when the flux is unidirectional, or about 10,000 gausses at 25 cycles and 8,000 gausses at 60 cycles, although these figures will vary according to the construction and duty rating of the electromagnet. The length of the magnetic circuit generally varies with the reluctance of the air gap. The longer the cores of a horseshoe electromagnet, for instance, the longer will be the coil and the greater will be the magnetizing force at the coil ends for attracting the armature through the solenoid effect. Likewise, the smaller will be the extent of the magnetic leakage from the core ends back to the half cores of opposite polarity, but the greater will be the time constant.

The coil is designed to have the maximum current density (about 900 to 1,500 amp. per square inch) that shall not cause the coil to heat at too great a time-rate, and its coil-wall thickness is made as small as practicable in order to keep the length of conductor as small as possible, as well as to augment the flow of heat to the coil surfaces.

By keeping the length of the magnetic circuit a minimum when the air gap is small, the number of ampere-turns required becomes a minimum. By reducing the average length of wire per turn to a minimum, the number of heating watts are kept at a minimum. However, the cooling surface must be consistent with the average length of wire per turn, so that the coil may cool as rapidly as it heats and the maximum coil temperature shall not become sufficient to damage the insulation.

Cheapness and compactness should always be borne in mind, and this requires that the maximum amount of iron or steel and the minimum amount of copper be employed, on account of their relative costs. However, if the cost of operating is to be considered, the ratio of the amount of copper to that of iron may have to be increased. This also depends upon the duty required of the electromagnet.

In general, the higher the voltage which is to be applied to a coil of fixed dimensions, the smaller must be the cross-section of the wire and the thinner will be the insulation on the wire. This is compensated for to a certain extent by the fact that the mechanical pressures on the insulation are not so great for fine as for coarse wires. Furthermore, the actual voltage between turns may not be much greater in the former case than in the latter, because there are so many more layers of fine wire. But the finer wires have smaller space factors than do the coarser

wires, so that less copper and more insulation is contained in the coil space filled with fine wire than with coarse wire. When the wire is fine, the copper heats at a greater time-rate for a given number of watts, or for a given current density, because the mass is smaller than when more copper is present in the coarse wire. The presence of the additional insulating material, in the case of the fine wire, delays the conduction of heat from the interior of the coil-wall to the cooling surfaces. In consequence, high-voltage electromagnets are inherently of a lower duty type than are low-voltage electromagnets.

The foregoing shows the importance of correct design of electromagnets in general. The application of electromagnets to mechanical devices, as well as those generally outlined in this treatise, is becoming so general that special designs are required in nearly every case of application. For instance, the length and the diameter or side of the core, armature, or plunger, the position of the working air gap, length of coil, coil-wall thickness, proper cooling surface, and so forth are all dependent upon the specific conditions under which the electromagnet is to be used.

The following rules are useful in estimating and checking. Wherever an air gap is referred to, the magnetic flux and the pull are assumed to act straight across the air gap.

1. The product of the pull in kilograms and the cross-section of the air gap in square centimeters is equal to 0.000 000 040 58 times the square of the total magnetic flux in maxwells.

2. The product of the pull in pounds and the cross-section of the air gap in square centimeters is equal to 0.000 000 089 46 times the square of the total magnetic flux in maxwells.

3. The product of the pull in pounds and the cross-section of the air gap in square inches is equal to 0.000 000 013 87 times the square of the total magnetic flux in maxwells.

4. The pull in kilograms for each square centimeter of air gap is equal to 0.000 000 040 58 times the square of the magnetic flux density in gausses.

5. The pull in pounds for each square centimeter of air gap is equal to 0.000 000 089 46 times the square of the magnetic flux density in gausses.

6. The pull in pounds for each square inch of air gap is equal to 0.000 000 013 87 times the square of the magnetic flux density in maxwells per square inch.

7. The square of the magnetic flux density in gausses is equal to 24,645,000 times the pressure in kilograms per square centimeter, or to 11,179,000 times the pressure in pounds per square centimeter.

8. The square of the magnetic flux density in maxwells per square inch is equal to 72,122,000 times the pressure in pounds per square inch.

9. In a solenoid-and-plunger, for each thousand ampere-turns per inch of coil length, a maximum pull of about 9 lb. may be expected for each square inch of plunger in the core hole of the solenoid.

10. The total pull of a plunger electromagnet, horseshoe electromagnet, and so forth is equal to the sum of the air-gap pull and the solenoid pull.

11. In any case, the average mechanical force in pounds is equal to 4.426 times the product of (*a*) the square of the current strength and (*b*) the ratio $\frac{\text{difference in self-inductance}}{\text{difference in length}}$.

12. The cross-section of wire is approximately halved when the A.w.g. (B. & S.) number is increased by 3, and the cross-section is approximately doubled when the gage number is decreased by 3. Likewise, the diameter is about halved when the gage number is increased by 6, while the diameter is about doubled when the gage number is decreased by 6.

13. The resistance of copper changes about 1 per cent for each corresponding change in temperature of 2.5° C., or 4.5° F.

14. The number of r.m.s. active volts required to produce a given number of r.m.s. ampere-turns in an alternating-current electromagnet is somewhat (say, 20 per cent) greater than the equivalent continuous active voltage required.

15. The ratio of the active voltage to the number of ampere-turns is equal to the product of the resistance per unit length of wire and the average length of wire per turn.

16. The size of wire to use is found by dividing the active voltage by the product of the number of ampere-turns required and the average length of wire per turn. This gives the resistance per unit length of wire to use.

17. The current density will vary from about 900 amp. per square inch to about 1,500 amp. per square inch, depending upon the thickness of the coil wall, the cooling surface, and the space factor.

18. The ratio of (*a*) the ampere-turn gradient in ampere-turns per inch of coil length to (*b*) the current density in amperes per square inch of bare copper is equal to the product of the coil-wall thickness and the space factor.

19. To change from one active voltage to another, when the active voltage is to be doubled, halve the cross-section of the wire by increasing the gage number by 3. To halve the active voltage, double the cross-section of the wire by decreasing the gage number by 3.

20. To change from one frequency to another, when the frequency is doubled, halve the number of turns by doubling the cross-section of the wire. When the frequency is halved, double the number of turns by halving the cross-section of the wire. Adjustments must be made for variations in the space factor.

21. When the product of the impressed voltage and the frequency is constant, no change need be made in the cross-section or number of turns of wire.

22. The cross-sections of adjacent sizes of wire in the A.w.g. have the constant ratio 1.26, that is, the cross-section of each decreasing gage number is 26 per cent greater than the adjacent smaller size.

23. The resistance of copper changes 26 per cent with an increase in temperature of 66 C. deg., or 118 F. deg.

24. Calculate the size of wire for a given number of ampere-turns on the basis of a temperature of 20° C. (68° F.) and then use the next larger size of wire (in cross-section) in order to obtain that number of ampere-turns at the final temperature.

25. An alternating-current electromagnet requiring 26 per cent more ampere-turns than when operated on direct current may be wound with one size finer (in cross-section) wire than that required for direct current. A series magnet is here meant.

26. In order to maintain a constant number of ampere-turns per active volt, the cross-section of the conductor must be increased in direct proportion to any increase in temperature or in the average girth (average length of turn) of the coil.

27. The number of watts per square inch of cooling surface (3.1416 × outside diameter × length of coil) varies from 0.5 to 1, unless asbestos-covered wire is used.

28. The number of watts per ampere-turn is equal to the number of active volts per turn, or to the product of the amperage and the resistance per turn.

29. The number of turns is equal to the ratio of the length of wire to the average girth of the coil, or to the ratio of (*a*) the product of the coil cross-section and the space factor to (*b*) the cross-section of the copper wire in square inches.

30. The resistance is equal to the product of the length of wire and the resistance per unit length, or to the product of the resistance per turn and the number of turns.

31. If the coil always consisted of bare copper wire without interstices, the cross-section of the conductor would vary inversely with the number of turns, the length of conductor would vary directly with the number of turns, and the resistance and the self-inductance would vary directly as the square of the number of turns.

32. The number of ampere-turns is equal to the ratio of the active voltage to the resistance per turn, regardless of the number of turns.

33. Assuming no magnetic leakage, and considering the air-gap pull only, the number of ampere-turns required per centimeter length of air gap is equal to 3,951 times the square root of the ratio of the pull in kilograms to the cross-section of the air gap in square centimeters. The number of ampere-turns may be increased, say, 50 or 100 per cent to allow for magnetic leakage.

34. Assuming no magnetic leakage, and considering the air-gap pull only, the number of ampere-turns required per inch length of air-gap is equal to 2,661 times the square root of the ratio of the pull in pounds to the cross-section of the air gap in square inches. The number of ampere-turns my be increased, say, 50 to 100 per cent to allow for magnetic leakage.

285. Practical Design.—The slide rule and the planimeter are exact instruments when the variations in the materials, voltages, temperatures, and so forth encountered in the manufacture and operation of commercial electromagnets is considered. Common sense and experience are the guides that are to be backed up with figures. A good factor of safety and sufficient tolerance should be allowed for economy of manufacture and positive operation. The resistance of a coil is a check on the number of turns for a given average girth. But the average

girth and the cross-section of the wire vary, the latter being due to worn dies and stretching during respooling and winding. Furthermore, unless very accurate tension devices are employed in winding, the wire will not be uniformly stretched.

The thickness of the insulation varies regardless of how carefully the wire is insulated. Coil spaces should be made sufficiently large so that the required number of turns of even oversize wire will go in the allotted space. While a high space factor is desirable, it must be remembered that more bare wire will go into a given winding space than will a properly insulated wire. This rule holds for all kinds of insulation. Since all magnet wires of the same general class do vary some, the constant multiplication of this small variation, as turn upon turn is wound into a coil, makes a noticeable difference in the size and resistance of the wound coil.

286. Universal Electromagnets.—Small shunt magnets, especially, are sometimes required to operate on either direct or alternating current. In such cases, the resistance is made relatively great so that the impedance shall approximate the resistance, and the phase angle shall be small. Such electromagnets are designed for use in shunt with the line, and have relatively feeble pulls. The best form of universal shunt electromagnet is that provided with a series resistor, so that the latter may be placed in series when the electromagnet is to operate on direct current.

Series electromagnets are inherently of the universal type, as they may be operated on either direct- or alternating-current circuits and produce nearly as strong pulls with alternating current as with direct current.

287. Electromagnet in Series with Impedance.—In series circuits, an electromagnet will receive the maximum number of volt-amperes when its impedance is equal to the impedance in series with it. By this method, two apparatuses having equal impedances may be connected in series in a circuit containing a third impedance for comparison with one another, because each will receive the same number of volt-amperes. When two apparatuses having different impedances are connected in series with a third impedance, if the sum of the impedance of one of the apparatuses and the third impedance is equal to the impedance of the second apparatus, the latter will receive the maximum number of volt-amperes. Comparative tests for relative efficiency may be conducted by this method.

288. The Efficiency of the Electromagnet.—Under this title Leo Schüler published an article in "*Elektrotechnische Zeitschrift*," which was republished in the *Electrician*.[1] It is stated in the article that a number of authors[2] have already treated the "balance of energy" of the electromagnet, but that they have endeavored to simplify the problem by assuming constant field strength or constant current strength for the whole stroke.

Such assumptions are certainly not admissible, as under practical conditions a continual change of all values takes place from the moment of closing the circuit up to the completion of the stroke; during the first few moments after switching on, as long as the armature remains at rest, the current and the field will build up according to the well-known exponential curve, the rate of increase depending upon the dimensions of the magnet, the number of turns and the resistance of the coil, and the pressure applied. When the stroke begins, *i.e.*, the air gap decreases, the current will also decrease, as the magnet cannot absorb at any moment more current than is necessary for the production of the magnetic field; the current would therefore decrease nearly in proportion to the decrease of the air gap if the magnetic field itself remains constant during the stroke. This, however, cannot be the case as long as the applied e.m.f. is not fully absorbed by the ohmic resistance of the coil. The further performance of the current during the stroke will, therefore, depend upon the relative speed of the increase of the field and the decrease of the air gap, while the increasing saturation of the iron has also to be taken into consideration. If, for example, the conditions are such that, even after the completion of the stroke (air gap = 0), the saturation of the iron is low, then the current must attain at that moment a very low value corresponding to the small number of ampere-turns necessary for the magnetization of the iron core.

A mathematical formula representing the performance of the current during the stroke would have to take into consideration not only the electrical and magnetic dimensions of the apparatus but also weights to be lifted, and the mass to be accelerated. I have not tried to solve the problem on a purely mathematical basis, as the necessary formula, if it can be found at all, would certainly be too complicated for practical application. I shall, however, describe a "step-by-step" method, which solves the problem in a very simple manner. The principle of the method is to subdivide the time of the stroke into a number of small time intervals, and to calculate the values of armature speed, air gap, saturation, and current for one interval, then for the next, and so on, until the air gap 0 is attained, *i.e.*, the stroke completed.

[1] London, Nov. 14, 1913, p. 238.

[2] Emde (*Elektrotech. Z.*, 1908, S. 817); Schiemann, Ilmenau (*Elektrotech. u. Maschinenbau*, 1913, S. 11); Kalisch (*Archiv für Elektrotech.*, 1913, S. 458).

The value e represents the mean e.m.f. during the time interval t. The magnetic leakage may be neglected; it is true that the leakage greatly affects the attracting power, and therefore the absolute value of the energy produced, but the influence is not so pronounced on the ratio input : output. It is, however, possible to take the leakage into consideration by this method, but this would complicate matters, and I will therefore neglect it in the present description. I assume that the magnetic flux is the same in the air gap, the core, and the frame . . . If the friction is to be taken into account, a corresponding amount must be subtracted from P; I shall, however, assume that the movement of the armature takes place without friction.

It is also stated that it is assumed that practically no eddy-current loss takes place.

Calculations are made and tabulated for both direct and alternating currents. It is shown that

The mechanical output of an alternating-current magnet varies in the ratio of about 1:3, according to the point on the pressure curve in which the apparatus is connected to the supply circuit.

In the accompanying illustrations taken from the foregoing article, W is the distance in centimeters passed through during time t, V the velocity of the armature in centimeters per second, i the current strength in amperes, r the resistance in ohms, and N the magnetic flux in the core.

The efficiency of the electromagnet is defined as the ratio of (1) the mechanical energy delivered to (2) the electric energy supplied to the coil from the moment the coil is switched on up to the instant of the completion of the stroke. The mechanical energy consists of two components, that is, the kinetic energy and the potential energy (neglecting friction).

When only the lifting work is considered as useful, then the accelerating work appears on the debit side of the balance sheet; when, however, the purpose of the apparatus is the production of percussion effects (electric hammers), the accelerating work represents the useful work, and in that case the arrangement will be such that none or very little lifting work is produced.

The calculated overall efficiency for the direct-current magnet was 46 per cent, and the efficiency for the lifting work alone was 32 per cent.

The same electromagnet is then assumed to be supplied with alternating current of 25 cycles and 2,500 volts. It is first

assumed that the magnet is switched on at the maximum point of the pressure curve.

The lifting work is 44 watt-sec. The accelerating work is 31.5 watt-sec., totaling to 75.5 watt-sec. By planimetering the i^2r curve, an ohmic loss of 9 watt-sec. is found.

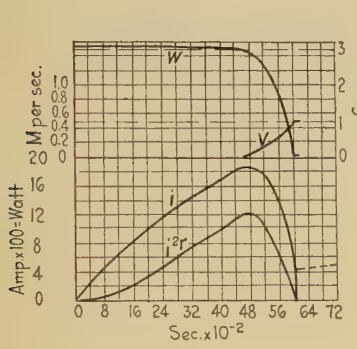


FIG. 455.

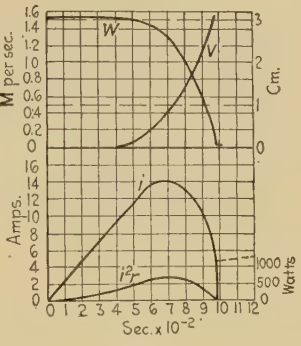


FIG. 456.

FIG. 455.—Vertical lift with 110 volts direct current, with a net load weighing 750 kg.

FIG. 456.—Vertical lift with 110 volts direct current, with a net load of 185 kg.

By opening the circuit when the current strength is equal to 0, the overall efficiency is found to be 89.6 per cent—52 per cent for the lifting work and 37 per cent for the accelerating work.

When the current is switched on at the zero point of the pressure curve, the lifting work is the same as before, the accelerating work is 162 watt-sec., and the total output is 206 watt-sec.

The ohmic losses are found to be 55 watt-sec., and the efficiency is 79 per cent overall, only 17 per cent for the lifting, but 62 per cent for the accelerating.

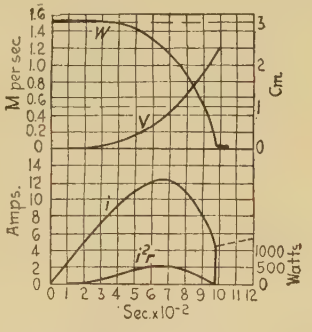


FIG. 457.—Horizontal displacement with 400 volts direct current; the net load weighing 370 kg.

It is to be noted that solenoid-and-plunger, and plunger electromagnets in general, must be differently calculated than in the above article, wherein the pull is based on Maxwell's hypothesis. In a solenoid-and-plunger the pull is entirely due to leakage, which has been entirely neglected in the article.

However, the pulls at different points may be calculated by proper equations or measured for various current strengths from an actual electromagnet. Then the method as described amounts to the following:

1. The average self-induced voltage during a small time interval t while the self-inductance is increasing is equal to the ratio of the difference in the number of weber-turns to t, or to

$$\frac{\text{difference in weber-turns}}{\text{difference in time}}.$$

2. From (1) it follows that the difference in the number of weber-turns is equal to the product of the average self-induced voltage and the time interval t.

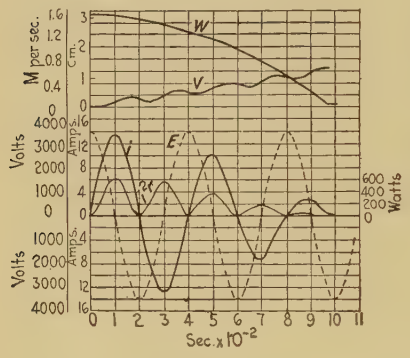


FIG. 458.

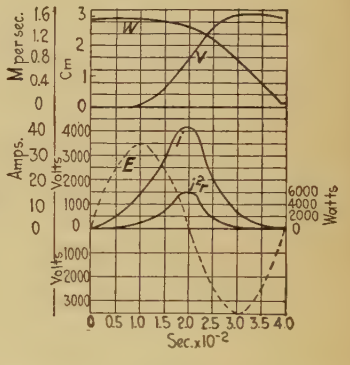


FIG. 459.

FIG. 458.—Vertical lift with 2,500 volts at 25 cycles. Weight of load 150 kg. Current switched on at maximum point of pressure curve.

FIG. 459.—Same as Fig. 458 but switched on at zero point of pressure curve.

3. The average pull in kilograms is then calculated for the movement (page 214) by multiplying either (*a*) the product of the average self-induced voltage and the time interval t or (*b*) the difference in the number of weber-turns by (*c*) 5.099 times the ratio of the average current strength in amperes to the difference in length in centimeters constituting the movement.

4. The mechanical force is equal to mass $\times$ acceleration;

$$\text{mass} = \frac{\text{weight}}{\text{gravitational constant}^1}, \text{ and acceleration} = \frac{\text{increase in velocity}}{\text{time interval}}.$$

[1] g = 980.597 cm. per second per second = 32.172 ft. per second per second = 386.06 in. per second per second.

5. From (4) it follows that the increase in velocity during the time interval t is equal to $\frac{\text{force}}{\text{mass}} \times$ time interval.

6. When the force acts in the vertical upward direction it is necessary, due to gravity, to subtract the weight from the total force of the magnet. The increase in velocity during the time interval t is then equal to $\frac{\text{force} - \text{weight}}{\text{mass}} \times$ time interval.

7. The velocity of the armature at the end of the time interval t is equal to the sum of the velocity at the beginning of time interval t and the increase in velocity during that time interval, the average velocity during the time interval t being equal to one-half the sum of the initial and final velocities for that time interval.

8. If the velocity is in centimeters per second, the distance (W in the figures) in centimeters traversed by the armature or plunger during the time interval t is equal to the product of the average velocity and the time interval t.

9. The new position of the armature or plunger now being known, the resulting difference in the number of flux-turns and the new pull are calculated.

10. A current-time curve is now constructed, the zero point is connected with the point i_2 found for the first time interval, and the value of i is then found by extrapolation for the middle of the following time interval.

11. The value of the self-induced voltage is then found by subtracting the product of the current i and the resistance r from the impressed voltage.

The accuracy of the extrapolation has little influence on the result, especially when ir is small as compared with E. It will further be seen that any mistakes can at once be detected and corrected when the calculation proceeds, as the e extrapolated must check with the e given by the formula $e = E - ir$ after the mean value i for the time interval in question is found.

Figures 460 and 461 are from a test made by B. C. Batcheller[1] with a constant-current, direct-current plunger electromagnet by measuring the time on a chronograph disk.

A series of oscillograph tests made some years ago showed similar results.

1 "Experiments with a Plunger Electromagnet," *Elec. World*, vol. 65, p. 1037, 1915

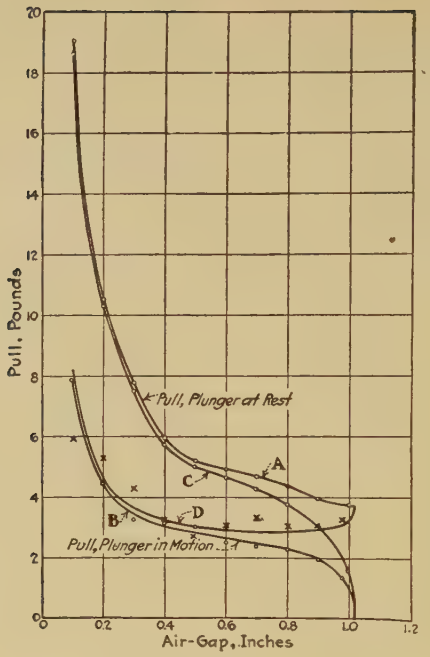


FIG. 460.—Relation between air gap and pull.

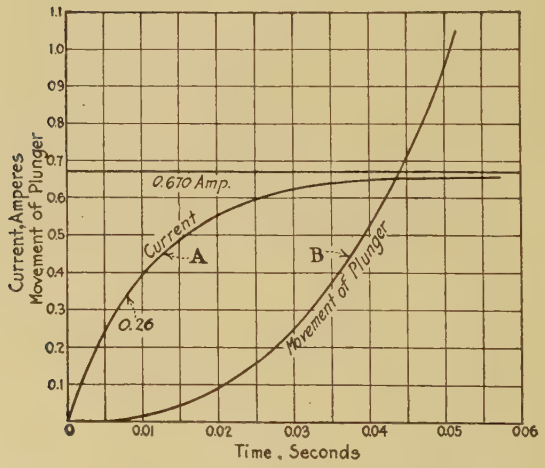


FIG. 461.—Current strength for various plunger movements.

Very accurate data are obtained from oscillograms. Figure 462 shows how the current is reduced as the plunger accelerates, and again builds up after the plunger strikes the stop of a direct-current, constant-voltage shunt-plunger electromagnet. In his discussion of a paper[1] by C. P. Steinmetz, C. M. Davis explains the operation of a plunger electromagnet as outlined in Fig. 462, which

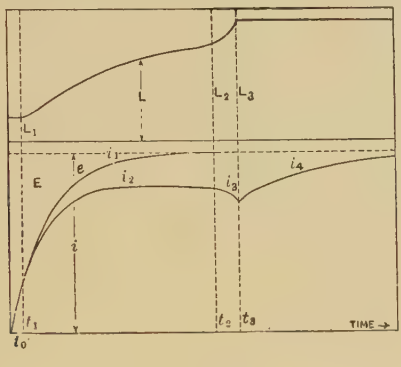


FIG. 462.—Effects due to accelerating and stopping the plunger.

is here quoted, the figure number being changed to conform to the text.

I wish to offer an explanation of the operation of a solenoid when used to actuate an oil switch by means of a toggle mechanism. The plunger of the solenoid acts on the middle joint of the toggle, moving it until the two arms are brought a little beyond the point where they are in the same straight line.

The electrical operation may be outlined by reference to Fig. 462. The plunger in its initial position gives the solenoid the inductance L_1, and the current, starting at time t_0, would rise along the transient curve i_1. After time t_1 the current has risen to produce sufficient flux to start the plunger and it begins to close the toggle. In doing so the inductance of the solenoid increases, and an increase of inductance means a decrease in the rate at which the current rises in the solenoid. Thus at t_1 the current begins to rise along another transient curve indicated by i_2. At t_2 the arms of the toggle have come into line, which suddenly releases the load and the joint breaks back to lock the switch mechanism; the plunger then suddenly jumps to its final position, reaching it at t_3, and with it the inductance increases from L_2 to L_3, which may mean an actual decrease of current, indicated by i_3. From t_3 the current rises along a curve i_4 determined by L_3 and the current at t_3.

[1] "Mechanical Forces in Magnetic Field," *Trans.* A.I.E.E., 1911.

The power at any instant is the product of the current and the induced voltage at that instant, and the induced voltage is proportional to the ordinates e above the curve; hence the energy consumed by the solenoid is the area product above and below the curve. Now the area above the curve is constant, since the total voltage induced E is proportional to the total number of lines of force cut which is constant. Thus the work done by the electrical energy supplied depends only on the area under the curve, and hence the longer time the solenoid acts the more work it can do.

The above quotation is selected because it checks with results obtained some years later.

In his paper, just referred to, Dr. Steinmetz shows:

The efficiency of the electromagnet, therefore, is the difference between maximum and minimum voltage, divided by the maximum voltage; or the difference between maximum and minimum volt-ampere consumption, divided by the maximum consumption; or the difference between maximum and minimum inductance, divided by the maximum inductance.

CHAPTER XXV

PERMANENT MAGNETS

This subject is so broad that an entire book, rather than a single chapter, could be devoted to it. However, an attempt is made to treat the subject in a general manner.

289. Structure of a Magnetic Material.—The structure of a magnetic material may be visualized as a rather dense group of molecules, each (or the great majority) having at least one electron as a satelite. A magnetic material may, therefore, be thought of as a sort of coil consisting of a vast number of current-conducting turns, each having a molecule within it, and which always has a definite axial relation thereto. When the electronic turn (the orbit of the electron) is reversed in position, the molecule reverses with it. This coil has no individual or common core hole.

Let a great number of single-layer coils of exceedingly fine insulated wire be wound upon non-magnetic cores of the smallest practicable diameter, and then made up into a group of parallel coils to resemble an ordinary wire core. When an electric current flows in all of the coils, a bar permanent magnet will be simulated, but the turns of wire will be fixed in position and cannot move like the electronic orbits surrounding the molecules of a magnetic material.

But the molecules of a magnetic material can lie in various positions with respect to one another so as to form the best individual magnetic circuits within the structure of the material, and are only straightened out, so to speak, with their electronic orbits all lined up in ranks and files, like the turns of the above group of coils, under the influence of a magnetic field acting straight across (or through) them.

The magnetic flux density in gausses in a magnetic material is equal to 12.567 times the intensity of magnetization. But the magnetic flux density in the above group of coils may be the same as that in an inferior magnetic material. Consequently, the ferromagnetic or structural flux density may be considered as an electronic magnetizing force for the magnetization of other

magnetic substances placed in its magnetic field. This electronic magnetizing force can never be increased beyond the point of saturation where all of the individual electron currents unite to produce the maximum amount of magnetic flux in the direction of the magnetizing field. The greater the number of electron currents per file and the greater the number of ranks, for a given length and cross-section of material, the greater will be the electronic magnetizing force, commonly known as the structural flux density, and the stronger will be the field of the permanent magnet.

The above conception is of a practical nature but it has a scientific foundation. There are a number of theories of magnetism, and these have been well summed up by Duschman.*

In order to produce a good permanent magnet, the inherent density of the magnetic energy of the magnetic material must be great and there must be the equivalent of much friction between the molecules. Then, when the magnetic material is placed in a strong magnetic field the molecules can be lined up in ranks and files by means of a suitable magnetizing force so that their electronic currents shall act concertedly to produce the maximum density of magnetic flux in the common direction, and the equivalent of friction will prevent them from again forming local groups (which they tend to do), and thus produce what is termed demagnetization.

In this connection, it is well to bear in mind that the molecules and their electronic turns always tend to assume such positions as to prevent the appearance of magnetism in a magnetic material, but are forced into various other positions through the influence of a magnetizing field, such positions depending upon the strength and direction of the magnetizing field with respect to the magnetic material and the resultant of the forces acting upon the individual molecules after the magnetizing field has been discontinued. It is possible to magnetize a bar crosswise as well as lengthwise. Hence, if the bar is magnetized crosswise when it is desired that it shall be magnetized lengthwise, it is no fault of the magnet that it does not come up to expectations.

Permanent magnets depend upon the residual or remanent magnetism for their usefulness. The term retentiveness refers to the property possessed by a magnetic material of retaining mag-

* Duschman, S., "Theories of Magnetism," *Gen. Elec. Rev.*, vol. 19, pp. 351, 666, 736 818, 1083, 1916.

netism, that is, magnetism in the ordinary sense. The remanence is the structural flux density of a permanent magnet, sometimes called the residual induction B_r. The coercive force is the reversed magnetizing force required to reduce the remanence to zero and thus demagnetize the specimen.

The remanence indicates how well the molecules lie in plaec and how densely the ranks are formed. The coercive force is a measure of the force required to move them back to their magnetically neutral positions and, therefore, is a measure of the permanence or permanent quality of a permanent magnet. In a series of magnets having equal remanences, the magnet possessing the greater coercive force would possess the greater retentiveness.

The electronic currents associated with the molecules of magnetic materials at normal temperatures are eternal and require no energy other than that inherent to the structure of the magnetic material to maintain magnetic fields permanently. However, the magnetic energy of a permanent magnet may be decreased by permitting it to attract to it a piece or iron, for instance, in which case magnetic energy is converted into work, while mechanical energy is converted into magnetic energy and stored in the permanent magnet when the iron is forcibly removed therefrom.

290. Uses of Permanent Magnets.—Permanent magnets are used where their magnetic fields, as well as their tractive and portative properties, can supplant the magnetic fields produced in solenoids and electromagnets in general by electric currents, thereby saving space and cost, as well as insuring constant field strength in many cases. However, a permanent magnet is useful for tractive and portative purposes only where the magnetic material attracted to and held by it is readily removable without decreasing the magnetism. Therefore, permanent magnets are largely used where magnetic fields of constant or nearly constant strength are desired.

Some of the uses of permanent magnets and the names of the devices wherein they are used are:

Magnetos—for supplying strong magnetic fields.

Electric motors—of the motor-relay type, wherein the field is supplied by a permanent magnet, and of the small synchronous type.

Watt-hour meters, and other electrical instruments having rotating disks—for producing eddy currents in the disks upon which their fields react, thus producing a braking action.

Field magnets—for voltmeters, ammeters, and moving-coil galvanometers in general.

Polarized relays and apparatus.

Loud speakers, head sets, and telephone receivers in general.

Magnetic separators—called mill magnets.

The moving elements of stationary-coil galvanometers.

Magnetic compasses.

Relays—to replace retractile springs.

Toys, and so forth.

291. Factors Affecting the Quality of a Permanent Magnet.—These may be classified as follows:

1. Quality of steel: (*a*) Chemical content. (*b*) Conditions under which the billets are made. (*c*) Rolling conditions.

2. Annealing.

3. Forming: (*a*) Shape and dimensions. (*b*) Furnace conditions. (*c*) Heat treatment during forming or forging.

4. Hardening: (*a*) Furnace conditions. (*b*) Temperature. (*c*) Quenching. (*d*) Further heat treatment.

5. Mechanical operations following heat treatment.

6. Magnetizing: (*a*) Methods. (*b*) Magnetizing force.

7. Maturing: (*a*) Natural ageing. (*b*) Mechanical shock. (*c*) Reheating. (*d*) Partial demagnetization.

8. Conservation: (*a*) Safeguards. (*b*) Shape. (*c*) Dimensions.

9. Uniformity: (*a*) Structural. (*b*) Magnetic. (*c*) Mechanical.

10. Cost: (*a*) Class of service. (*b*) Weight. (*c*) Shape. (*d*) Production.

292. Quality of Steel.—Specific steels are treated in Chap. XVIII, page 309. The chemical content of a steel gives it its identity, but the conditions under which the billets are made and rolled determine, to a great extent, the character and uniformity of its structure. According to J. O. Arnold, the permeability of a magnetic material is inversely proportional to its carbon content, but the amount of permanent magnetism retained is directly proportional to its carbon content, the amount of hardenite ($Fe_{24}C$) present determining the permanence of a magnet. Hence, the coercive force is dependent upon the carbon content but the remanence is not dependent upon it.

293. Annealing.—Steels contain such structural constituents as ferrite, pearlite, hardenite, cementite, and so forth; the amount of each may be controlled by the temperature of the magnetic material with the aid of microscopic examination. Consequently,

magnet steels must be uniformly annealed in the aggregate, as well as in each individual case, as the standard methods of the magnet manufacturer depend upon the uniformity of the steel. Otherwise, the permanence may vary over a wide range.

294. Forming.—Each lot of steel is given a preliminary test. Bars may be sheared cold to the required lengths or, in the cases of small U-shaped or C-shaped magnets, as those used in radio head sets, they are stamped or blanked out. Hot forged magnets may be made from initially unannealed steel. Bars may be heated in fuel-oil furnaces so arranged that the products of combustion are kept from the steel, and they may then be forged in a vertical press or else in a bull-dozer, in one heat if possible.

The forging should be done with the minimum amount of working of the material and the temperature should be kept at a minimum. After forging, the steel should be normalized by raising its temperature to about 950° C. and then lowering to about 750° C. for a time, after which it is allowed to cool. In some cases, the pieces are allowed to cool in the air, but if they are to be drilled they may be placed in mica dust to prevent air-hardening.

295. Hardening.—There are two ways of hardening steel: (1) by heating it to a high temperature and then suddenly cooling it, and (2) by subjecting it to hydraulic pressure as it slowly cools. The former method is the one usually adopted. For short bars, the best results are obtained by making them glass-hard by heating the steel to a brilliant red and then quickly plunging the steel into water, brine, oil, or mercury, the latter being the best quenching medium for this purpose. However, the steel becomes very brittle and often splits during the process. The hydraulic method causes the steel to be hardened more uniformly than in the quenching method, since, in the latter case, the outer crust may become hard while the inner portion is still plastic. However, a laminated magnet may be no more powerful than a solid magnet of equal weight and length, although each lamination may be quenched much more quickly than the larger solid piece.

The best temperature for hardening, except in the cases of short bars, is just above the recalescence point, or the temperature where the steel changes from the magnetic to the non-magnetic state. Too high temperatures may affect the structure of the steel, depending upon the time of application and the history of its former heat treatment. The steel may be heated to

about 950° C. for, say, 5 min., the temperature then lowered to about 700° C., and the steel immediately quenched in water, brine, or oil. Each specimen has its proper quenching temperature, the above temperatures being approximate. The temperature may be 1,000° C. in certain cases, as when the steel is heated in a bath of $BaCl_2$.

The time-rate of cooling in oil is less than in water and the steel does not become so hard when quenched in oil as in water. Nevertheless, the relatively soft oil-hardened steel may have the greater coercive force, although the remanence may be lower. If the permanence of the magnet is the principal requisite, as in cases where cast-iron magnets are used in galvanometers and similar apparatus where weight is not objectionable and general efficiency is not a matter of importance, the coercive force is the more important. However, in other cases, the remanence may be the more important. For instance, the coercive force of hardened cast iron is about 75 per cent of that of hardened tungsten steel, while the remanence of the hardened cast iron is only about 30 per cent of that of the hardened tungsten steel. Even with the greatest care in hardening, the coercive force and the remanence may vary, although their product may be nearly constant.

The volume resistivity of steel increases with the hardness, but the usual methods of measuring hardness are employed.

In the manufacture of forged permanent magnets, the forgings are ground to proper dimensions, tumbled to remove all scale before hardening, and are then heated in a furnace similar to that for forging and provided with a pyrometer so that the temperature may be accurately checked. The hardening is done in a darkened room so that the operator may use his own judgment aside from the information obtained from the pyrometer, the forgings being held in special tongs during hardening to prevent springing or warping.

In the wholesale manufacture of permanent magnets for small apparatus this distortion is often very marked and the magnets may be unevenly hardened due, no doubt, to their being heated and quenched in batches with consequent occasional overlapping of the pieces.

When hardness is relieved by drawing the temper, the remanence (as well as the maximum induction) is increased but the coercive force is reduced. This effect is independent of whether the steel is water-hardened or oil-hardened, but is the more pro-

nounced when it is oil-hardened. There is a distinct difference between hardening and tempering, and the effects due to retarded quenching are directly contrary to those due to tempering, as just noted. When the drawing temperature is about 315° C., oil-hardened steels are softened to a less extent than are the water-hardened steels, while the greatest magnetic changes are in the oil-hardened steels. Under these conditions the two steels are about the same.

296. Mechanical Operations Following Heat Treatment.—After hardening, the pieces are usually tumbled to remove all scale and they are ground if necessary. They may then be plated, japanned, or otherwise finished, although these operations may be performed after magnetizing or after maturing.

297. Magnetizing.—Permanent magnets are best magnetized by magnetizing forces produced by electric currents. Long bars may be placed within the core hole of a solenoid about twice as long as the bar. Short bars may be placed between the polepieces of strong electromagnets. U-shaped magnets of the forged variety may be magnetized by using two coils with the abutting ends of two magnets meeting at the middles of the coils. The combination resembles a two-coil plunger electromagnet.

The author adheres to the view that the magnetizing force due to a current in the turns of a solenoid is constant when the current strength is constant whether or not any magnetic material is in the core hole or otherwise adjacent to the solenoid. The molecules of a magnetic material can swing around so that the direction of the magnetic flux due to each electron current may be varied and the field strength of the magnet thereby changed, but the directions of flow of electricity in the turns of conductor in a solenoid are fixed and, therefore, the magnetizing force is fixed when the current strength is constant.

The principle may be illustrated by magnetizing the same magnet, or two similar magnets, in two different manners. When U-shaped magnets are magnetized in coils as above described, the molecules are forced by the magnetizing force into positions which tend to correspond to the shape of the magnet, that is, the electronic currents tend to line up so that the resulting magnetic flux shall have a direction lengthwise of the steel. When similar magnets are placed across the polepieces of an electromagnet, the magnetic field of the electromagnet acts across the U as well along its contour. The permanent magnets may be demagnetized by

establishing a strong alternating current in the coils and gradually reducing it. The permanent magnets magnetized by the coils will be found the better.

The method described on page 301 sometimes may be employed. Long U-shaped magnets with their sides close together may be pulled between movable polepieces of a powerful electromagnet, beginning at the neutral-polarity plane. The so-called stroke and touch methods are resorted to in some cases, but an electromagnet is the magnetizing medium, except where small objects are to be magnetized.

There appears to be a slight advantage in turning the current on and off a few times and jarring the magnet with a wooden mallet during the process. The time of magnetization is of little importance, but the field strength is of the greatest import. Coils or electromagnets with relatively few turns may be connected in direct-current circuits with an inverse-time current relay in series for the purpose of tripping a circuit breaker after the current strength has attained a safe maximum. However, the circuit should not be opened too suddenly, as the eddy currents in the steel have a demagnetizing effect. Arcing contacts of carbon are preferably employed. By proper methods of magnetizing in each individual case, it is unnecessary to apply the magnetizing force more than once for each magnet and the jarring may be omitted.

At least one of the poles is usually marked immediately after magnetizing for polarity identification, or the magnet may be previously marked and magnetized accordingly. The latter method is preferable, as a permanent mark may be made during the forging process. Where magnets are magnetized after assembly in apparatus, no marking is necessary.

Magnets with long air gaps, as those for magnetos, should have soft-iron keepers placed across the poles after they are magnetized and before they are removed from the magnetizing electromagnet. Special devices for this purpose are on the market.

Magnets with short and broad air gaps, as certain brake magnets for watt-hour meters, do not necessarily have to be provided with keepers, as their air gaps remain open in service. Neither do they require careful handling, except as hereinafter noted. While in service, they are subject to the demagnetizing influences of eddy currents.

298. Maturing.—Permanent magnets age with time. Lamont made an investigation lasting eleven years, and found that the losses, as given in decimal parts of the whole magnetic moment (pole strength × length) varied from 0.0174 in the year 1847 to 0.0063 in 1858. Temperature, mechanical shock or vibration, and demagnetizing magnetic fields hasten the maturing of most magnetic materials.

The magnetism of a newly made magnet appears to consist of a permanent part and a temporary part. Maturing consists of removing the temporary part by one or more of the above-mentioned methods. In general, it appears that magnetic materials having the greatest coercive force are least affected by shock; that there is a self-annealing process within the steel from the time it is hardened and regardless of the time at which it is magnetized; that a loss in coercive force up to 20 per cent occurs when a magnet is drawn over the face of an iron plate; and that glass-hard magnets are not so susceptible to shocks as are others.

Magnets may be matured or artificially aged by placing them in oil at a temperature of about 120° C. for about one-half hour and then submitting them to vibration. In some cases, only one vigorous rapping is sufficient. The remanence is thus reduced to about 70 or 80 per cent of its original value, the heating accomplishing the greater part of the maturing when heating is followed by vibration.

Magnets for telephone receivers, magnetos, and the like, where the maximum field strength is required regardless of ageing, require no maturing. However, in the cases of permanent magnets used in indicating and recording instruments, it is imperative that they shall be matured for constancy. This may be accomplished as above before assembly, or after assembly by holding near it a coil of relatively large diameter, like a looped cable, energized with alternating current, the coil being gradually moved near the instrument until the maturing and calibration are simultaneously accomplished.

299. Conservation.—In the cases of permanent magnets having relatively short air gaps, or when the magnets are magnetized after assembly, keepers are not generally required. However, in the cases of magneto magnets and the like, it is highly important that keepers be constantly kept across the poles from the moment they are magnetized until they are in place on the apparatus. This is accomplished by sliding a soft-iron keeper having

a cross-section of at least one-half that of the magnet across the poles immediately after the magnet is magnetized and before its removal from the electromagnet, and sliding it off again as the magnet is placed in position during assembly. The keeper should never be slammed on. If, for any reason, such a magnet has to be removed from the apparatus, the keeper should again be placed in position as the magnet is removed.

A permanent magnet should never be drawn over the surface of a piece of iron or steel; neither should it be subjected to any demagnetizing influence, as a current-carrying conductor, electromagnet, or permanent magnet, nor should it be subjected to constantly changing temperature.

The retentiveness of a permanent magnet is largely dependent upon its shape and dimensions. The dimension ratio is the ratio of length to diameter or thickness of the magnet. Upon the dimension ratio depends the self-demagnetizing effect of the magnet. Since the poles are nearer together in short than in long magnets, short magnets tend to become demagnetized unless made glass-hard. When the dimension ratio is about 20 or greater, the self-demagnetizing effect is approximately constant. The demagnetizing coefficient varies inversely as the dimension ratio. In the case of a uniformly magnetized toroidal or ring permanent magnet of uniform structure, the dimension ratio and the demagnetizing coefficient have no meaning, since the electronic magnetizing force or structural flux density is uniform and there is no magnetic leakage. However, if an air gap is introduced in such a permanent magnet, the demagnetizing effect will be proportional to the length of the air gap up to a certain limit. If there was no magnetic leakage, the demagnetizing effect would be directly proportional to the length of the air gap. The demagnetizing coefficient is reduced by decreasing the length of the air gap and by increasing its cross-section, that is, by increasing its permeance (and reducing the reluctance). The remanence increases regularly with the dimension ratio.

Two or more permanent magnets having different qualities should never be mounted together in any manner if the maximum magnetic field is to be obtained. Even when their dimension ratios are similar their coercive forces may be, or may become, dissimilar, thus making a difference in their remanences, in which case the one will form a partial shunt magnetic circuit for the other. Even when the proper relations exist between two

or more magnets to be used together, the rubbing or touching of the part of one against a dissimilar part of another in assembly may produce detrimental effects. However, the use of one or more keepers, which should be pulled straight away from the magnets upon removal, is beneficial. This general statement is of a precautionary nature. Large number of compound and parallel magnets are constantly being made and have given excellent results.

300. Uniformity.—The structure of a permanent magnet should be uniform, but it cannot be uniformly hardened, that is, the interior does not cool as quickly as the exterior. Therefore, there is a limit to the thickness, say, $\frac{3}{8}$ or $\frac{7}{16}$ in., when the steel is 3 in. or greater in width. In a U-shaped permanent magnet, the outer length is greater than the inner length, so that the inner portion may act as a partial shunt magnetic circuit for the outer portion. It is better to use solid magnets where possible rather than compound magnets of either the parallel or leaf types. Experience has proved that there is a limit to the number of leaves which may be used without actually decreasing the strength of the magnet.

Magnetizing permanent magnets in such a manner that the structural flux shall follow the contour of the magnet after the magnetizing force has been removed gives the greatest magnetic uniformity. Magnetizing a U-shaped magnet by placing it on the polepiece of an electromagnet may cause portions of the magnet to be magnetized nearly crosswise, especially in regions back from the poles.

Distortion in hardening tends to prevent a close contact between the magnet and its coacting polepieces of soft iron or steel, thus tending to increase the reluctance and cause an uneven distribution of the magnetic flux, as to the armature of a magneto.

301. Cost.—It may be taken as a criterion that the best permanent magnet is the most expensive. A fairly good permanent magnet may be made from cast iron by quenching at 1,000° C. Such a magnet has been found by Campbell to have a remanence of about 215 and a coercive force of about 50. On the other hand, tungsten steel is superior to most other commercial magnetic materials. However, chromium steel is probably used more than any other magnetic material in the manufacture of permanent magnets.

The production of large quantities of small permanent magnets in the face of keen competition calls for automatic or semi-automatic means for forming, hardening, and so forth. Commodities of a general nature of which permanent magnets form parts must strike a balance between high quality and excessive cost.

The cost, therefore, is dependent upon the weight of steel, the labor cost, and the percentage of rejections. The better the magnet the less will be the weight of steel used. The labor cost is dependent upon the difficulty experienced in forming magnets on edge and giving them complex shapes, the number of operations required, and the difficulty of machining magnet steel even in the annealed state. Milling and tapping operations are especially difficult. The shrinking and warping of magnets in hardening is the cause of the greater percentage of rejections, and the excessive wear on tools, as well as their shrinkage in the hardening process, is an important item of cost. Naturally, the smaller the lot the greater is the cost per piece.

302. Standards of Quality.—The density of magnetic energy is proportional to the product of the induction and the magnetizing force. The ratio of the induction to the magnetizing force is the permeability. The residual induction or remanence B_r is dependent upon the reversed magnetizing force or the coercive force H_c, for its maintenance. The total magnetic flux is directly proportional to the product of the remanence and the cross-section of the magnet. The coercive force is proportional to the length of the magnet. Therefore, it would seem that the product B_rH_c would be a good gage of the quality of a magnet steel. This is the view of J. W. Esterline.[5]

J. A. Mathews[4] takes the view that the ratio $\frac{B_r}{H_c}$ is the better gage for the magnetic quality of magnet steels. He states that the coercive force alone is not a sufficient gage of permanence, and that $\frac{B_r}{H_c}$ indicates the average remanence removed per demagnetizing unit of coercive force. Using an Esterline permeameter, the ratio $\frac{B_r}{H_c}$ becomes almost constant when the magnetizing force $H = 200$ or more, thereby permitting comparisons of tests made with different values of H.

Reference to a hysteresis loop, as on page 293, or to a section thereof, as in Fig. 463, shows that the remanence decreases very slowly with respect to the demagnetizing force when the latter is started at zero and gradually increased. Therefore, the remanence is fairly stable for moderate demagnetizing forces. However, as the demagnetizing force is increased, the remanence decreases at an increasingly rapid rate, becoming equal to zero when the coercive force, demagnetizing force, or reversed magnetizing force is about 60 in this case.

FIG. 463.—Remanence and coercive force.

303. The Effects of Air Gaps.—The illustrations above cited are due to closed ferromagnetic circuits. The effect of an air gap is to demagnetize the magnet partially, as previously noted, and the extent of the demagnetization is proportional to the ratio of the length of the magnet to the length of the air gap. Consequently, this ratio should not be reduced to a point where the magnetization becomes unstable.

In certain types of apparatus, as in magnetos, currents in coils or disks tend to demagnetize the magnet, and an additional factor of safety should be allowed for this effect. However, such magnets are usually made sufficiently long to provide the proper ratio. This is the reason why keepers should be kept on such magnets when not mounted on the apparatus which has a short air gap.

When the demagnetization of a permanent magnet is reduced to the point where the remanence becomes unstable, it will not recover.

304. Some Forms of Permanent Magnets.—The bar and horseshoe permanent magnets are familiar, the latter being flat as though bent on edge. Magneto magnets are of the horseshoe form but are formed with their widest parts parallel. Some general outlines are shown in Fig. 464. The magnets used in certain loud speakers and telegraph and other relays have special shapes. A novel magnet (Fig. 465) is successfully used for protecting vending machines from iron slugs. The magnet is so designed that the slug is attracted to the poles as it moves into the machine. Since the field of the magnet decreases as the slug moves towards the neutral-polarity plane, it naturally follows the magnet to approximately that point and then falls into a separate slot. This belongs to the magnetic-separator class.

305. Cobalt Steels for Permanent Magnets.—Under this title, J. F. Kayser[3] has shown that straight, short bar magnets

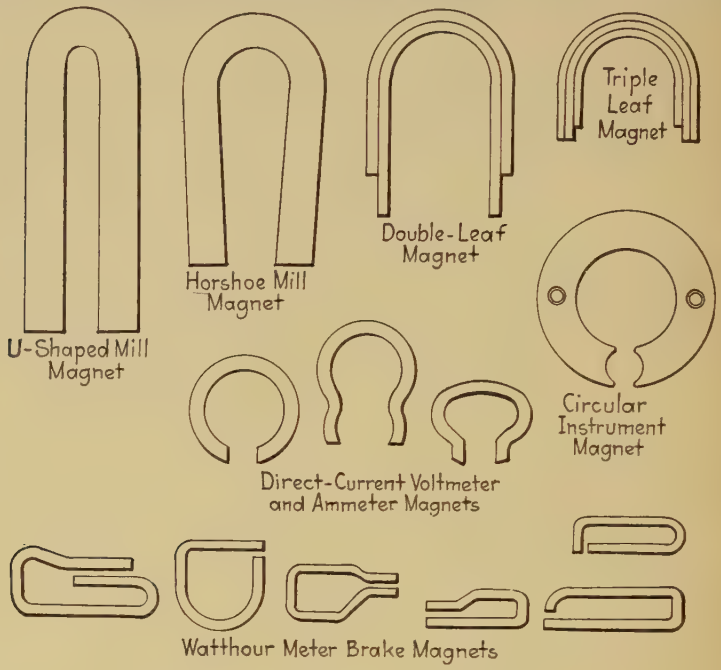


FIG. 464.—Forms of permanent magnets.

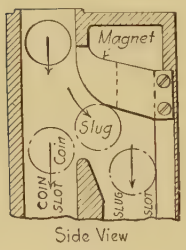


FIG. 465.

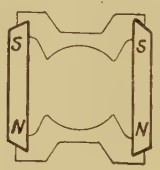


FIG. 466.

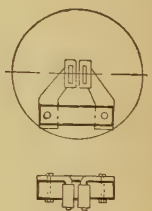


FIG. 467.

FIG. 465.—Vending-machine protection magnet.
FIG. 466.—Straight magneto magnets.
FIG. 467.—Straight receiver magnet.

are now replacing bent permanent magnets in magnetos, telephone receivers, measuring instruments, and the like. Figures

466 and 467 respectively illustrate the improvements in magnetos and telephone receivers due to the use of such steels. Such methods are impracticable with tungsten and chromium steels.

Steels of the cobaltcrome type rely solely on air hardening, and hence do not give trouble during heat treatment in so far as cracking or distortion is concerned. The hardening temperatures vary considerably according to composition, and, whatever type of steel is dealt with, it is essential that the temperature control shall be very efficient, and it is always necessary to work within $\pm 10°$ C. of the required temperature.

Magnet steels of the cobaltcrome type contain from 1 to 18 per cent cobalt, 9 to 15 per cent chromium, and carbon above 1 per cent, the optimum percentages giving coercive forces of 200 to 250 (see also Art. 211, page 313).

In the article above quoted it is stated:

The pioneer movement to utilize the new steels was led by E. A. Watson, a director of the M. L. Magneto Syndicate, Limited, Coventry, who took up a very different attitude, and, as soon as small quantities of the new steels first became available towards the end of 1919, he began to design new ignition apparatus. As a result of a great deal of intense application to the subject, the first ignition apparatus using cobalt steel magnets was ready to place on public exhibition at the Motor Car Show in 1920. The development of magnetos fitted with cobalt magnets has since been so rapid that at the recent Cycle Show over 30 per cent of the magnetos exhibited were fitted with cobalt steel magnets, and 50 per cent of the makers showing motor cycles were fitting such magnetos.

306. Leviated Magnet.—To the author, the extreme test for the retentiveness of a permanent magnet is to make it in the form of a bar, place it over the poles of a horseshoe permanent magnet, and cause the bar magnet to remain in suspension above the poles of the horseshoe magnet. This has been tried many times but, until lately, the author has never heard of its successful accomplishment. The conditions for leviation demand a coercive force so great that a straight magnetized bar shall not be demagnetized when placed in a magnetic field that is sufficient to produce a repelling force at least as great as the weight of the leviated magnet. This test also indicates the magnitude of the remanence.

F. H. Glew[6] states:

Presumably all interested in magnetism have tried to keep a magnet in suspension by the repulsion of like poles balancing its weight. The experiment fails through inability of the small magnet to resist having its poles reversed, or diminished in strength, by the intense field necessary for leviation . . . The particular steel used by me I owe to the kindness of W. H. Glaser, who tells me it contains 15 per cent cobalt and is known as "cobalt chrom" steel.

The solid rod was 2½ in. long by 3/16 in. diameter, making the dimension ratio 13.33, and it weighed 12 grams. In making the test, the leviated magnet was placed in the bottom of a glass cell about 3 in. deep and open at the top with 1-mm. clearance between the glass walls and the rod, so that the latter might move freely. The cell may be made from old photograph plates and adhesive tape.

Holding the cell vertically, it is lowered slowly towards the poles of a horseshoe magnet, held vertically. If the poles of the horseshoe magnet are of the same sign as the opposing poles of the magnetic rod, the latter will rise and oscillate up and down in its enclosure. When properly adjusted in the field of a good magnet, the rod will remain permanently poised about half an inch above the poles, which should be about the same distance apart as the poles of the leviated magnet.

If an electromagnet is used, the leviated magnet must be brought downward towards it very carefully, or the strong field of the former may demagnetize and even attract the latter. However, if care is taken "a much greater distance of separation may be obtained."

As stated at the beginning of this article, here appears to be the nucleus of testing permanent magnets of any shape in terms of their weights. The tests described in the foregoing articles are based on closed ferromagnetic circuits. But a permanent magnet is not very useful unless it has an air gap or consequent poles. A test hereby suggested, although there is really nothing new in the method, is to place a magnet in a specially prepared balance and then note how much the magnet can be leviated in terms of its weight in a given repelling magnetic field. The suggested method really amounts to counterbalancing the magnet and then weighing the force of repulsion, noting the tendency of any instability during the test, wherein the magnetic field of the electromagnet may be varied.

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INDEX

A

Absolute temperature, 9
  zero, 9
Acceleration relay, 143
Action, delayed, 40
  drop-hammer, 76
  generator, 122, 333
  pile-driver, 76
  time-limit, 76
  transformer, 333
Active current, 352
  volt per ampere, 7
  voltage, 422
Adjustable magnetic shunts, 304
Admittance, 352
Ageing of steel, 314
A.I.E.E. standards, 120, 139
Air, as a heat conveyor, 9
  as an insulator, 9
  cylinders, 156
  in series with steel, 329
Air-cored reactor, 16
  solenoid, 173, 384
Air gap, air gaps, 29, 316
  cone-shaped, 67
  effects, 445
    on fields, 325
    on leakage, 316
    on pull, 227, 427
  saturation curve with, 329
  series, 329
Air-gap pull, 62
Air-hardening, 437, 447
Alarms, 145
Algebraic addition, 340
  subtraction, 7, 340
Alloy steel, 1, 310
Alternating current, 4, 21, 330
Alternating-current apparatus, 270
  electromagnets, 47
  mechanisms, 389
Alternating-current phenomena, 330
  resistance, 346
  solenoids, 369
  sounder, 69
  telegraph relays, 145
  values, 145
Alternation, 23
Aluminum coils, 108
  oxide insulation, 108
Ampere, 6
  per active volt, 7
Ampere-turn, Ampere-turns, 13
  additional, 62
  gradient, 382, 422
  per volt, 51, 422
Ampere-weber-turns, 15
Angle, phase, 347
Angular velocity, 350, 383
Annealing, 310, 436
Annular electromagnet, 44
  lifting magnet, 105
Apparatus, 119
Arcing, 88
  contacts, 119
Arc-lamp generator, 159
  magnet, 389
Areas under curves, 203, 272, 275
Armature, armatures, 34, 122
  biased, 97
  effects of moving, 378
  floating, 69, 389
  hole through, 68
  massive, 84
  momentum of, 68
  pivoted, 60
  rotary, 60, 94
  slidable, 69
  tuned, 399
  velocity of, 429
  Z-type, 60
Arrester, lightning, 121
Asbestos, 10

Ascending logarithmic curve, 259
Assymetric coils, 64
Attraction between solenoid and plunger, 185
Automatic compensator, 127
control, 125
generator-voltage regulators, 144
motorstarters, 125
regulators, 131
switch, 138
Autotransformer, 256
Auxiliaries, mechanical, 399
Auxiliary relay, 140
switch, 120
Axle generator system, 132

B

Back iron, 43
Balance, magnetic, 392
Balance wheel, electric, 69
Balanced solenoids, 221
Balanced systems, 365
Ball on core, 78
Band brake, 149
Bar electromagnet, 46, 90
Bar permanent magnet, 12, 317
Bath, quenching, 437
Bellows, 127, 142
B-H curves, 290
Biased armatures, 98
Bismuth, 101, 309
Blow field, force of, 137
Blow, hammer, 35, 65
Blow-out, magnetic, 119, 136
Brake, brakes, 11, 51, 149
magnet, 389
solenoid, double-plunger, 236
Braking, dynamic, 123, 129
Butt contacts, 130

C

Cable-coil, 306
with separable rings, 306
Capacitance, 11, 250
e.m.f. of, 348
of condenser, 361
Car lighting, 135
Car-switching magnet, 117
Carbon-break circuit breakers, 136
Carbon-pile resistor, 134
Carbon steel, 310
Cast iron, 310
Cast steel, 310
Characteristics of a.c. magnets, 371, 427
systems, 365
Characteristics, varying the, 83
Charcoal iron, 310
Charge, 4, 12
Chromium (chrome) steel, 310
Chuck, chucks, magnetic, 101, 118
Circuit, circuits, electric, 6, 237
alternating-current, 336, 347
classification, 237
coupled, 254
direct-current, 240
metallic, 7
natural frequency of, 332
oscillating, 332, 363
parallel, 248, 347, 355
phenomena in, 237
primary, 25
secondary, 25
series, 7, 240
short, 10
shunt, 237
Circuit, circuits, magnetic, 13, 281
classification, 286
electromagnetic, 286
ferromagnetic, 286
length of, 419
relations, 283
series, 297
series-parallel, 305
shunt, 297
Circuit breakers, 121, 136
quick-acting, 137
solenoid-operated, 136
Clapper-type electromagnet, 58
Clock, Warren, 403
time, 158
Clockwork, 142
Clutch-brake, 111, 149
Clutch, Farrel, 111
Clutches, magnetic, 108
Cobalt steel, 446

Cobaltchrome steel, 310, 447
Code-calling systems, 145
Coercive force, 309, 435
Coil, Coils, 1, 9
  air-cored, 16
  aluminum, 108
  assymetric, 64
  average girth of, 10
  cable, 306
  cooling of, 10
  damping, 303
  edge-wound, 9, 128
  field, 122
  form for, 10
  heating of, 420
  induction, 67, 255
  length, 10
  loading, 314
  section, 141, 388
  series, 38
  shading, 71, 80, 395
  short-circuited, 84
  shunt, 38
  square-cored, 34
  toroidal, 281
  tripping, 136
  wall, 10
Collar on plunger, 56
Combined control, 126
Commercial solenoids and electromagnets, 232, 390
Compass, magnetic, 326
Compensator, 127
Components of current, 352
  of power, 239
  of voltage, 239, 341
Compound, insulating, 44
Compound-wound motor, 122
Concentration, 112
Condensance, 10, 351
Condenser, condensers, 11, 29
  capacitance of, 361
  charged, 251, 330
  discharging, 253
  leaky, 253
  parallel (or shunt), 253
  series, 87
  synchronous, 109
Condensive susceptance, 354
Conductance, electric, 5, 7
  effective, 353
Conduction-current leakage, 10
Conductor, conductors, electric, 1, 5
  thermal, 6
Coned plungers and stops, 67, 227
Cone-shaped air gaps, 67
Connection, connections, parallel, 47, 248, 355
  series, 47, 240
  series-parallel, 252
  shunt, 47
Conservation, 441
Constant-current solenoids, 176
  electromagnets, 61
Constant-voltage electromagnets, 64
  generator, 308
Construction, 232, 389, 398
Contact, electrical, 10
Contactor, contactors, 121
  parts of, 391
Contacts, 130
Continuous current, 4
Continuous-duty electromagnet, 39, 50
Continuous voltage, 4
Control, 1, 120, 126
  solenoid for gags, 157
Controller, controllers, 121
Cooling surface, 419
  of coils, 10
Copper around core, 84
  collars on cores, 87
  disk on end of core, 84
Core, cores, 2
  copper around, 84, 87
  disk on end of, 84
  enlarged ends, 67
  heating of, 49
  helical, 90
  laminated, 49
  losses, 310, 380
  slotted, 49
  solid, 49
Core hole, 2
Core loss, 310
Core-type transformer, 404
Cosine of phase angle, 347
Coulomb, 6

Coulomb, per dielectric volt, 11
per second, 6
Counter, 73
Counter e.m.f., 122
voltage, 21
Coupled circuits, 254
Coupling, magnetic, 110
brake, 111
Current, currents, 3
alternating, 21, 71, 330
characteristics of magnets, 371
components of total, 352
constant, 61
continuous, 21
density, 419
direct, 4, 21, 71
displacement, 4, 10
eddy, 26, 48
electromagnets, 51
electron, 89, 434
harmonic, 330
high-frequency, 10
in-phase, 352
inrush, 122
lagging, 352
leading, 352
leakage, 10
-limit control, 126
-limiting phenomena, 50
line, 122
oscillating, 4
permanent magnets and, 90
phase, 365
positive, 89
primary, 25
quadrature (reactive), 352
relay, 139
resonance, 363
reversed, 86
secondary, 25, 255
effects of, 378
strength, 3, 6
surges, 122
-time characteristics, 259
wave, 80, 296
Cushion, magnetic, 67
Cut-off relay, 141
Cycle, 23
Cylinders, air, 156

D

Damping coil, 303
Damping, electrical, 24
mechanical, 69
Daraf, 11
Dash pot, 127, 150
Definite-time relay, 140
Delayed action, 40
Delta system, 365
Demagnetizers, 104
Demagnetizing coefficient, 442
effect, 1, 181
Density, current, 232, 419
flux, 13, 283
of energy, 29, 169, 204
Design, 418, 420, 423
Devices, 147
Diamagnetic substances, 309
Dielectric, dielectrics, 1, 11
constant, 12
flux, 4
hysteresis, 5
Differential magnetic balance, 393
relay, 140
secondary, 393
solenoid, 180, 199
Dimension ratio, 442
Direct current, 4, 21, 90
Direct-current electromagnets, 47, 87, 221
Direct voltage, 4, 21
Directional relay, 139
Directions of current and flux, 90
Disk on core, 77, 84
Displacement current, 4
Displacement, horizontal, 427
Disruption, electrical, 11
mechanical, 10
Distortion of waves, 296
Divided bar electromagnet, 180
Double-acting switch, 129
Double frequency, 32, 47
Double-plunger solenoid, 236
Double-pole contactor, 129
Double-solenoid brake, 151
Drop, active-voltage, 7
Drop-hammer action, 76
Duty ratings, 50, 87
Dynamic braking, 123, 129

Dynamo, principle of, 91
steel, 310
Thomson arc-light, 159
Dyne, 170

E

Eddy currents, 26, 48
losses due to, 50
rotation due to, 402
Edge, magnetic, 101
Edge-wound coils, 9, 128
Effective conductance, 353
susceptance, 354
values, 342
Efficiency of electromagnet, 28, 425, 432
Elastance, 11
Electric circuit, 237
hammers, 404
pressure, 6
Electrical particles, 4
sheets, 49, 311
Electricity, 3
quantity of, 6
Electrodynamometer, 97
Electrolytic iron, 314
Electromagnet, electromagnets, alternating-current, 47, 87, 369, 389
annular, 44
as an armature, 95
bar, 46
classification, 38, 41
condenser in series with, 87
constant-current, 61
constant-voltage, 375
continuous-duty, 50
current, 51
direct-current, 47, 87, 221
efficiency of, 425, 432
for constant flux, 308
forces of, 61
forms, 31
general, 414
heavy-duty, 87
high-frequency, 82
horseshoe, 46
in general, 414
in series with impedance, 424
influenced by a permanent magnet, 93
Electromagnet, initial pull of, 47
intermittent-duty, 50
Joule's, 90
long-range, 47, 73
low-frequency, 82
magnetic-cushion type, 46
Marcel Deprez, 73
momentary-duty, 50
movable-core, 56
plunger, 45, 62, 225, 370
polarized, 92
polyphase, 79, 398
Portative, 41, 101
practicability of, 414
pushing, 72
push-pull, 99
quick-acting, 81
quick-release, 85
quick-response, 85
reactor in series with, 84
in shunt with, 84
repulsion, 74, 99
resistor in series with, 85
in shunt with, 85
ring, 44
rotary, 60
sealing-pull of, 47
selection of type, 417
semi-toroidal, 46
series, 51
short-range, 39
shunt, 52
single-phase, 79, 389
slow-acting, 81
slow-release, 85
slow-response, 84
stationary-core, 56
test characteristics, 370
three-phase, 80
toroidal, 44
tractive, 34, 45
two-coil, 47
two-phase, 80
universal, 424
uses of, 147
voltage, 52
Weiss, 159
with permanent magnet, 92, 95
Electromagnetic induction, 14, 256
inertia, 14

Electromagnetic mass, 14
momentum, 15
Electromagnetism, 13, 21
Electromotive force, 3
Electron, electrons, 3
bound, 4
current, 89, 434
flow, 90
free, 3
orbit of, 3, 433
Electronic magnetizing force, 433
Electrostatic repulsion, 4
Elementary metals, 3, 15
E.m.f. (see voltage), 3
active, 7, 239
counter, 122
of capacitance, 11, 348
of self-induction, 348
reenforcing, 22
Energy, 2
conversions, 339
density of, 29, 170
electric, 5, 7, 11
forms of, 26
in magnetic circuit, 418
kinetic, 26
in armature or load, 74
magnetic, 2, 18, 169
mechanical, 2, 8, 18
potential, 26
reconversions, 238
storage of magnetic, 14, 169, 418
and conversion, 271
electric, 11
in solenoid-and-plunger, 206
thermal, 5, 27
transferences of, 238
English system of units, 8
Equalizers, 37, 63
Equivalent single phase, 367
delta connection, 368
Erg, 170
Ether, the, 13
Extrapolation, 429

F

Factor, factors, conversion, 8
linkage, 183
power, 108, 348
Factor, space, 423
Farad, 11
Ferromagnetic substances, 1, 309
linkage factor, 183
Ferromagnetism, 4, 21, 91
Field coil, 122
Field, magnetic, 3, 122, 223, 317
direction of, 90
effect of air gap, 318
mechanical forces in, 18
of bar permanent magnet, 317
of plunger electromagnet, 223, 326
of solenoid, 186
rotating, 81
strength, 308
Field-protective relay, 144
Filament breaker, 157
Filing figures, 316
Filler, non-magnetic, 101
Fishing tool, electric, 118
Floating armatures, 389
stops, 66
Floor-type mounting, 390
Flux density, densities, 178, 287
at saturation, 192
electromagnetic, 178
ferromagnetic, 178
summary of, 287
Flux, fluxes, magnetic, 2, 282
across air gap, 308
air-gap, 374
conception of, 326
constant, 308
construction of paths, 328
density, 13, 283
electromagnetic, 21
ferromagnetic, 21
leakage, 374
names for, 21
ratio of, 385
relative values, 374
reversal of, 304
separating, 382
sinusoidal wave, 296
structural, 21, 385
with constant current, 294
Flux-turns, 14
Force, electromotive (e.m.f.),3
magnetizing, 6

Force, magnetomotive (m.m.f.), 14
forces, mechanical, 3, 8, 18
and volt-amperes, 377
average, 19, 33
between solenoids, 170
characteristics, 61, 369
double-frequency, 47
due to change in self-inductance, 208
limits of, 418
of blow field, 137
of electromagnets, 61
of plunger electromagnets, 62
pulsating, 32
radial from poles, 328
ratio to voltage, 213
Force-distance characteristics, 63
Forces and pressures, 29
Forging, 437
Form, coil, 10
Forming permanent magnets, 437
Forms of electromagnets, 31
Frame, 61
Free electrons, 3
Freezing of valves, 156
Frequency, 10, 23
changing, 422
double, 32, 47
effects on electromagnets, 82
meter, 302
natural, 24, 332
quadruple, 55
Fuse, 121

G

Gags, control for, 157
Galvanometer, 95
Gauss, gausses, 13
per ampere-turn, 161, 164
per gilbert, 167
Generator, generators, 7, 404
action, 122, 333
axle system, 132
constant-voltage, 308
Thomson's first arc-light, 159
voltage regulator, 144
Geometric addition and subtraction, 7, 340
Gilbert, 14
Gilberts per unit length, 283
Girth of coil, average, 10
Gradient, gradients, ampere-turn, 422
m.m.f., 283
voltage, 11
Gravitational constant, 8, 428
Growler, 393

H

Hammer blow, 35, 65
Hammer, hammers, electric, 404
definitions, 410
equivalent 200-lb., 412
induction, 410
power, 409
Schüler's, 406
Syntron, 407
Trombetta's, 413
Hand magnet, 105
Hand, magnetic, 117
Hard iron wire, 314
Hardening magnet steel, 310, 437
Harmonic current, 330
motion, 69, 331
Hazeltine's coned-plunger electromagnet, 228
Heat, 5, 9
flow of, 10, 419
losses, 6
Heating power, 343
due to short-circuited turns, 403
of coils, 419
of cores, 49
Heavy-duty electromagnet, 87
Helix, 2, 9
Henry, 14
Henry-amperes (weber-turns), 385
Hesitating-control relay, 143
High-frequency currents, 10
High-frequency electromagnets, 82
Hinged electromagnet, 58
Hold-down magnet, 117
Hole through armature, 68
Horizontal displacement, 427
Horseshoe electromagnet, 46, 90
Humming, 36

Hysteresis, magnetic, 5, 292
dielectric, 6
effects of, 378
losses, 50

I

Impedance 50, 351
and condensance, 356
components of, 352
in parallel, 355
in series with electromagnet, 424
per phase, 367
Impressed voltage, 7, 341
components of, 352
Increasing logarithmic quantities, 265
Independent magnetic chuck, 102
Induced voltage, 14, 256
Inductance, mutual, 25
self, 14
Induction, change in, 295
coil, 67
electromagnetic, 14, 256
e.m.f. of, 348
hammer, 410
magnetic, 20, 288
motor, 122, 410
mutual, 25
self, 14
Inertia, electromagnetic, 14
Initial pull, 47, 65
Inrush of current, 122
Instantaneous relay, 140
Instruments, measuring, 401
Insulating compound, 44
materials, 1
properties, 5
resistance, 10
Insulation, electric, 1
aluminum oxide, 108
thickness of, 424
Interlock, 391
Interpolation, 264
Interrupters, 67
Inverse-time relay, 140
Iron, 310, 314
flux density and saturation, 192
losses, 50
Iron, tramp, 117
Iron-clad solenoid, 61, 221
magnetic-cushion type, 46
Isthmus, 301

J

Jigs, magnetic, 101
Joint resistance, 249
self-inductance, 249
Joule, joules, 7, 8, 15
per microfarad, 12
per second, 27
Joule's electromagnet, 90

K

Kilogram, 171
Kilogram-centimeter, 8
Kilogram per square centimeter, 171
Kinetic energy, 26
in armature, 74
operating counter, 73
Krämer alternating-current magnet, 395
K. S. magnet steel, 313

L

Lagging current, 352
Laminated parts, 48
contacts, 130
structure, 311
Laminations, 48
Lamp-filament breaker, 157
Leading current, 352
Leads, 10
Leakage, current, 10
Leakage flux, 374
Leakage, magnetic, 63, 160, 316
from plunger, 188, 202
Leaky dielectric, 11
Legs, 43
Length-rate of change, 65, 208
Lever, magnetic, 67
Leviated magnet, 447
Lifting magnets, 105, 131
Hazeltine's, 226
Lighting, railway-car, 133

Lightning arrester, 121
Limit, time-, 76
  current-, 127
Line relay, 141
Linkage factor, 183
Linkage, magnetic, 3, 160
  mechanical, 63
Load, 1
  kinetic energy in, 74
  magnetic, 36, 41
Loading coils, 314
Locking relay, 140
Lock-out switch, 128, 303
  magnet, 128
Lodestone, 1
Logarithmic current-time characteristics, 259
  curves, 259, 272, 275
  quantities, 265, 267
  tables, 263
  voltage-time, 259
Long-range electromagnets, 47, 73
Loss, core, 310
Losses, eddy-current, 50
  heat, 6
  hysteresis, 50
  in solid cores, 380
Loud speakers, 94, 95, 97
Low-frequency electromagnets, 82
  relays, 393
Low-voltage release, 121

M

Magnet, magnets, 1
  bar, 12
  brake, brakes, 51, 149, 389, 446
  car-switching, 117
  hold-down, 117
  instrument, 446
  leviated, 447
  lifting, 105
  lock-out, 128
  magneto, 446
  mill, 446
  permanent, 1, 433
  receiver, 446
  steel, 310
  surgical, 157
Magnet motor, 403
Magnetic blow-out, 119
  brakes, 111
  chucks, 101
  circuits, 13, 281, 286
  clutches, 108
  compass, 1, 326
  couplings, 110
  cushions, 45, 67, 225
  edge, 101
  energy, 2, 26
    and volt-amperes, 379
  field, 3, 122, 223
    rotating, 81
  field strength, 13, 308
  flux, 2, 326
  flux density, 13, 283
  hand, 105
  hysteresis, 5, 292
  induction, 20, 288
  jig, 101
  leakage, 63, 160, 316
  lever, 67
  linkage, 160
  load, 36, 41
  lock-out switch, 128, 303
  materials, 1, 309, 433
  packing plate, 118
  phenomena, 281
  reluctance, 282
  screens, 102, 304
  separators, 112
  shunt, 304
  spacers, 304
  spectra, 316
  susceptibility, 287
  switches, 127
Magnetic energy in solenoid-and-plunger, 206
Magnetic properties, 1, 5, 315
  effects of temperature on, 315
Magnetism, 1
  remanent, 291
Magnetization, curves, 311
  effects on length, 315
  of plunger, 202
Magnetized particles, 328
Magnetizing force, 6, 13, 176
  along axis of solenoid, 161

Magnetizing force, around a current, 285
effects of changes in, 296
of toroidal solenoid, 284
Magnetizing permanent magnets, 439
Magnetomotive force (m.m.f.), 14, 91, 282
gradient, 283
influencing, 91
opposing, 14, 17
reenforcing, 17
relation to ampere-turns, 14
Manganese, 310
steel, 44
Marcel Deprez electromagnet, 73
Mass, 8, 84
Massive armature, 84
Master switch, 120, 126
Materials, 2, 309, 433
Matter, 2, 3
and electricity, 3
and energy, 2
Maturing permanent magnets, 441
Maximum pull, 386
values, 25, 344
Maxwell, 13
Maxwell per gilbert, 17
Maxwell-turns, 14
Maxwell's hypothesis, 177
Mechanical auxiliaries, 399
disruption, 10
energy, 2, 8, 18
force, 18, 61
pressures, 10
Mechanism, mechanisms alternating-current, 389
indicator, 78
repulsion, 77
reversing, 72
rotating, 400
signal, 68
time-stamp, 158
Megadaraf, 11
Megavolt per coulomb, 11
Mercury motor, 91
Mesh system, 365
Metals, elementary, 3, 15
Meter, meters, frequency, 302
watt-hour, 401
Metric system, 8
Mho, 7
Microcoulomb, 11
Microfarad, 11
Microscopic examination, 436
Mill magnet, 446
Millihenry, 14
Miscellaneous electromagnetic devices, 147
Molybdenum magnet steel, 310
Momentary-duty electromagnets, 50
Momentum, electromagnetic, 15
of armature, 68
Motion, accumulated, 363
harmonic, 69
of wire in magnetic field, 91
Motor, motors, 122
induction, 122, 410
magnet, 403
mercury, 91
straight-line, 74, 410
synchronous, 109, 404
Motorstarter, motorstarters, 121, 124
portative magnet on, 43
Movable-core electromagnets, 56
Multiple-disk brake, 149
Mutual inductance, 25
induction, 25

N

Names of electromagnetic devices, 147
Natural frequency, 24, 332
Neck of the bottle, 300
Negative electric charge, 4
Net pull, 191
Neutral-polarity plane, 160, 188
of plunger, 187
Neutral relay, 141
Noise, 390
Non-magnetic filler, 101
stop, 89
Non-polar relay, 141
North-seeking pole, 89
Notching relay, 140

O

Oblique approach, 37, 66
Ohm, 7, 51
Oil-hardening, 438

Oil-immersed switch, 130
Oil pots, 142
One-coil horshoe electromagnet, 58
Open-frame electromagnet. 58
Open-phase relay, 140
Opposing m.m.fs., 14
Opposing voltages, 22
Oscillating current, 4
  circuit, 332, 363
Oscillation constant, 332, 363
Oscillations, damped, 251
Oscillograms, 408
Oscillograph, 96
Overload, protection from, 123

P

Packing plate, magnetic, 118
Parallel connections, 47, 248, 355
  circuits, 347
Particles, electrical, 4
  magnetized, 328
Parts of brake magnet, 391
  of contactor, 392
Pendulum, electric, 70
Permalloy, 313
Permanence, 436
Permanent magnet, magnets, 91, 433
  as armature, 94
  and electric currents, 91
  fields of, 317
  forged, 437
  forms, 446
  influencing electromagnet, 93
  leviated, 447
  magnetizing, 300
  normalizing, 437
  quality of, 243, 436
  quenching, 310, 437
Permeability, permeabilities, 283, 287
  and saturation, 289
  and susceptibility, 287
  curves, 290
  high, 311
  initial, 313
  of magnetic materials, 314
Permeance, 16, 282
  constant, 314
  per square centimeter, 167
Phantom sections, 166
Phase, 79, 365
  angle, 347
    method of finding, 358
  compensation, 359
  currents, 365
  difference, 79, 335
  -failure protection, 122
  relations, 335, 342
  relay (open), 140
  -reversal protection, 122
  splitting the 71, 363
  voltages, 365
Phase-rotation relay, 100, 139
Phonic wheel, 60
Pile-driver action, 76
Pivoted-magnet type, 95
Planimeter, 203
Plug, 222
Plunger, plungers, attraction of solenoid, 185
  coned, 64, 67, 227
  construction of, 393
  current strength for various movements, 431
  effects of movement, 378
  flat-faced, 190
  in motion, 430
  magnetization of, 203
  short, 184
  starting and stopping, 431
  stationary, 221
  tapered, 64, 227
  variable, 194, 216
  varying the cross-section, 216
  velocity of, 215
  working together, 222
Plunger electromagnets, 45, 62
  arc-lamp type, 389
  construction of, 232
  force characteristics, 63
  Hazeltine's coned plunger, 228
  magnetic fields of, 325
  strong sealing pull, 45
  two-coil, 57
  with laminated cores, 57
  with open-frame, 223
  with yoke, 57
Polar relay, 140

Polarity, 89
  characteristics, 79
  of electromagnet, 40
Polarity-directional relay, 139
Polarized electromagnets, 59, 92, 98
Polepieces, 35, 67
  hollow, 108
  spherical, 108
  tapered, 67
Poles, 89, 188, 327
  consequent, 319
  radial forces from, 328
Polyphase electromagnets, 80
  systems, 364
Portative electromagnet, 34, 41, 101
  with shell, 44
  with two branches, 43
Positive electric charge, 4
  electric currents, 89
Potential energy, 26
Potentiometer, 254
Pound per square inch, 172
Powdered iron, 314
Power, 27
  components of, 239
  electric, 27
  electrothermal (heating), 343
Power-directional relay, 139, 145
Power factor, 108, 348
Power hammers, 409
Power relay, 139
Precautions, 414
Primary circuit, 25
Primary current, 25
Pressure curve, 428
Pressure, electric, 6, 10
  mechanical, 10, 170
  net, 217
Pressures and forces, 29
Properties, magnetic, 1, 5
  insulating, 5
Protection, 121
  from hard material, 114
  from low voltage, 123
  from overload, 123
  from phase failure, 122
  from phase reversal, 122
  mechanical, 43
  of motors, 122
Protective apparatus, 119
  device, 303
  relays, 139
  switch, 53, 232
Proton, 3
Pull, air-gap, 62, 227
  changing the, 63
  initial, 47, 66
  maximum, 386
  maximum net, 192
  net, 191
  sealing, 47, 70
  solenoid, 61
  uniform, 67
  unlimited, 289
Pull characteristics of test magnet 371
Pulley, magnetic, 116
Pulls through short air gaps, 227
Pulsating forces, 32
Pushing electromagnets, 72
Push-pull electromagnets, 99

Q

Quadrature components, 348
Quadruple frequency, 55
Quality of permanent magnets, 243, 436
  standards of, 44
Quantity of electricity, 6, 11
Quarter-phase system, 365
Quenching, 310, 437
Quick-acting electromagnets, 81
Quick-acting relay, 140
Quick action through quick release, 87
Quick release, 85
Quick response, 85

R

Radial mechanical forces, 328
Radian, 333
Radius, average, 161
Railway car lighting, 132
Range, ranges, 20, 37
  of solenoid-and-plunger, 185
Ratings, duty, 50

Reactance, 350
and impedance, 355
and resistance, 355
condensive, 350
inductive, 350
per phase, 368
Reaction, resistance, 7
self-inductance, 14
Reactive component, 354
Reactor, reactors, 16, 29, 403
air-cored, 16
and condenser, 250
in parallel, 249
in series, 84
in shunt, 85
protective, 121
Real sections, 166
Recalescence, 437
Reclamation, 114
Reconversions of energy, 238
Recorder, time, 158
Reenforcing m.m.f., 17
voltage, 22
Regulators, 131, 141
Relay, relays, 139
alternating-current, 145, 393
electric protective, 139
low-frequency, 393
phase-rotation, 139
polarity-directional, 139
polarized, 393
split-phase, 394
telegraph, 141, 145, 399
telephone, 393
Release, 86
Reluctance, 282
Reluctivity, 283
and saturation, 291
Remanence, 309, 435
and coercive force, 444
Remanent magnetism, 88, 291, 434
Remote control, 1, 129
Repulsion, electrostatic, 4
electromagnet, 74, 97
electromagnetic, 32
Resistance, electric, 5
alternating-current, 346
and capacitance, 342
and condensance, 356
Resistance and impedance, 355
and reactance, 355
and self-inductance, 341
insulating, 10
insulation, 10
joint, 249
per phase, 366
temperature effects, 421
Resistivity, 438
Resistor, resistors, 29
and condenser, 253
carbon-pile, 134
in parallel, 248
in series, 85
in shunt, 85
starting, 126
Resonance, 363
Response, 86
Retarding torque, 152
Retentiveness, 21, 442
Reversal, reversals, of flux, 304
of motors, 123
Reversed current, 86
Reversible solenoid, 100
Reversing mechanisms, 72
Rheostat, 121
Ring, rings, electromagnet, 44
on cable, 306
on core, 77
shrink, 403
vortex, 281
Ringer, movement, 95
party-line, 98
Room temperature, 9
Root-mean-square, 24
Rotary electromagnet, 60
armature, 60, 94
Rotating magnetic field, 81
mechanisms, 400
Rotation, due to eddy currents, 400
due to repulsion, 77
of current around a magnet, 91
Rotor, 122
Runner, 410

S

Safety solenoid, 156
Saturation curve, normal, 329
steel and air, 329

Saturation and reluctivity, 291
and permeability, 289
effects in solenoids, 189
in pure iron, 192
Sawmill solenoids, 154
Sealing pull, 70
Screen, magnetic, 102, 304
Secondaries, differential, 392
Secondary circuit, 25
effect of short circuit, 84
Secondary current, currents, 25
effects of, 255, 378
shrink ring heated by, 403
Secondary voltage, 25
Section, sections, coil, 388
relay coil, 141
solenoid, 162, 166
solenoid divided into, 388
Self-demagnetizing effect, 442
Self-induced voltage, 14, 209
and weber-turns, 345
Self-inductance, 14
air-cored solenoids, 173, 384
average, 211
joint, 249
length-rate of change, 208
reaction, 14
solenoid-and-plunger, 207
Self-induction, 14
e.m.f. of, 348
Self-restoring drop, 68
Semi-toroidal electromagnet, 46
Separable steel ring, 306
Separating the fluxes, 382
Separator, magnetic, 112
Series circuits, 347
coil, 38
connections, 240
electric circuit, 7
lock-out switch, 128, 303
magnetic circuit, 297, 305
motor, 122
-parallel connections, 252
tractive electromagnets, 51
turns, 9
-wound solenoids, 235
Shading coils, 71, 80, 395
Sheets, electrical, 49, 311
Shoe brake, 149
Short air gaps, 227
Short circuit, 10
Short-circuited turns, 76, 84, 256, 393
heating effects, 404
Short-pull brake solenoid, 236
Shunt coils, 38
connections, 47
-field protective relay, 144
lock-out switch, 303
magnetic, 66, 301, 304
magnetic circuit, 297
motor, 122
switch, 127
tractive magnet, 52
-wound solenoid, 231
Signal mechanism, 128
relay, 144
Sine of phase angle, 347
Single-layer solenoid, 161
Single-phase brake magnets, 389
electromagnets, 79, 389
operation of two-phase, 395
reduction to equivalent, 367
relations, 335
Sinusoidal magnetic flux, 296
Skin effect, 346
Skull-cracker ball, 107
Slidable magnetic shunt, 67
Slotted cores and tubes, 49
Slow-acting electromagnet, 81
relay, 141
Slow-release, 87
electromagnets, 85
relay, 141
Slow-response, 84
Solenoid, solenoids, 1, 12, 45, 160
air-cored, 16, 384
alternating-current, 369
and plunger, 45, 57
attractions between solenoids and plungers, 185, 188
balanced, 221
brake, 236
automatic, 150
characteristics of, 61, 202
constant-current, 176
construction, 232
control for gags, 157

Solenoid, definition, 2
demagnetizing effect, 181
differential, 180, 199
dimensions, 232
divided in sections, 388
double-acting, 57
double-plunger, 221, 236
effect of length on pull, 190
of saturation, 189
finite, 164
force characteristics, 61
force-distance curves, 236
general statements 219
in sawmills, 154
iron-clad, 45, 61, 221
lifts at full load, 236
magnetic-cushion type, 225
magnetic field of, 186, 223
magnetizing force along axis, 161
m.m.f. factor, 167
net attraction, 185
pull, 211, 216
pressures and forces between, 170
range, 185
relative lengths, 183
reversible, 100
rotary, 59
rules for, 217
safety, 156
sections, 162
self-inductance of, 173, 207, 210, 384
series-wound, 235
short-pull, 236
shunt-wound, 234
single-layer, 161
six-section, 164, 168
solid core, 381
standard, 232
steel-clad, 45
stored magnetic energy in, 206
temperature rise of, 232
ten-section, 388
test characteristics, 190, 381
trimmer-table, 155
twelve-section, 388
unlimited pull of, 289
valve-operating, 155
variable plungers, 194
Solenoids with plungers, 176
work characteristics, 218
done by, 203
Solenoid-operated brakes, 136
Solid cores, 49
Sounder, alternating-current, 69
Space factor, 423
Spacer, magnetic, 304
Sparking, 88
Spectra, magnetic, 316
Spiral-frame electromagnet, 85
Split-phase magnets, conditions, 394
construction, 396
Splitting the phase, 71, 393
Spring-closed magnetic switch, 129
Spring, retractile, 85
on alternating-current magnet, 389
Stamp, time, 158
Standard brake magnets, 390
Standard equipment, 389
Standards of A.I.E.E., 120, 139
Standstill test of hammer, 413
Star system, 366
Starting electric motors, 122
resistors, 126
Stationary-core electromagnets, 46, 56
Stator, 122
Steel, steels, 1, 310, 444
Sticking of armatures, 88
Stop, stops, 222
coned, 229
floating, 66
non-magnetic, 89
position, 371, 377
Storage of energy, 271
of magnetic energy, 418
Stored electric energy, 11
magnetic energy, 14, 169, 379
Straight-line motors, 74, 410
Striker magnet, 93
Stroke, finding, 418
Structural flux density, 192
Surface, cooling, 419, 422
Surges, current, 122
Surgical magnet, 157
Susceptance, 354, 364
Susceptibility, magnetic, 287

Swedish charcoal iron, 310
Switch, switches, 120
alternating-current, 130
automatic, 138
condenser across, 253
double-acting, 129, 130
fuse-plug socket, 138
high-voltage, 130
lock-out, 128, 302
magnetic, 127
series lock-out, 303
shunt lock-out, 302
oil-immersed, 130
protecting, 53, 232
remote-control, 129
reverse, 130
shunt, 128
Switching apparatus, 119
cars, 117
Symbols, general, 348
graphic electric, 239
Synchronous motor, 109, 404
condenser, 109
Syntron electric hammer, 407

T

Tangent of phase angle, 356
Tapered plungers, 64
Telegraph relays, 141, 145, 399
Telephone receivers, 98, 446
relays, 59, 140, 393
transmission, 314
Temper, effects of, 438
Temperature, 6, 436
absolute, 9
effects on magnetic properties, 315
on resistance, 421
on size of coil, 9
relay, 139
rise, 232
room, 9
Terminals, 10
Test characteristics, alternating-current, 371
and direct-current, 376
Test characteristics, analysis of, 376
comparison of, 197
coned plunger, 230
Test characteristics, differential solenoid, 199
electric hammer, 413
flat-faced plunger, 229
horseshoe electromagnet, 227
iron-clad solenoid, 225
lifting magnets, 226
magnetic-cushion, 225
solenoids, 190, 236
ten-inch solenoid, 190
various plungers, 194, 216
with solid core, 381
Test electromagnets, alternating-current, 376
and direct-current, 376
Thermal energy, 5
conductors, 6
Thomson's first arc-light generator 159
Three-phase electromagnets, 80, 398
systems, 375
type, 59
Time-acceleration control, 126
Time constant, 68, 83, 173, 261
devices, 158
Time-limit action, 66, 76
Time-rates of change, 209, 272, 294
Toggle, 37, 71
equalizer, 63
Toroidal coil, 281
electromagnet, 44
Torque, retarding, 152
Track relay, 145
Tractive electromagnets, 34, 45, 47
Tractive-portative electromagnets, 38
Tramp iron, 117
Transferences of energy, 238
Transformer action, 333
Transformer, relay, 145
Transformers, 404
Trigonometric relations, 362
Trimmer solenoids, 155
Trip coil, 390
Trip-free relay, 140
Tripping coil, 136
Trombetta's first electric hammer, 413

Tube over core, 84
Tubes, slotted, 49
Tubular electromagnets, 57
Tuning fork, electric, 69
Turn, turns, average length of, 10
  groups of, 162
  series, 9
  short-circuited, 256, 393, 404
Two-coil electromagnet, 46, 57, 394
  reversing mechanisms, 72
Two-phase electromagnets, 80, 398
  system, 365
  type, 59
Two-plunger solenoids, 222
Types of electromagnets, 34
  selection of, 417

U

Under-voltage protection, 121
Uniform pull, 67
Uniformity of steel, 443
Units, Metric and English, 8
Universal electromagnets, 424
Unlimited pull of solenoid, 289
Uses of electromagnets, 147

V

Valve-operating magnet, 155
Valves, freezing of, 156
  magnetic, 152
  solenoid-operated, 152
Vandepoele's electric hammer, 411
Variable resistor, 134
Velocity, angular, 350
  of armature, 429
  of plunger, 215
Vending-machine magnet, 446
Vertical lift, 428
Vibration, 37, 77
  mechanisms, 69
Volt, 6
  per ampere, 7
  per mil, 11
  per millimeter, 11
Volta-mperes, 27, 339, 377
Volt-coulomb, 7
Volt-second, 13
Voltage, voltages, active, 7, 348
  alternating, 21, 332
  changing, 422
  components, 239, 341
  continuous, 4
  counter, 22
  dielectric, 11
  direct, 4, 21
  drop, 7
  electromagnets, 52
  gradient, 11
  impressed, 7, 348
  induced, 14, 91, 256, 333
  opposing, 22
  phase, 366
  quadrature, 348
  ratio to angular velocity, 383
  reenforcing, 22
  regulator, 133, 144
  relay, 139, 144
  resonance, 363
  secondary, 25
  self-induced, 14, 209, 345
  -time characteristics, 259
    relations, 296
  to neutral, 365
  wave, 80
    distortion, 296
Volume resistivity, 438
Vortex ring, 281

W

Warren clock, 403
Watt, 27
Watthour meter, 401
Wattmeter, 97
Watt-second, 7
Wave, waves, current, 80, 338
  distortion of, 296
  three-phase, 335
  two-phase, 334
  voltage, 80, 338
Weber, 13
  per gilbert, 17
Weber-turns, 14, 345, 385

Weight, 8
Weiss electromagnet, 159
Wheel, phonic, 60
Winding, coil in sections, 388
Wire, wires, hard iron, 314
  insulation on, 2, 424
  length per turn, 10, 419
Work, 2, 101
Wrought iron, 310

Y

Y connection, 367
  for polyphase magnets, 398
Y system, 366
Yoke, 43

Z

Z-armature, 60

G.E. Supply Catalog
Page 404

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